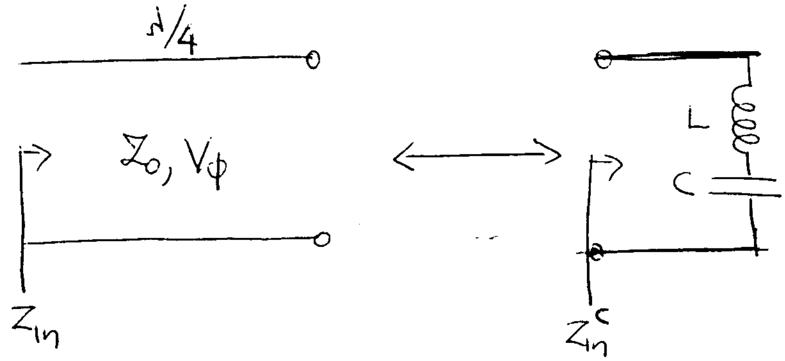
Problem 1:



For the lumped-element case:

$$Z_{in}^c = j\omega L + \frac{1}{j\omega C}$$

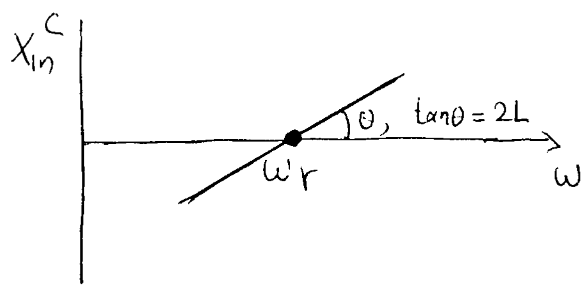
At resonance $\omega = \omega_r = 1/\sqrt{LC}$ and $Z_{in}^c = 0$.

Let us expand Z_{in}^c in a Taylor's series about $\omega = \omega_r$:

$$Z_{in}^c \cong Z_{in}^c(\omega_r) + \left.\frac{\partial Z_{in}^c}{\partial \omega}\right|_{\omega_r} (\omega - \omega_r) + \cdots$$

$$= 0 + \left(jL + \frac{j}{\omega_r^2 C} \right)(\omega - \omega_r)$$

Hence, $Z_{in}^c \cong \left(jL + j\frac{1}{\cancel{C}} L\cancel{C} \right)(\omega - \omega_r) = 2jL(\omega - \omega_r) \quad \ldots (1)$



Now consider the distributed network case:

$$Z_{in} = -j Z_0 \cot\beta l = -j Z_0 \frac{\cos(\beta l)}{\sin(\beta l)}$$

when $\beta_r l = \pi/2$, $Z_{in}(\beta_r l) = 0$ since the open becomes a short at input

Let us define then the resonant frequency from $\frac{\omega_r l}{V_\phi} = \pi/2$

i.e $\boxed{\omega_r = \frac{\pi V_\phi}{2l}}$, $l = \lambda/4$ ...... (2)

Expand $Z_{in}(\beta l)$ about $\beta_r l = \pi/2$, i.e: Let $\beta l = \pi/2 + \Delta\beta l$

$$\Rightarrow \cos(\beta l) = \cos\left(\frac{\pi}{2} + \Delta\beta l\right) = -\sin(\Delta\beta l) \simeq -\Delta\beta l$$
$$\sin(\beta l) = \sin\left(\frac{\pi}{2} + \Delta\beta l\right) = \cos(\Delta\beta l) \simeq 1$$

and $\Delta\beta l = \frac{(\omega - \omega_r) l}{V_\phi}$, hence

$$Z_{in}(\omega) \simeq j Z_0 \Delta\beta l = j \frac{Z_0 l}{V_\phi}(\omega - \omega_r) \quad \ldots \quad (3)$$

Comparing (1) and (3) we see that they behave in a similar way. Also, $2L \equiv \frac{Z_0 l}{V_\phi} \Rightarrow \boxed{L = \frac{Z_0 l}{2V_\phi},\ l = \lambda/4}$

and $\omega_r = \frac{\pi V_\phi}{2l} = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{C = \frac{1}{L\omega_r^2} = \frac{8l}{Z_0 \pi^2 V_\phi}}$

where L, C are the equivalent series inductance and capacitance.

