Coupled-line metamaterial coupler having co-directional phase but contra-directional power flow

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A coupled-line coupler comprising a microstrip line, edge-coupled to a negative-refractive-index line, exhibits co-directional phase but contra-directional Poynting vectors on the lines, leading to backward coupling. A key feature of this coupler is that it can support complex coupled modes. The resulting exponential field decay enables enhanced coupling with moderate line lengths and spacing. An example 3 dB device has been implemented and tested at 3 GHz.

Introduction: Theoretical studies of left-handed or negative-refractive-index (NRI) media date back to work done by Veselago in the 1960s [1]. Such NRI media were implemented using periodically loaded transmission lines and enabled the demonstration of focusing from a regular to an NRI medium by Iyer and Eleftheriades [2]. A similar approach was adopted by Caloz and Itoh and led to interesting devices including a backward-coupled-line coupler implemented using two identical NRI lines [3]. A different metamaterial backward-coupled-line coupler (MS/NRI) using one regular microstrip (MS) line, edge coupled to an NRI line, was introduced in [4] and demonstrated superior performance when compared to its regular (MS/MS) counterpart of equal length, line spacing and propagation constant. In this Letter we present an extended theory compared to [4], as well as simulation and experimental results for a 3 dB device. Specifically, it is demonstrated that arbitrary backward coupling is a direct consequence of the formation of a stopband for the coupled-mode system at the frequency where the ω–β dispersion curves of the isolated lines intersect.

Theory: The geometry of the MS/NRI coupler is shown in Fig. 1. Using the coupled mode theory in [5], it is possible to show that the propagation constants of the coupled modes (γ) excited in the two coupled transmission lines are given by:

$$\gamma^2 = \frac{1}{2}(a_1 + a_2) \pm \frac{1}{2}\sqrt{(a_1 - a_2)^2 + 4b_1 b_2}$$
$$a_1 = Y_1 Z_1 + Y_m Z_m,\ a_2 = Y_2 Z_2 + Y_m Z_m,$$
$$b_1 = Y_m Z_1 + Y_2 Z_m,\ b_2 = Y_m Z_2 + Y_1 Z_m \qquad (1)$$

The quantities Z_1 and Y_1 correspond to per unit length impedances and admittances of line 1, respectively, and those with subscripts 2, refer to line 2. Coupling between the two lines is taken into account by Z_m and Y_m which represent the per unit length inductive and capacitive coupling.

The propagation constant of isolated line 1 is given by $\sqrt{(-Y_1 Z_1)}$ and that of line 2 by $\sqrt{(-Y_2 Z_2)}$ where [2]:

$$Z_1 = j\omega L_0, \quad Z_2 = j\omega L_0\left(1 - \frac{1}{\omega^2 d L_0 C}\right),$$
$$Y_1 = j\omega C_0, \quad Y_2 = j\omega C_0\left(1 - \frac{1}{\omega^2 d C_0 L}\right) \qquad (2)$$

L_0 and C_0 are the per unit length inductances and capacitances, respectively, of the transmission line segments in the coupler, whereas L and C are the loading shunt inductance and series capacitance of the NRI unit cell, and d is being the cell length (see Fig. 1).

When the ω–β dispersion curves of the isolated lines cross (i.e. when they have equal propagation constants: $Y_1 Z_1 = Y_2 Z_2$), it can be seen from (1) that $Z_2 b_1 - Z_1 b_2 = 0$. Hence at this frequency, which will be denoted by $\omega_{\mathrm{equal}\_\beta}$, the ratio b_1/b_2 equals Z_1/Z_2. Moreover, if one chooses L and C in (2) such that $L/C = L_0/C_0$ (closing the NRI line bandgap [2]), it is evident that $Y_1 = -Y_2$ and $Z_1 = -Z_2$ at $\omega_{\mathrm{equal}\_\beta}$. Using the observations above, a_1 equals a_2, and the ratio b_1/b_2 (hence the product $b_1 b_2$) is negative in (1). This gives rise to complex conjugate γs, creating a stopband for the coupled mode system. The coupled modes decay as they propagate forward and power is not lost due to radiation or substrate losses. This implies that the Poynting vectors on the two lines must be contra-directional so that power continuously 'leaks' from one line to the other adjacent to it.

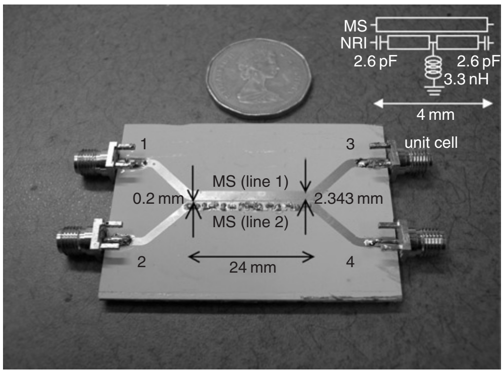

Fig. 1 *3 dB MS/NRI coupled line coupler (six unit cells long) constructed on 50 mil Rogers TMM4® ($\varepsilon_r = 4.6$) substrate*

