Resonance Cone Formation, Reflection, Refraction, and Focusing in a Planar Anisotropic Metamaterial

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***Abstract*—The material studied is a planar wire-grid network loaded with orthogonal capacitors and inductors. Over a ground plane and excited by a localized single-frequency source, this metamaterial exhibits conical high-field regions called "resonance cones." The cones exhibit specular reflection from boundaries and, with two such materials interfaced, the cones exhibit negative refraction as well as subwavelength focusing into a spot smaller than 1/20th of a free-space wavelength. Moment-method simulations and experiments show good agreement.**

***Index Terms*—Anisotropic metamaterial, backward refraction, negative refractive index, planar metamaterial, resonance cone, subwavelength focusing.**

I. Introduction

THE present work stems from two fields of research, namely antennas in anisotropic plasmas and microstructured "metamaterials." In the study of antennas in anisotropic (magnetized) plasmas, resonance cones are well known as conical high-field regions that extend outward from antennas under (lossless) conditions such that two of the three diagonal elements of the plasma permittivity matrix have opposite signs. Key early analyses were those of Bunkin [1], Kogelnik [2], and Kuehl [3]. Fisher and Gould [4] were first to measure the high-field regions and also appear to have been the first to call them "resonance cones." Turning briefly to mathematics, we know that the vacuum wave equation in space-time is a hyperbolic partial differential equation whose characteristic surface is known as the "light cone." On the other hand, for anisotropic plasmas it was pointed out by Balmain [5] that the time-reduced quasistatic partial differential equation for the scalar potential can also be hyperbolic, but in the space coordinates alone, showing that the associated conical characteristic surfaces are just the locations of the resonance cones already mentioned. In the same paper, Balmain associated an outward power flux with the region between the particular near-field resonance cones that extend from the antenna extremities, as depicted in Fig. 1. Later, Balmain and Oksiutik [6] noted that the plasma negative and positive permittivities could be interpreted, respectively, in terms of inductors and capacitors.

Manuscript received September 11, 2002; revised November 1, 2002. This work was supported by the Natural Sciences and Engineering Research Council of Canada. This work is based on a paper with the same title and authors which was presented June 17, 2002, at the 2002 IEEE AP-S International Symposium and USNC/URSI National Radio Science Meeting, San Antonio, TX (abstract in *URSI Digest*, p. 45).

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Digital Object Identifier 10.1109/LAWP.2002.807565

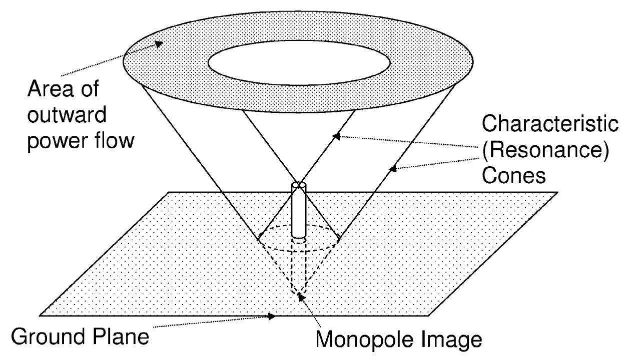

Fig. 1. Monopole antenna in highly anisotropic (resonant) plasma, showing characteristic cones (resonance cones) extending from antenna ends.

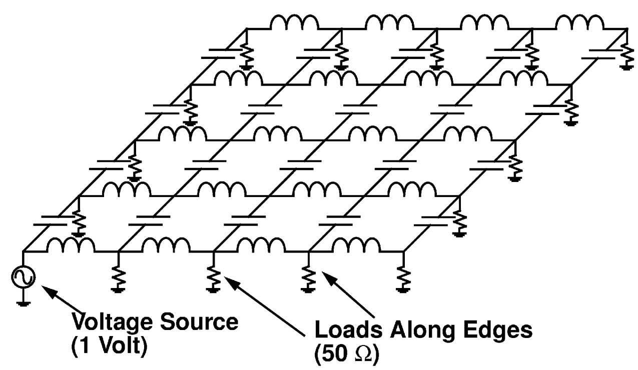

Fig. 2. Uniform anisotropic planar L-C grid over ground, with corner feed and resistive edge loading.

Regarding such circuit elements as physical reality, we can view a physical inductor-capacitor (L-C) network as a material, a "metamaterial," that could exhibit resonance cone phenomena. If the L-C network is a planar, square-celled wire grid series-loaded with orthogonal inductors and capacitors and located over a ground plane, as shown in Fig. 2, then this grid emerges as a finite-size, two-dimensional resonance-cone metamaterial that is practical to construct and feasible to test. Still looking at Fig. 2, and now thinking in terms of circuit theory, one can see the ultimate simplification, that the resonance cone directions are just the directions of the zeros in reactance measured across the grid surface.

