Compact Linear Lead/Lag Metamaterial Phase Shifters for Broadband Applications

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***Abstract*—A compact one-dimensional phase shifter is proposed using alternating sections of negative refractive index (NRI) metamaterials and printed transmission lines (TL). The NRI metamaterial sections consist of lumped element capacitors and inductors, arranged in a dual TL (high-pass) configuration. By adjusting the NRI-medium lumped element values, the phase shift can be tailored to a given specification. Periodic analysis is applied to the structure and design equations are presented for the determination of the lumped element parameters for any arbitrary phase shift. To validate the design, various phase shifters are simulated and tested in coplanar waveguide (CPW) technology. It is demonstrated that small variations in the NRI-medium lumped element values can produce positive, negative or 0° phase shifts while maintaining the same short overall length. Thus, the new phase shifter offers some significant advantages over conventional delay lines: it is more compact in size, it exhibits a linear phase response around the design frequency, it can incur a phase lead or lag which is independent of the length of the structure and it exhibits shorter group delays.**

***Index Terms*—Broadband, metamaterials, negative refractive index, periodic structures, phase shifters.**

I. Introduction

RECENTLY, there has been a strong interest in the development of composite artificial dielectric media that exhibit simultaneously negative electric permittivity ϵ and magnetic permeability μ. Such metamaterials have been shown to exhibit a negative refractive index (NRI) [1]–[3], where the index of refraction is defined as the ratio of the speed of light to the phase velocity in the medium. In NRI metamaterials, the direction of the Poynting vector is antiparallel to the direction of phase velocity and, thus, the vectors $\vec{E}$, $\vec{H}$ and $\vec{k}$ form a left-handed triplet. These materials were therefore termed left-handed media (LHM) [4]. Similarly, conventional positive refractive index (PRI) media could be termed right-handed media (RHM) since $\vec{E}$, $\vec{H}$ and $\vec{k}$ form a right-handed triplet.

Composite media that exhibit simultaneous negative ϵ and μ were originally developed using thin wire strips and split-ring resonators [1], [2]. Their size however renders them impractical for the physical realization of microwave circuits. More recently, a compact two-dimensional dual transmission line (TL) network was developed that exhibits simultaneous negative ϵ and μ, and has been shown to experimentally demonstrate negative refraction and focusing of guided waves [3], [5]. The dual TL is realized by periodically loading a conventional transmission line with lumped element series capacitors (C_o) and shunt inductors (L_o), where the loading elements dominate the propagation characteristics. The inherent advantages of this approach are its simplicity and the ease with which it lends itself to the fabrication of planar microwave circuits. In addition, the circuits do not explicitly rely on resonances, and are therefore capable of achieving a negative refractive index over a large bandwidth with small losses [5].

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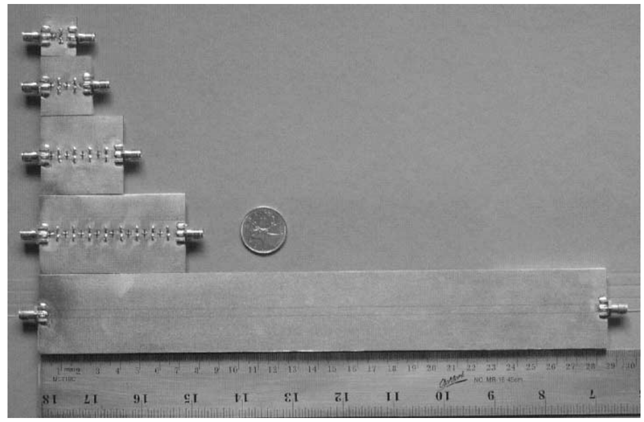

Fig. 1. Single-stage, two-stage, four-stage, and eight-stage 0° phase shifter circuits compared to a conventional 360° TL at 0.9 GHz.