Problem 2 :

(a)

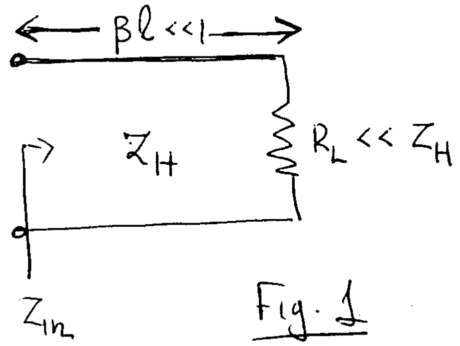


Fig. 1

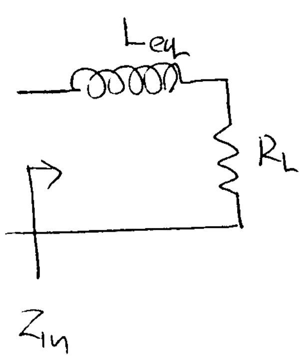
From the impedance transformation:

$$Z_{in} = Z_H \frac{R_L + jZ_H \tan\beta l}{Z_H + jR_L \tan\beta l} \simeq \frac{Z_H R_L + jZ_H^2 \beta l}{Z_H + \underbrace{jR_L\beta l}_{\text{small}}} \simeq R_L + jZ_H \beta l$$

I.e for $\beta l \ll 1$ and $R_L \ll Z_H$

$$Z_{in} \simeq R_L + j\omega\left(\frac{Z_H l}{V_\phi}\right) \equiv R_L + j\omega L_{eq}$$

This shows that the lumped equivalent circuit to Fig. 1 is



where $\boxed{L_{eq} = \frac{Z_H l}{V_\phi}}$

(b)

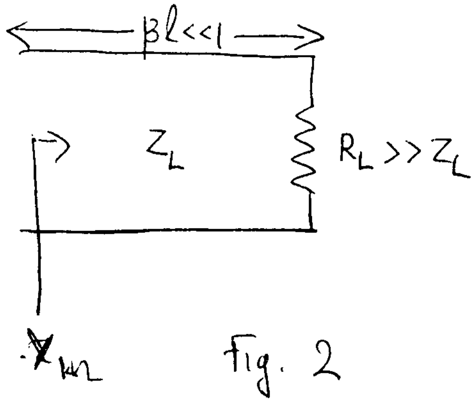


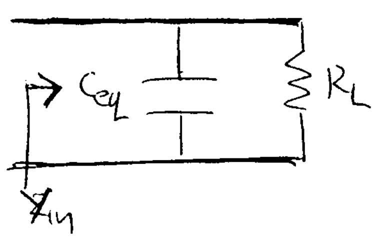
fig. 2

Since we are trying to show equivalence to a shunt element, it is more convenient to use admittances:

$$Y_{in} = \frac{1}{Z_L} \frac{Z_L + jR_L \tan\beta l}{R_L + \underbrace{jZ_L \tan\beta l}_{\text{very small}}} \simeq \frac{1}{Z_L} \frac{Z_L + jR_L \beta l}{R_L}$$

$$\Rightarrow \quad Y_{in} \simeq \frac{1}{R_L} + j\frac{\beta l}{Z_L} = \frac{1}{R_L} + j\omega\left(\frac{l}{V_\phi Z_L}\right) \equiv \frac{1}{R_L} + j\omega C_{eq}$$

This shows that the equivalent lumped circuit to fig. 2 is



where $\boxed{C_{eq} = \frac{l}{V_\phi Z_L}}$

These results can be generalized for complex loads as well. The corresponding conditions are $|Z_{Load}| \gg$ or $\ll$ $|Z_{Line}|$ depending on whether we would like to simulate a series-inductor or a shunt capacitor.

Problem 3 :

The normalized to $Z_1 = 50\Omega$ impedance at the load is

$$\bar{Z}_L = Z_L/50 = 2 + j1.5$$

Move 0.12λ towards the generator to reach $\bar{Z}_A = 1 - j1.3$

Since the inductance is in series

$$Z_B = Z_A + j30 \Rightarrow \bar{Z}_B = \bar{Z}_A + 0.6 = 1 - j0.7$$

On the other hand the capacitance is shunt connected. Therefore it is easier to use admittances.