The current research interest in metamaterials sprang from the pioneering theory of Veselago [7] on isotropic "left-handed"

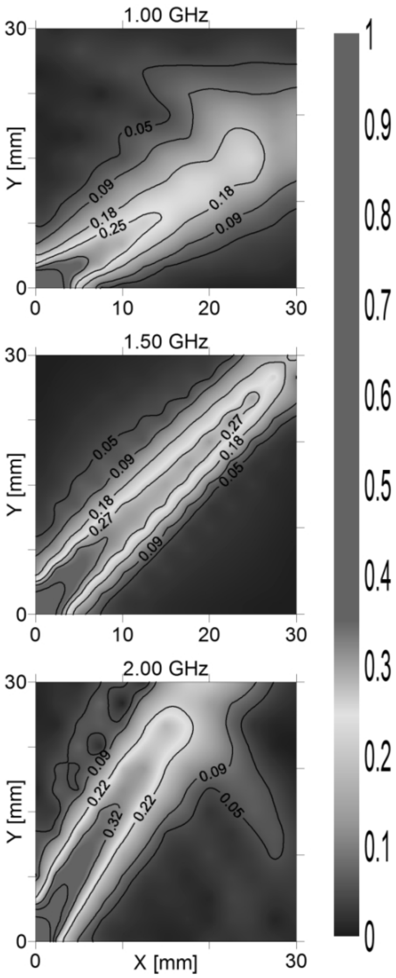

Fig. 3. Uniform-grid moment-method simulation displaying resonance cones at three frequencies. Contours show grid-to-ground voltages.

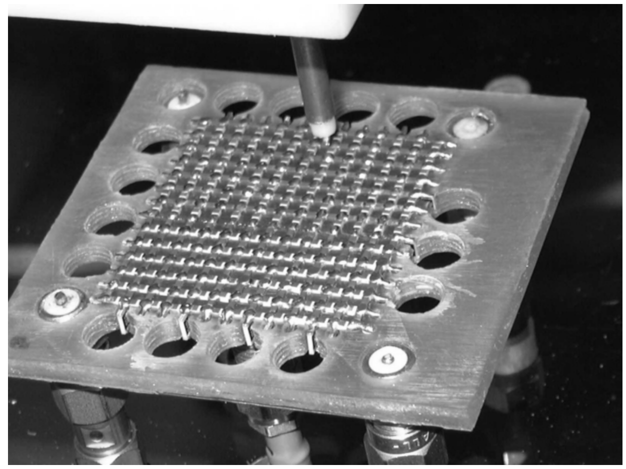

Fig. 4. Experimental setup showing a scanning open-ended probe over a nonuniform L-C grid with the transposed-element interface near the middle of the grid.

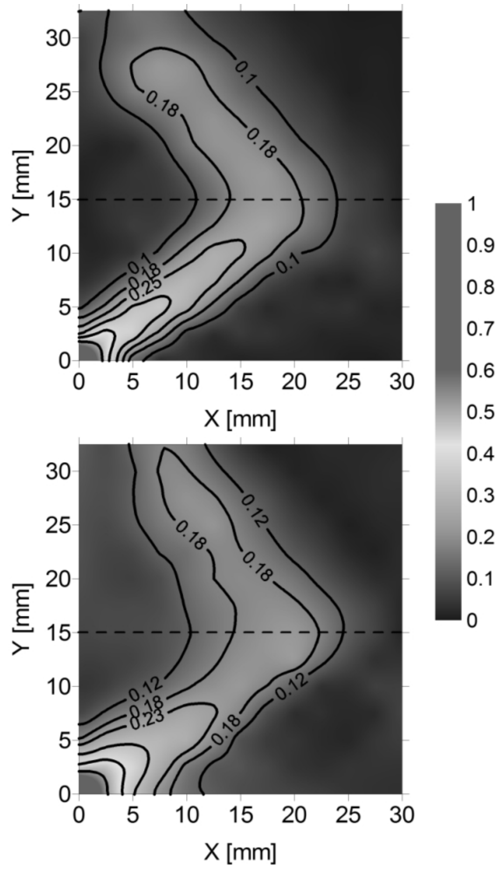

Fig. 5. Resonance-cone refraction at 1.2 GHz—simulation (top) showing contours of grid-to-ground voltage magnitude, and experiment (bottom) showing contours of normalized $|S21|$. Refraction occurs at the $y = 15$ mm interface between two grids with transposed (interchanged) L and C elements.

materials (having at the same time both negative permittivity and negative permeability), which led to the recent work of Pendry directed toward using these materials to make a "perfect" lens [8]. Such a lens requires negative refraction which takes place at a planar interface between left-handed and right-handed media or through a planar layer of left-handed material. The first clear measurement of such negative refraction was carried out at microwave frequencies by the team led by Smith at University of California, San Diego, using a volume distribution of cellular composite metamaterial with each cell made up of a resonant wire plus a resonant loop [9].