Extending the range of ϵ and μ of these artificial dielectrics, has prompted the inception of numerous new RF/microwave circuits and devices that exhibit desirable characteristics such as the leaky-wave antenna radiating its fundamental spatial harmonic presented in [6], [7]. This paper extends the concepts presented in [6], [7] to describe the development of a compact one-dimensional (1-D) phase shifter comprising sections of NRI metamaterial lines cascaded with sections of conventional PRI TLs that can be used to synthesize an arbitrary transmission phase. A related phase compensating idea was proposed recently in [8] to create subwavelength cavity resonators using metamaterials. The proposed phase shifter seen in Fig. 1 offers some significant advantages when compared to standard delay TLs: it is more compact in size, it can achieve a positive *or* a negative phase shift while occupying the same short physical length and it exhibits a linear, flatter phase response with frequency, leading to shorter group delays.

II. Theory

A. Proposed Structure

In the NRI metamaterial proposed in [5], the phase leads in the direction of group velocity, therefore incurring a positive phase shift with propagation away from the source. In addition, it is well known that in conventional PRI TLs the phase lags in the direction of positive group velocity, thus incurring a negative phase shift with propagation away from the source. It therefore

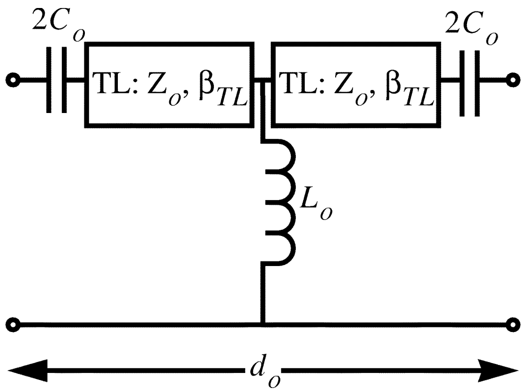

Fig. 2. 1-D phase shifter unit cell.

follows that phase compensation can be achieved at a given frequency by cascading a section of a NRI medium with a section of a PRI medium to form a unit cell with a given phase shift.

The proposed unit cell of the phase shifter structure is shown in Fig. 2 with total dimension d_o. The structure consists of a host TL medium periodically loaded with discrete lumped element components, L_o and C_o.

B. Network Parameter Analysis

The dispersion characteristics of the structure in Fig. 2 can be determined by following the periodic analysis procedure outlined in [7]:

$$cos(\beta_{\rm Bloch}d) = cos(\beta_{\rm TL}d)\left(1-\frac{1}{4\omega^2 L_o C_o}\right) + sin(\beta_{\rm TL}d)\left(\frac{1}{2\omega C_o Z_o}+\frac{Z_o}{2\omega L_o}\right) - \frac{1}{4\omega^2 L_o C_o} \qquad (1)$$

where $\beta_{\rm Bloch}$ is the Bloch propagation constant and $\beta_{\rm TL} = \omega\sqrt{LC}$ is the propagation constant of the host TL. In order to consider a series of cascaded unit cells as an effective periodic medium, the physical length of the unit cell must be much smaller than a wavelength, therefore restricting the phase shift per unit cell and the length of the TLs to small values ($\beta_{\rm Bloch}d \ll 1$ and $\beta_{\rm TL}d \ll 1$). Thus, an effective propagation constant $\beta_{\rm eff}$ can be defined for the medium [7]

$$\beta_{\rm eff} \approx \pm\omega\sqrt{\left(L-\frac{1}{\omega^2 C_o d}\right)\left(C-\frac{1}{\omega^2 L_o d}\right)}. \qquad (2)$$

Fig. 3 shows the dispersion relation for a unit cell with typical parameters $C_o = 15$ pF, $L_o = 20$ nH, $Z_o = 50\ \Omega$ and $\beta_{\rm TL}d = 0.28$ rad. It can be observed that the corresponding dispersion diagram exhibits a band structure with two distinct passbands and stopbands as in [5] and [7]. Expressions for the pertinent cutoff frequencies as indicated in Fig. 3 are as follows:

$$f_b = \frac{1}{4\pi\sqrt{L_o C_o}}, \quad f_{c1} = \frac{1}{2\pi\sqrt{LC_o}}, \quad f_{c2} = \frac{1}{2\pi\sqrt{L_o C}}. \qquad (3)$$