In principle one should be able to achieve complete backward transfer of power between the lines if they are made sufficiently long. In this case, the exponentially increasing solutions will have to be discarded and the line voltage expressions given in [5] can be cast in the following simplified form:

$$\begin{pmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ j & -j \\ \dfrac{1}{Z} & \dfrac{1}{(-Z^*)} \\ \dfrac{j}{(-Z)} & \dfrac{-j}{Z^*} \end{pmatrix} \begin{pmatrix} V_c^+ e^{-\gamma_c z} \\ V_\pi^+ e^{-\gamma_\pi z} \end{pmatrix} \qquad (3)$$

where $\gamma_c = \alpha + j\beta$ and $\gamma_\pi = \alpha - j\beta$ are the two conjugate modes (symmetric and anti-symmetric, respectively) and Z is the characteristic impedance of the symmetric mode on line 1 ($Z = Z_{c1} = (Z_1 Z_2 - Z_m^2)/\gamma_c(Z_2 - Z_m R_c)$ refer to [5]). The assumptions of closing the NRI line bandgap and operation at $\omega_{\mathrm{equal}\_\beta}$, allow simplification of expressions for R_c, R_π, Z_{c1}, $Z_{\pi 1}$, Z_{c2} and $Z_{\pi 2}$ parameters in [5] enabling (3) to take its present form. From (3), it follows that:

$$\frac{1}{2}\mathrm{Re}(V_1 I_1^*|_{z=0}) = \frac{\mathrm{Re}(Z)}{2|Z|^2}(|V_c|^2 - |V_\pi|^2) = -\frac{1}{2}\mathrm{Re}(V_2 I_2^*|_{z=0}) \qquad (4)$$

This indicates complete transfer of power from line 1 to line 2. Interestingly, from (4) it can be seen that the γ_c mode ($e^{-(\alpha+j\beta)z}$) carries power forwards on line (1) and backwards on line (2), while the opposite holds true for the γ_π mode. In fact, the symmetric mode, γ_c, is the dominant mode and the asymmetric mode γ_π is only excited to compensate for source and termination mismatches.

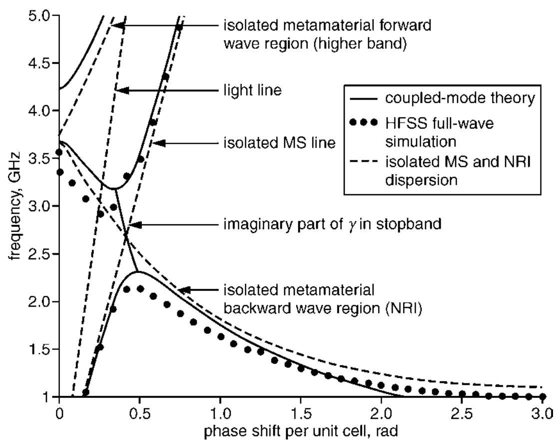

Fig. 2 *Coupled-mode dispersion diagram for MS/NRI coupled-line coupler*

Pierce [6] pointed out the existence of the coupled mode stopband and suggested that it occurs when a mode in a periodic structure couples to a higher-order backward-wave spatial harmonic. The unique feature presented here is that the coupling takes place between two lines involving their fundamental spatial harmonics. Indeed Ansoft HFSS® finite-element simulations and the theoretical dispersion relation (1) verify the formation of a stopband in the dispersion diagram at the location where the dispersion curves of the isolated lines meet (see Fig. 2).

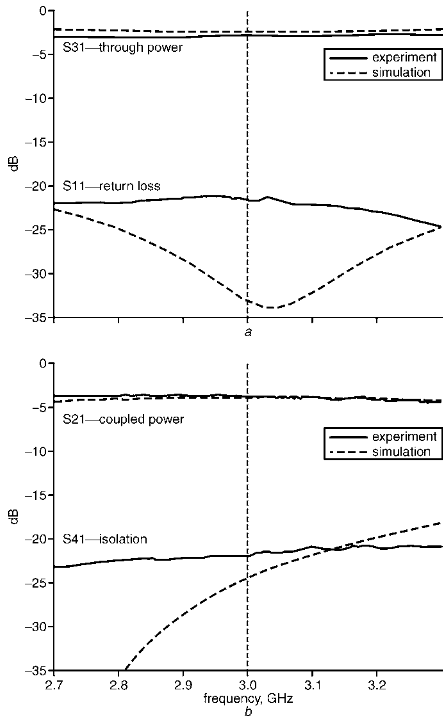

Fig. 3 *Simulation and experimental results for 3 dB MS/NRI coupled line coupler; return loss and through power, coupled power and isolation*

a Return loss and through power *b* Coupled power and isolation

Simulation and experimental results: A generalised version of (3) that includes reflected waves (i.e. both decaying and growing waves), along with Agilent's ADS, were utilised to design a 3 dB coupler that operates in the stopband and allows half of the power to be coupled to port 2 and the remainder portion to leave through port 3. Fig. 1 gives the dimensions and component values used in the design for operation at 3 GHz. Fig. 3 shows simulation and experimental results for the coupler described above. Measurements show a power split of −3.03 dB (through) and −3.68 dB (coupled) using the six cell-long coupler (24 mm) at the design frequency of 3 GHz (see Figs. 3*a* and *b*). The return loss and isolation is seen to be below −20 dB (see Figs. 3*a* and *b*). It is possible to achieve close to −0.5 dB coupling level using a coupler 12 cells long (48 mm).

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