II. Results for Resonance-Cone Media

For the present work with anisotropic materials, the simulations employed a full-electromagnetic, thin-wire moment-method program [10], which permits the insertion of lumped circuit elements in finite-length wire segments. The basic network simulated was as shown in Fig. 2, consisting of 12 cells $\times$ 12 cells, each cell 2.5 mm square, for a total grid size of 30 mm $\times$ 30 mm, with the grid 2.5 mm above the ground plane. The capacitance was 2 pF/segment and the inductance 5.6 nH/segment. Along the edges a 50-Ω resistor was connected between each node and ground. Each capacitor and each inductor had a 1.6-Ω damping resistor in series. Not shown in Fig. 2 is a 100-kΩ resistor from each nonedge grid intersection to ground to enable deduction of the grid-to-ground voltage from the computed resistor current for these nodes. Contour plots of grid-to-ground voltage magnitude for corner feed are shown in Fig. 3, which displays, in simulation, the formation of resonance cones and the way the cone orientation

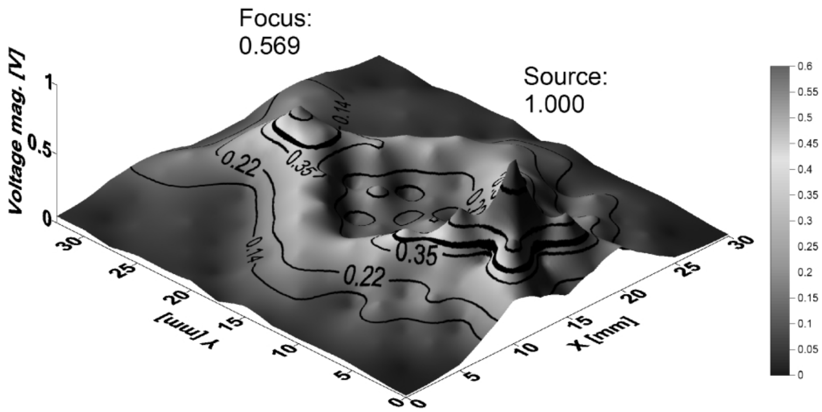

Fig. 6. Focusing simulation at 1.5 GHz. Heavy contour is at 0.707 of focal maximum of 0.569 V.

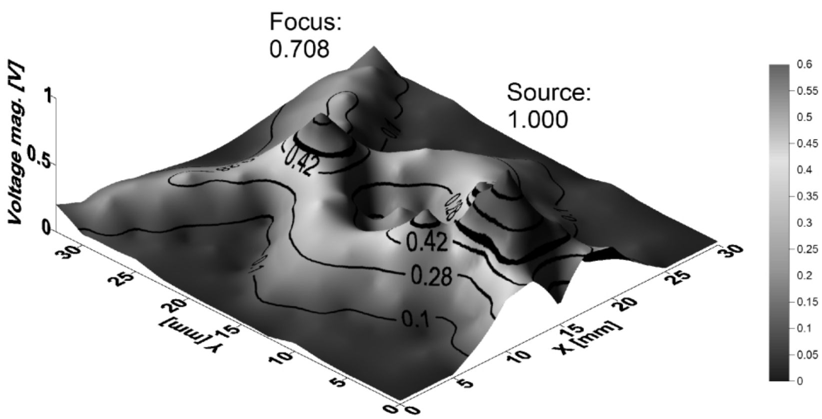

Fig. 7. Focusing simulation results at 1.7 GHz. Heavy contour is at 0.707 of focal maximum of 0.708 V.

scans with frequency. At 1.0 and 2.0 GHz, one can see weak specular reflection from the grid edges.

To study refraction, a different planar medium was created by interchanging (transposing) the inductors and capacitors in the grid, thus enabling a two-medium configuration with a linear interface between the original medium and the transpose medium. The interface is along a line through the center of a row of cells such that each half of the cell has the characteristics of the adjacent medium while the total capacitance and inductance along the cell's boundary remain constant, i.e., the segments perpendicular to the interface carry both a capacitance and an inductance. The physical realization of this setup is shown in Fig. 4 in which the grid elements are chip-type inductors and capacitors that have been soldered together. The vertical *E*-field is picked up by an open-ended coaxial probe positioned sequentially just above each conductor intersection, and the probe signal is fed to a network analyzer with output S21 which, when plotted on a linear scale, is approximately proportional to the grid-to-ground voltage. The magnitude $|S21|$ is then taken and normalized with respect to its value at the feed point, for comparison with the simulation data where the magnitude of the grid-to-ground voltage is 1 V at the feed point. The two-medium result for corner feed is shown in Fig. 5, which displays clearly the negative refraction of the resonance cone as it traverses the transposition interface. Notice that specular reflection from the interface is entirely negligible and there is no indication of transmission in the same direction as the incident cone.

