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Using Resonance Cone Refraction for Compact RF Metamaterial Devices

K.G. Balmain* A.A.E. Lüttgen† P.C. Kremer†

Abstract – **The metamaterial under consideration is a planar wire-grid network loaded with orthogonal capacitors and inductors over a ground plane. When excited by a single-frequency source, conical high-field regions called “resonance cones” are observed. When two or more such metamaterials interface, resonance cones exhibit refraction, which in the case of multiple metamaterial areas can result in containment of the cones by multiple refractions. In this paper, the principles of resonance cone refraction and containment are investigated by simulation in application to a compact spectrum analyzer and a low-profile antenna.**

1 INTRODUCTION

The present work is rooted in two fields of research, namely metamaterials and antennas in anisotropic plasmas.

The current research interest in metamaterials is based on the pioneering paper by Veselago [1] on the theory of materials having at the same time both negative permittivity and negative permeability, which he called “left-handed”. More recently Pendry [2] proposed using these materials for a “perfect” flat lens. Such a lens requires negative refraction at a planar interface between left-handed and right-handed materials or through a planar layer of left-handed material. The first clear measurement of negative refraction at microwave frequencies was carried out by Smith et al. [3] using a split-ring resonator/wire medium.

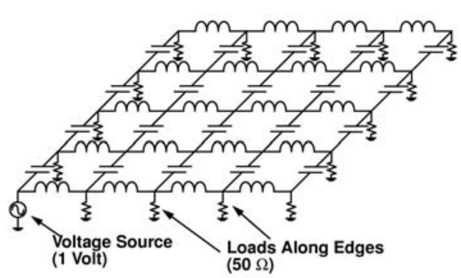

Figure 1: Uniform anisotropic L-C grid over ground with resistive edge loading.

In lossless anisotropic plasmas, resonance cones are well known as the conical high field regions extending from antennas when two of the three diagonal elements of the permittivity tensor of the plasma have opposite signs. Fisher and Gould [4] appear to be the first ones to call them “resonance cones” and to measure the high field regions. It was shown by Balmain [5] that quasistatic theory for small antennas predicts significant power flow along the resonance cones. In a later paper, Balmain and Oksiutik [6] noted that the positive and negative permittivities of the plasma could be interpreted in terms of capacitors and inductors, respectively.

These circuit elements were regarded as physical reality in the form of a square-cell, planar wire grid loaded with orthogonal inductors and capacitors over a ground plane as in Fig. 1, by Balmain et al. [7]. This amounts to a two-dimensional metamaterial.

2 RESONANCE CONES IN L-C GRIDS

Resonance cones, which are predicted by circuit theory as the directions of the point-to-point zeros in reactance across the surface of an L-C grid, were detected and their refraction and focusing were investigated in [7]. The structure depicted in Figure 1 is a uniform metamaterial, i.e. all inductors and capacitors are oriented in the same direction throughout the grid. In non-uniform grids, which consist of two contiguous areas in which the orientation of the inductors and capacitors is opposite, negative refraction of resonance cones occurs as reported in [7]. Along the edges in Figure 1 a 50-Ω resistor was connected between each node and ground. From each non-edge node a 100-kΩ resistor was connected to ground to enable computational monitoring of the grid-to-ground voltage throughout the structure. All simulations in this paper employed a full-electromagnetic, thin-wire moment method program [8] which permits insertion of lumped elements in finite-length wire sections.

2.1 Uniform Metamaterials

Simulation results for a uniform metamaterial with 2.5mm by 2.5mm cells are shown in Figure 3 of [7]. At a certain frequency which is close to the resonance frequency for each cell, the resonance cone extends along the diagonal. At lower frequencies it sweeps toward the direction of the

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*The Edward S. Rogers Sr. Dept. of Elec. & Comp. Engineering, University of Toronto, 10 King’s College Rd, Toronto, M5S 3G4, Canada, e-mail: balmain@waves.utoronto.ca, tel.: +1 416 9783127, fax: +1 416 9713020.
† The Edward S. Rogers Sr. Dept. of Elec. & Comp. Engineering, University of Toronto, 10 King’s College Rd, Toronto, M5S 3G4, Canada.

inductors while at higher frequencies it sweeps toward the direction of the capacitors. If one monitors the node voltages along the edges of a metamaterial as in Figure 1 while sweeping from low frequencies to high frequencies, the voltage maximum would move up along the right edge toward the upper right corner, then along the upper edge toward the upper left corner. A setup like this can be used as a compact spectrum analyzer.