$\bar{Y}_B = 1/\bar{Z}_B = 0.67 + 0.47j$ which is found by moving to the exact opposite point of Z_B on its SWR circle.

Now, $Y_C = Y_B + Y_{cap}$, $Y_{cap} = \frac{1}{-j200} = 0.005j \Rightarrow \bar{Y}_{cap} = Y_{cap}/Y_0 = Z_0 Y_{cap}$

$$= 50 Y_{cap} = 0.25j$$

$\Rightarrow \bar{Y}_C = \bar{Y}_B + \bar{Y}_{cap} = 0.67 + 0.72j$.

But the input-line is normalized to $Z_2 = 150\Omega$. Hence I should first denormalize $\bar{Y}_C$ to get $Y_C = \frac{1}{50}(0.67 + 0.72j) = 0.0134 + 0.0144j$

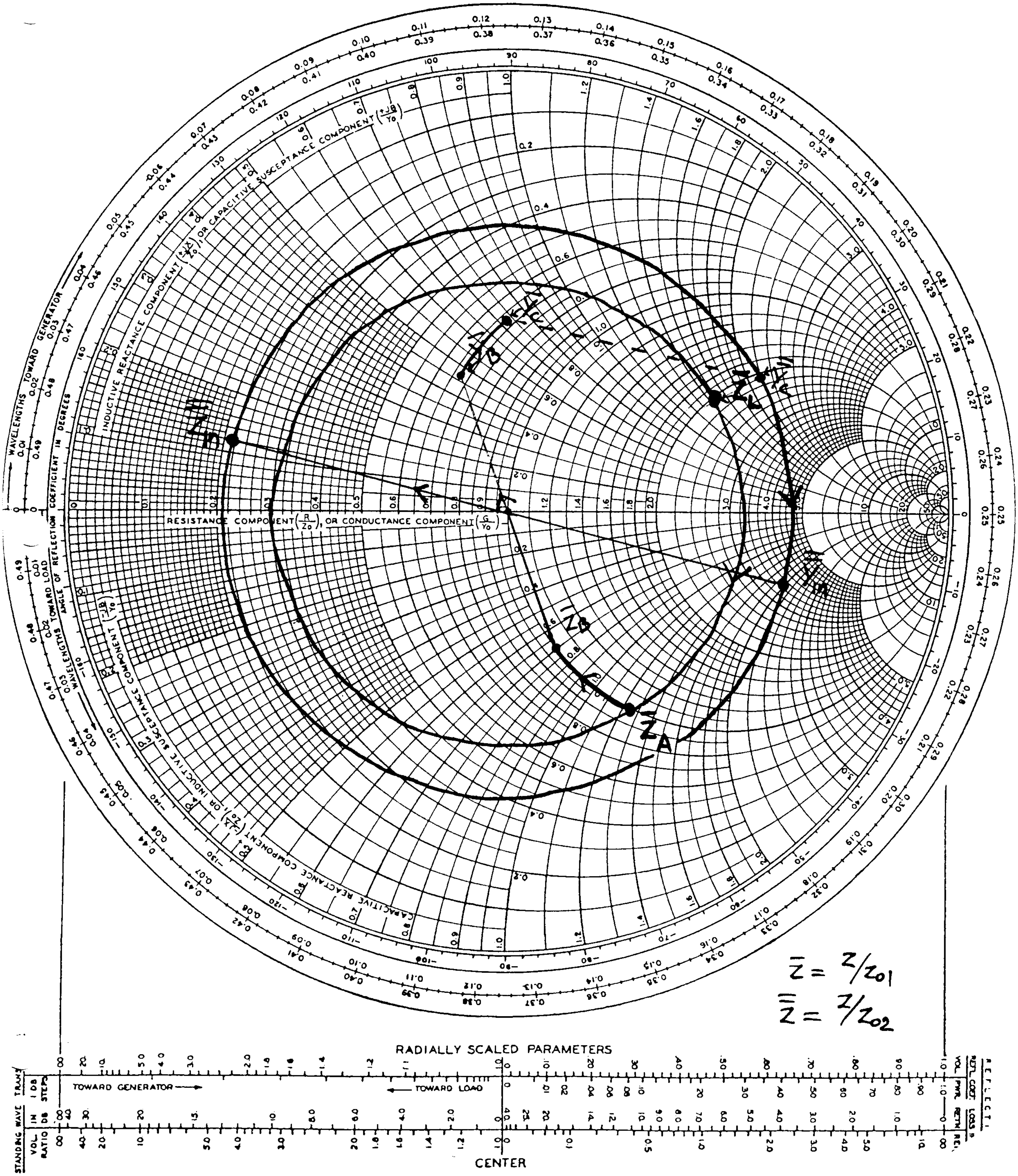
Normalize to $Z_2 = 150\Omega \Rightarrow \bar{\bar{Y}}_C = 150 Y_C = 2.01 + 2.16j$.