To study focusing, the feed point in the setup of Fig. 4 was moved from the corner to the seventh segment from left in the third row from bottom of grid-to-ground segments, i.e., it was moved from coordinates ($x = 0$, $y = 0$) to ($x = 15$ mm, $y = 5$ mm). This point is located close to the middle of one of the two planar metamaterials. The simulation results are shown at two frequencies in Figs. 6 and 7, and the corresponding measurements are in Figs. 8 and 9. In all cases, it is easy to see the cones emanating from the feed point (the "source"), the backward refraction at the $y = 15$ mm interface, and the cones merging at the "focus." The focal region is taken to be bounded by the contour that is at 0.707 of the focal region maximum, so this contour may be termed the "half-power" contour. It is worth noting that at both frequencies the experimental focal region boundary lies within a square measuring 1/25th of a free-space wavelength on a side, so the phenomenon properly may be termed "subwavelength focusing." Further, it happens that the measured focal

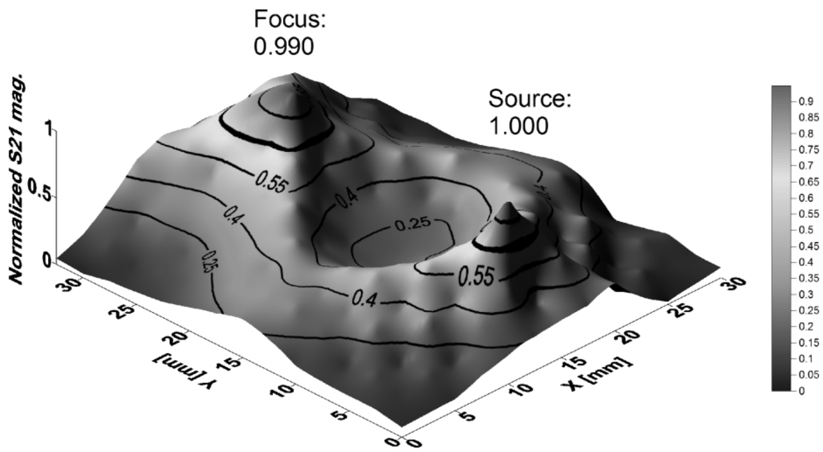

Fig. 8. Focusing measurements at 1.5 GHz. Heavy contour is at 0.707 of focal maximum of 0.990 (units of normalized $|S21|$).

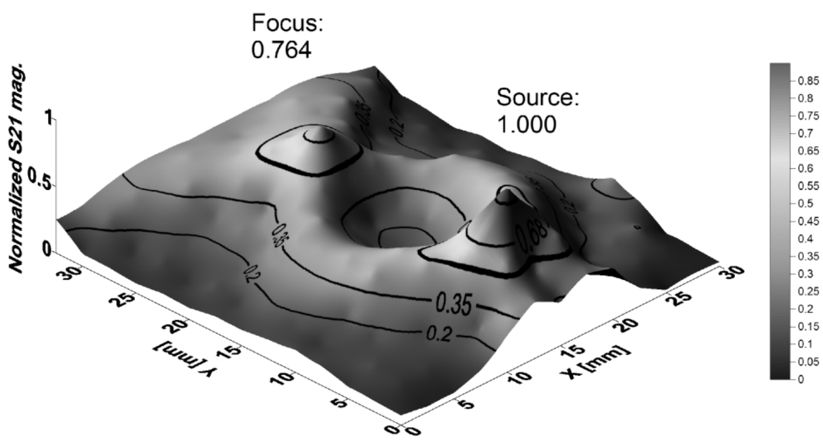

Fig. 9. Focusing measurements at 1.7 GHz. Heavy contour is at 0.707 of focal maximum of 0.764 (units of normalized $|S21|$).

peak signal of Fig. 8 is within 1% of the source peak, so that one can compare directly the half-power contours of the two peaks to get a self-consistent notion of the focal spreading that occurs, spreading no doubt largely due to the use of "off-the-shelf" components with a nominal Q of 27 and a $\pm 10\%$ distribution of L and C values.

III. Conclusion

Although the simulations and experiments described here employ lumped L-C loads, corresponding results have been obtained with equivalent distributed loads. Suggested applications include passive spectrum analyzers and antenna multiplexers.

Acknowledgment

The authors would like to thank Prof. G.V. Eleftheriades for stimulating discussions.

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