2.2 Two Interfacing Planar Metamaterials

For the study of refraction, the two-medium setup with the same parameters from [7] where in the lower part of this setup, which contains the feedpoint, the element orientation was the same as in Figure 1, while their orientation was interchanged in the upper part. The transition region between the two media consisted of a row of cells such that each half of the cell had the characteristics of the adjacent medium. An experimental setup with these properties has been built (Figure 4 of [7]) and simulation and experimental results are shown in Figure 5 of [7].

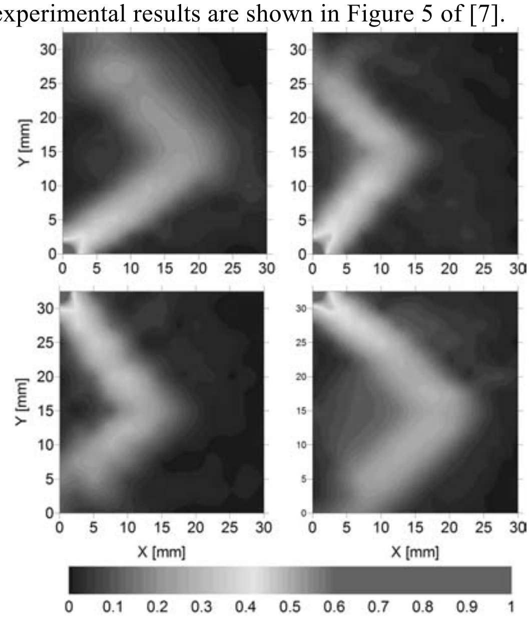

Figure 2: Refracted resonance cones for lower-left corner feed (top) and upper-left corner feed (bottom) at 1.2 GHz (left) and 1.6 GHz (right).

If the source frequency is very low, the resonance cone in such a setup will sweep too close to the lower edge and not reach the transition region and therefore it will not be refracted. At frequencies which are still low but for which refraction occurs, the refracted cone in the upper medium will eventually reach the upper edge. Raising the frequency further will result in cones that reach the part of the left edge above the boundary. Two such examples are displayed in the upper part of Figure 2.

Sweeping the frequency up will therefore lead to voltage maxima along the edges moving from the upper right corner to the left toward the upper left edge and eventually downward along the left edge to the transition region.

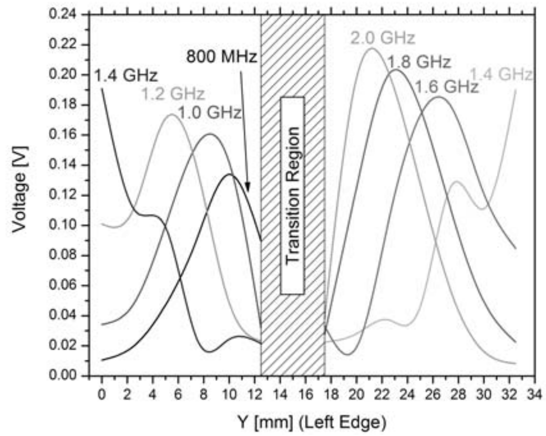

Figure 3: Left-edge node voltage for both feedpoints.

The feedpoint can be moved to the upper left corner. At low frequencies, since cones tend to sweep toward the inductors, refraction will occur and the refracted cone will reach the lower left edge. At higher frequencies the cone will move toward the capacitors and the refracted cone will instead reach the lower edge of the setup. Two examples are shown in the lower part of Figure 2. At even higher frequencies the cone will not be refracted.