Equating f_{c1} and f_{c2} causes the stopband between these two cutoff frequencies to close and the band becomes continuous. This corresponds to the following matching condition for the two media, which was originally derived in [5, Eq. (29), pp. 2706]:

$$Z_o = \sqrt{\frac{L_o}{C_o}} = \sqrt{\frac{L}{C}}. \qquad (4)$$

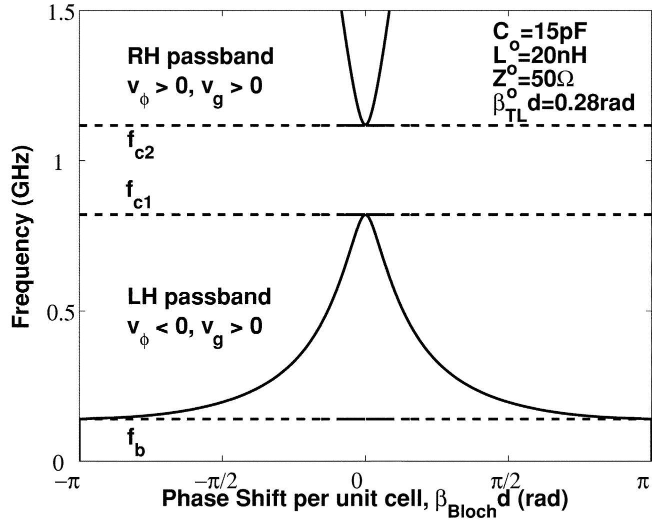

Fig. 3. Unmatched dispersion relation for the unit cell of Fig. 2.

Under this condition, it can be shown that the effective propagation constant of (2) simplifies to

$$\beta_{\rm eff} \approx \omega\sqrt{LC} + \frac{-1}{\omega\sqrt{L_o C_o}d}. \qquad (5)$$

This expression can be interpreted as the sum of the propagation constants of the host TL and a uniform backward wave L–C line. For a TL medium with periodicity d and phase shift $\phi_{\rm TL} = \beta_{\rm TL}d$, the total phase shift per unit cell, $|\phi_o|$, given that the matching condition of (4) is satisfied, can be written as

$$|\phi_o| = \beta_{\rm eff}d = \phi_{\rm TL} + \frac{-1}{\omega\sqrt{L_o C_o}}. \qquad (6)$$

Thus, (4) and (6) can be used to determine unique values for L_o and C_o for *any* phase shift, ϕ_o, given a TL section with intrinsic phase shift $\phi_{\rm TL}$ and characteristic impedance Z_o.

III. Theoretical Results

The unit cell of Fig. 2 was implemented in CPW technology at a design frequency of $f_o = 0.9$ GHz using the Agilent-Advanced Design System (ADS) microwave circuit simulator. The lumped element components and the TLs were assumed to be ideal. The TLs were designed with $Z_o = 50\ \Omega$, $\epsilon_r = 2.2$, dielectric height, $h = 20$ mils and length $d = 7.4$ mm. The dispersion relation for a unit cell with $\phi_o = 0°$ is shown in Fig. 4. The impedance matching condition of (4) has been satisfied and, therefore, the stopband has been closed. It can also be observed that at $f_o = 0.9$ GHz, the phase shift is 0°.

Based on the above parameters a $+10°$ phase shifter was designed consisting of four unit cells of total physical length $0.11\lambda_{\rm CPW} = 32$ mm, with loading element values of $C_o = 17.15$ pF and $L_o = 42.88$ nH. The resulting phase characteristic of the structure is shown in Fig. 5. It can be observed that the phase shift is $+10°$ at $f_o = 0.9$ GHz with a phase slope of $-95.7°$/GHz. In addition, the phase response around f_o is linear with respect to frequency, implying a constant group delay for the phase shifter. Also shown in Fig. 5 is the phase characteristic of a conventional $50\ \Omega$ TL of length $35\lambda_{\rm CPW}/36 = 275.7$ mm for comparison. The TL exhibits as expected a $-350°$ phase shift at f_o, which is equivalent to $+10°$, with a phase slope of $-389°$/GHz. In addition, its physical length is approximately nine times longer than the four-cell phase shifter line. Referring

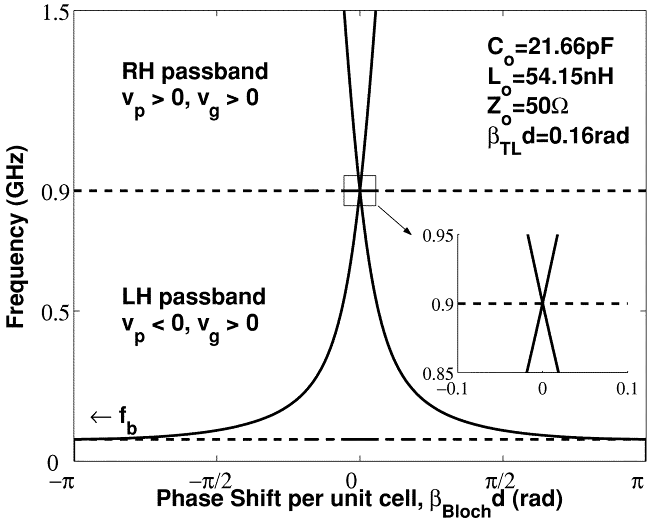

Fig. 4. Matched dispersion relation for the unit cell of Fig. 2.

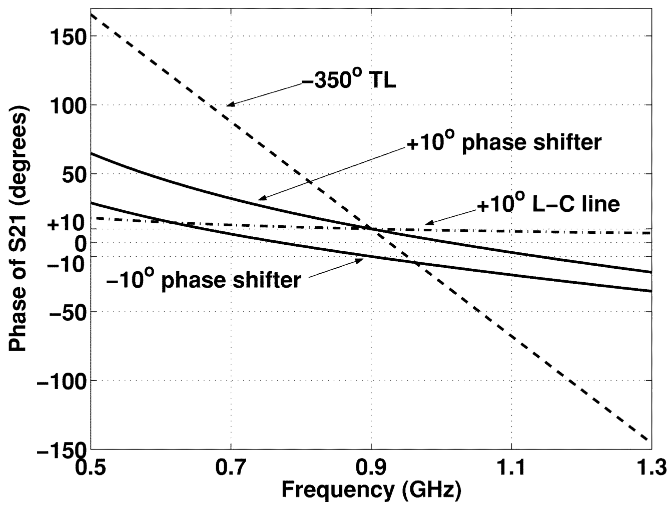

Fig. 5. Phase responses of the ideal four-stage $+10^\circ$ phase shifter, -10° phase shifter, $+10^\circ$ L-C line, and -350° TL.

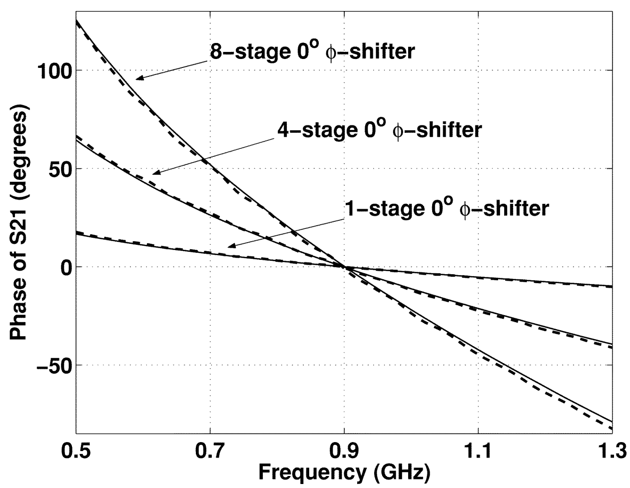

Fig. 6. Phase responses of one-, four-, and eight-stage 0° phase shifters $(- \; -)$ Measured, (—)ADS simulation.

to Fig. 1 one can obtain an impression of the relative sizes of the two structures. The proposed $+10^\circ$ phase shifter offers therefore two main advantages over its equivalent TL implementation: a $+10^\circ$ phase shift while occupying significantly less physical space than a conventional TL, and a shorter group delay which is desirable for applications that require broadband operation.