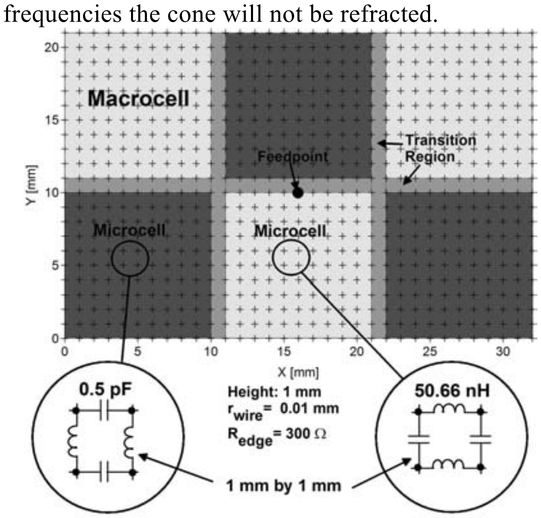

Figure 4: Metamaterial configuration designed for resonance cone containment at 1 GHz.

The setup described above can be used as a spectrum analyzer as well. If the feedpoint is kept at the lower left corner, the bandwidth has a lower limit since at very low frequencies refraction does not occur. Similarly, the bandwidth has a high-frequency limit when the feedpoint is at the upper left corner. Using the upper-left corner feedpoint for low frequencies and switching it to the lower-left corner for high frequencies, i.e. frequencies where the refracted cone would reach the lower edge, will result in cones that reach the left edge in all cases, giving rise to voltage maxima on the lower and upper half of this edge for low and high frequencies, respectively. Results for the edge voltage are shown in Figure 3 for both feedpoints.

Since in this case no refracted cones will reach the upper and lower edges, high fields will not occur in the area close to the right edge of the setup (see Figure 2). Therefore this method can be used to build a spectrum analyzer which is even more compact than the spectrum analyzer described in section 2.1, since this device needs to have only roughly two thirds of its width.

3 CONTAINED RESONANCE CONES

Figure 2 shows that any resonance cone in a corner-fed two-media metamaterial will eventually reach the boundary. It is however possible to construct an arrangement of multiple areas such that a resonance cone extending from a carefully chosen feedpoint will undergo multiple refractions and thus be "contained" in the metamaterial.

3.1 Resonance Cone Containment Example

A setup designed to keep resonance cones "contained" in the metamaterial is shown in Figure 4. It consists of six "macrocells", i.e. squares of metamaterial each with a particular "microcell" configuration of inductors and capacitors. The feedpoint is located in the middle of the array.

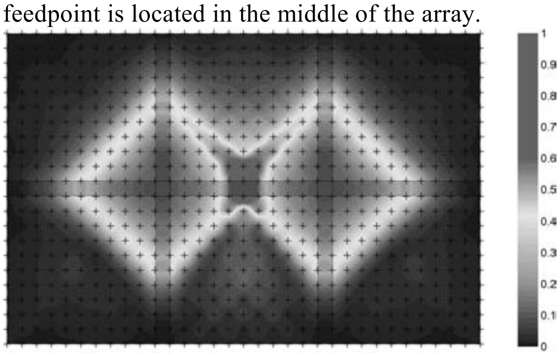

Figure 5: Two-square resonance cone pattern at 1 GHz for the configuration of Figure 4.

The setup in Figure 4 is designed for a frequency of 1GHz as the resonance frequency for each microcell, i.e. four resonance cones are excited, relative to the right-hand part of the middle transition regions they extend at angles of 45º, 135º, -135º, and -45º. A resonance cone excited by the source at the given feedpoint at an angle of 45º from the middle transition region, reaches the middle of the right transition region where it is refracted into the upper right macrocell. The refracted cone will reach the middle transition region, refract into the lower right square, and reach the right transition region again, where it is refracted toward the feedpoint. The other three cones undergo similar refractions, resulting in a closed two-square pattern of resonance cones as shown in Figure 5.

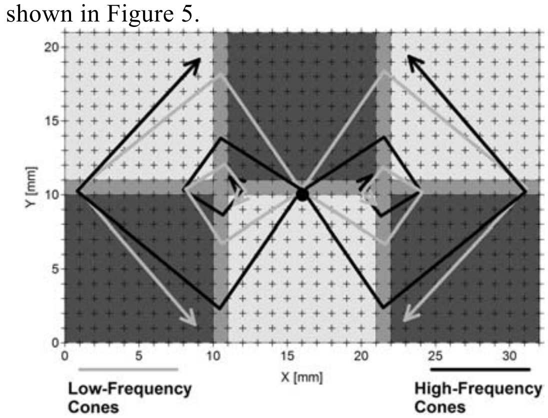

Figure 6: Schematic view of expected cone directions.