Also shown in Fig. 5 is the phase characteristic of a four-stage -10° phase shifter with loading element values of $C_o = 29.46$ pF and $L_o = 73.65$ nH. It can be observed that the -10° phase characteristic is similar to the $+10^\circ$ phase characteristic, with a phase slope of $-72.9^\circ/\text{GHz}$. The -10° phase shifter occupies the same physical length as the $+10^\circ$ phase shifter and has loading element values that are at a ratio of approximately $1.4:1$ compared to the ones used for the $+10^\circ$ phase shifter. This implies that a reasonable variation of the loading element values using tunable components can produce *either* a positive *or* a negative phase shift.

Fig. 5 also shows the phase characteristic of a 4-stage $+10^\circ$ uniform backward wave L-C line with loading element values of $C_o = 81.06$ pF and $L_o = 202.64$ nH. This corresponds to the structure of Fig. 2 with the TL sections removed. Therefore, setting ϕ_{TL} to zero in (6) implies that ϕ_o will always remain negative, corresponding to a phase advance for a positively traveling plane wave of the form $e^{-j\beta_{\text{eff}} d}$. Indeed, the absence of TLs implies that any combination of C_o and L_o values will produce a phase shift that will always be greater than 0° for this structure. This is verified by the phase characteristic of the L–C line in Fig. 5 that stays well above 0° at all times. Therefore, although the backward wave L–C line and the proposed structure of Fig. 2 can both provide positive phase shifts, the latter has the additional advantage of being able to provide a negative phase shift by simply varying the loading element values. More importantly, short broadband 0° phase shift lines can be realized which can be used in numerous applications.

IV. Simulated and Experimental Results

The 1-D phase shifter structures were constructed using CPW technology on a Rogers RT5880 substrate with $\epsilon_r = 2.2$ and dielectric height $h = 20$ mils. Standard size 0402 Panasonic ECJ-0EC capacitors were used for C_o with dimensions $L = 1$ mm, $W = 0.5$ mm, and $H = 0.5$ mm, and standard size 0603 Panasonic ELJ-RE/FJ inductors were used for L_o with dimensions $L = 1.6$ mm, $W = 0.8$ mm, and $H = 0.8$ mm. The $50\ \Omega$ TLs were designed to have a length of $d = 7.4$ mm, and a gap of $s = 0.6$ mm was created to accommodate the series capacitors, resulting in a unit cell of total length $d_o = 8$ mm. At 0.9 GHz the TLs were designed with a center conductor width of $w = 4$ mm and a gap between the center conductor and the ground plane of $g = 0.106$ mm. A thru-reflect-line (TRL) calibration was carried out in order to establish the reference planes at the input and output ports of the phase shifters. All lab measurements were carried out on a HP8753C series vector network analyzer. The simulated results were obtained by replacing the ideal models for the capacitors and inductors with their corresponding S-parameter files provided by their vendor. Since the S-parameter files were extracted directly from device measurements, they therefore contain all the parasitic values associated with each component.

Fig. 6 shows the simulated and measured phase responses of three 0° phase shifters with loading element values of $C_o = 12$ pF and $2L_o = 100$ nH. It can be observed that the experimental results match very closely the simulated results, with phase slopes of $-30.2^\circ/\text{GHz}$, $-122.1^\circ/\text{GHz}$ and $-237.5^\circ/\text{GHz}$ for the one-, four-, and eight-stage phase shifters, respectively. The measured insertion losses are 0.1,