At frequencies different from the "diagonal" frequency the resonance cones result in inward and outward spirals rather than squares, i.e. higher frequencies result in inward spirals in the upper half of the setup and outward spirals in the lower half. This pattern is reversed for low-frequency cones. This is indicated schematically in Figure 6.

3.2 Low-Profile Antennas Using Cone Refraction

Another significant result arises as a result of the generation of resonance cone radiation by the expedient of dropping inductors to ground from an anisotropic L-C grid [9]. In general, resonance cone radiation is degraded by interaction with the edges of the planar grid. This problem can be solved by using multiple refractions, as described in the previous section to divert the cones from the edges. This gives radiation patterns that do not suffer from unwanted edge interaction since, even as the frequency departs from the design frequency, the expanding cones become weak due to Ohmic losses or radiation before they reach the boundaries.

Figure 7 shows the node voltages for a six-square setup where each macrocell consists of 12 by 12 microcells carrying inductors of 50.66 nH and capacitors of 0.5 pF, and an inductance of 132 nH is connected to ground from each node. The size of the microcells is 1 cm by 1 cm by 1cm high with a wire radius of 0.1 mm. Since the wires contribute significantly to the inductance the resonance cones no longer form squares, but expanding and contracting spirals can be clearly identified in Figure 7. The Poynting vector in the grid plane is overlaid in the contour plot, indicating the power flow along the resonance cones. The inward spiraling cones form electromagnetic vortices located near the transition-region intersections.

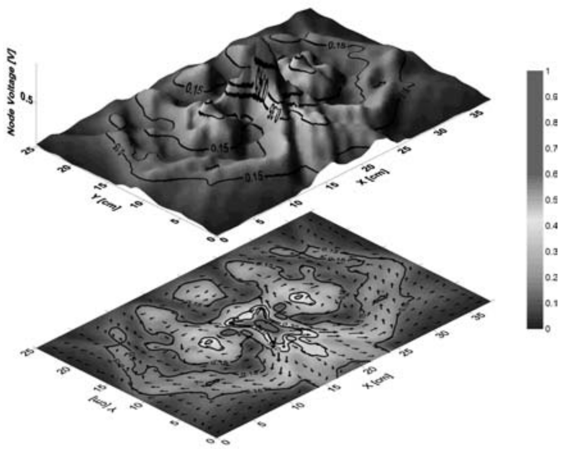

Figure 7: Node voltages and Poynting vectors, 1 GHz.

The radiation patterns as functions of ϕ for all angles θ are shown in Figure 8. All main lobes are in the ϕ=90º direction and the f/b ratio is 14.6 dB. The radiation maximum including all θ angles is close to ground and in the ϕ=90º direction.

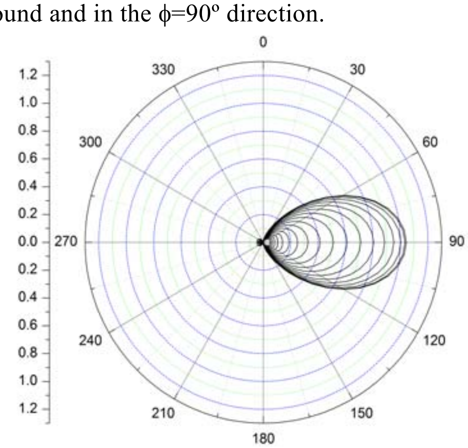

Figure 8: Power gain for the low-profile antenna.

4 CONCLUSIONS

It has been shown how the resonance cones on an L-C grid using refraction can function as a compact spectrum analyzer by switching the feedpoint for different frequency ranges. Another possible application of resonance cone refraction is their use in a low-profile antenna. Since this phenomenon is inherently not strongly resonant this could lead to broader operating bandwidths than for example patch antennas over ground.

Acknowledgments

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