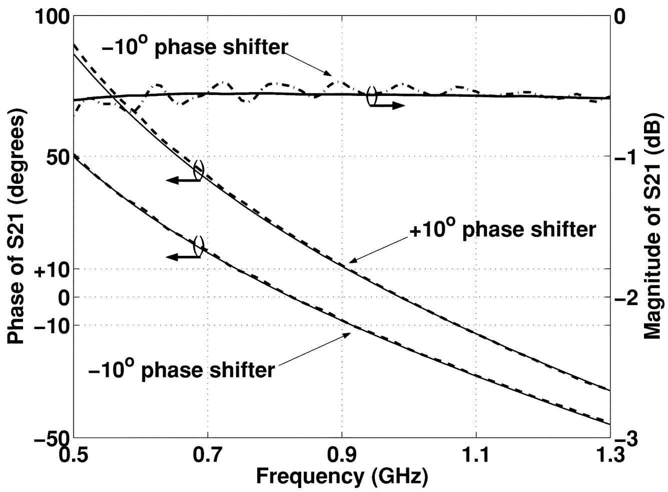

Fig. 7. Phase and magnitude responses of $+10^\circ$ and -10° four-stage phase shifters $(--)$ measured phase; $(-\cdot-)$ measured magnitude; (——) ADS simulation.

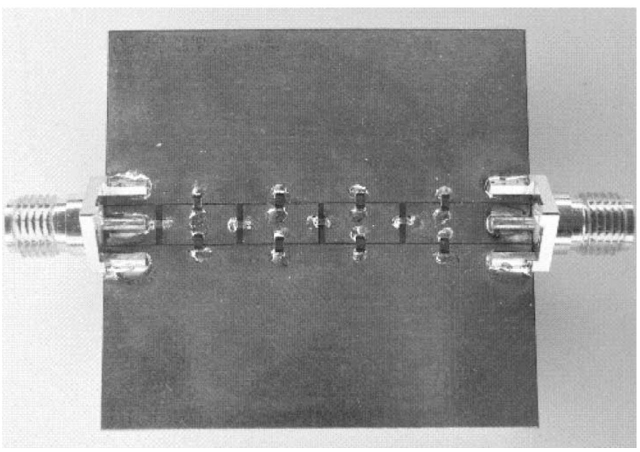

Fig. 8. Four-stage $+10^\circ$ phase shifter.

0.7, and 1.6 dB, respectively, at $f_o = 0.9$ GHz. It is therefore evident that any number of stages can be used in order to produce the desired phase shift, however, as the number of stages increases, so does the slope of the phase characteristic. For broadband applications, therefore, it is desirable to keep the number of stages to a minimum. Fig. 1 depicts the relative sizes of the compact one-, two-, four-, and eight-stage 0° phase shifters relative to a conventional 360° TL. It can be seen that the phase shifters offer a great size advantage, producing the same phase shift while occupying approximately 35 times less space in the one-stage case than the corresponding TL.

Fig. 7 shows the simulated and measured phase responses of a four-stage $+10^\circ$ phase shifter (shown in Fig. 8) with loading element values of $C_o = 12$ pF and $2L_o = 68$ nH and a four-stage -10° phase shifter with loading element values of $C_o = 15$ pF and $2L_o = 120$ nH. It can be observed that the experimental results correspond very closely to the simulated results with phase slopes of $-136.6^\circ/\text{GHz}$ and $-108.4^\circ/\text{GHz}$ for the $+10^\circ$ and -10° phase shifters respectively. Also shown in Fig. 7 is the magnitude response of the -10° four-stage phase shifter. The simulated and measured results correspond closely, however, as the number of stages is increased the phase shifter will begin to radiate, resembling the leaky-wave structure of [7]. Therefore, the phase shifters should be kept short in length to avoid unnecessary energy leakage. The measured insertion losses at $f_o = 0.9$ GHz are 0.4 and 0.5 dB, respectively, for the $+10^\circ$ and -10° phase shifters.

V. Conclusion

A new type of phase shifter using cascaded sections of NRI metamaterials and PRI TLs has been proposed, which offers some significant advantages over conventional delay lines and uniform backward-wave L–C lines. The phase shifter is compact in size, can be easily fabricated using standard etching techniques and exhibits a linear phase response around the design frequency. It can incur *either* a negative *or* a positive phase, as well as a 0° phase depending on the values of the loading elements, while maintaining a short physical length. In addition, the phase incurred is independent of the length of the structure. Due to its compact, planar design, the structure lends itself easily toward integration with other microwave components and devices. It is therefore ideal for broadband applications requiring small, versatile, linear phase shifters.

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