Move on the corresponding SWR circle 0.06λ toward the generator to reach $\bar{\bar{Y}}_{in}$.

Finally moving to the opposite point of the $\bar{\bar{Y}}_{in}$ SWR-circle, we obtain

$\bar{\bar{Z}}_{in} \cong 0.22 + 0.125j$, Hence $Z_{in} = \bar{\bar{Z}}_{in}(150) = 33 + j18.75$ $\underbrace{(33.7 + 17.52j)}_{\text{Exact solution.}}$

IMPEDANCE OR ADMITTANCE COORDINATES



$\bar{Z} = Z/Z_{01}$

$\bar{\bar{Z}} = Z/Z_{02}$

Problem 4 :

(5.9) Smith chart solution

1. plot $y_L = 1.4 + j2.0$ (ADMITTANCE CHART)
2. plot rotated $1+jb$ circle
3. add a stub susceptance of $-j0.1$ to y_L to move to rotated $1+jb$ circle
4. move $\lambda/8$ towards generator, to $1+jb$ circle
5. add a stub susceptance of $+j1.6$ to move to center of chart.
6. The open-circuited stub lengths are,

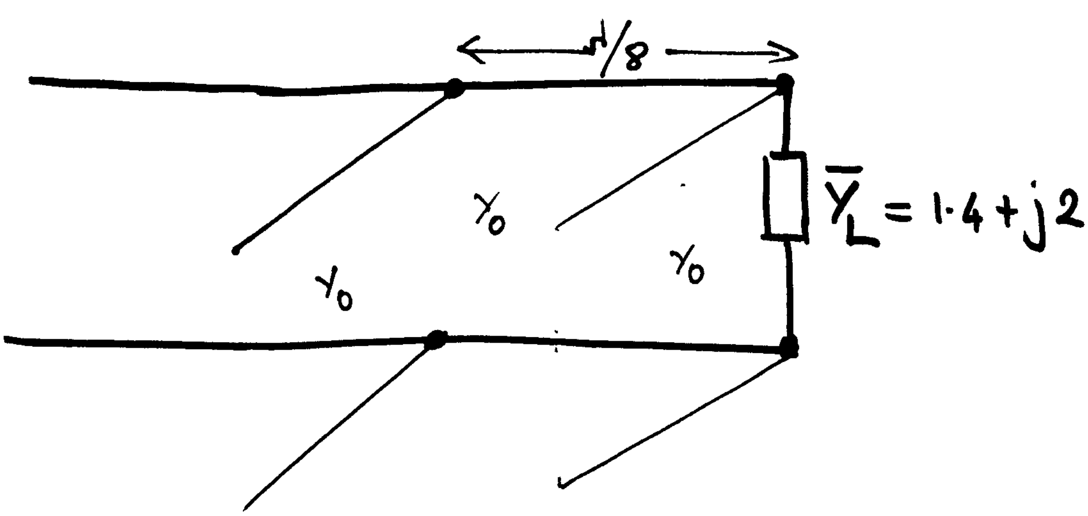
$$l_1 = 0.484\lambda \checkmark$$
$$l_2 = 0.161\lambda \checkmark$$

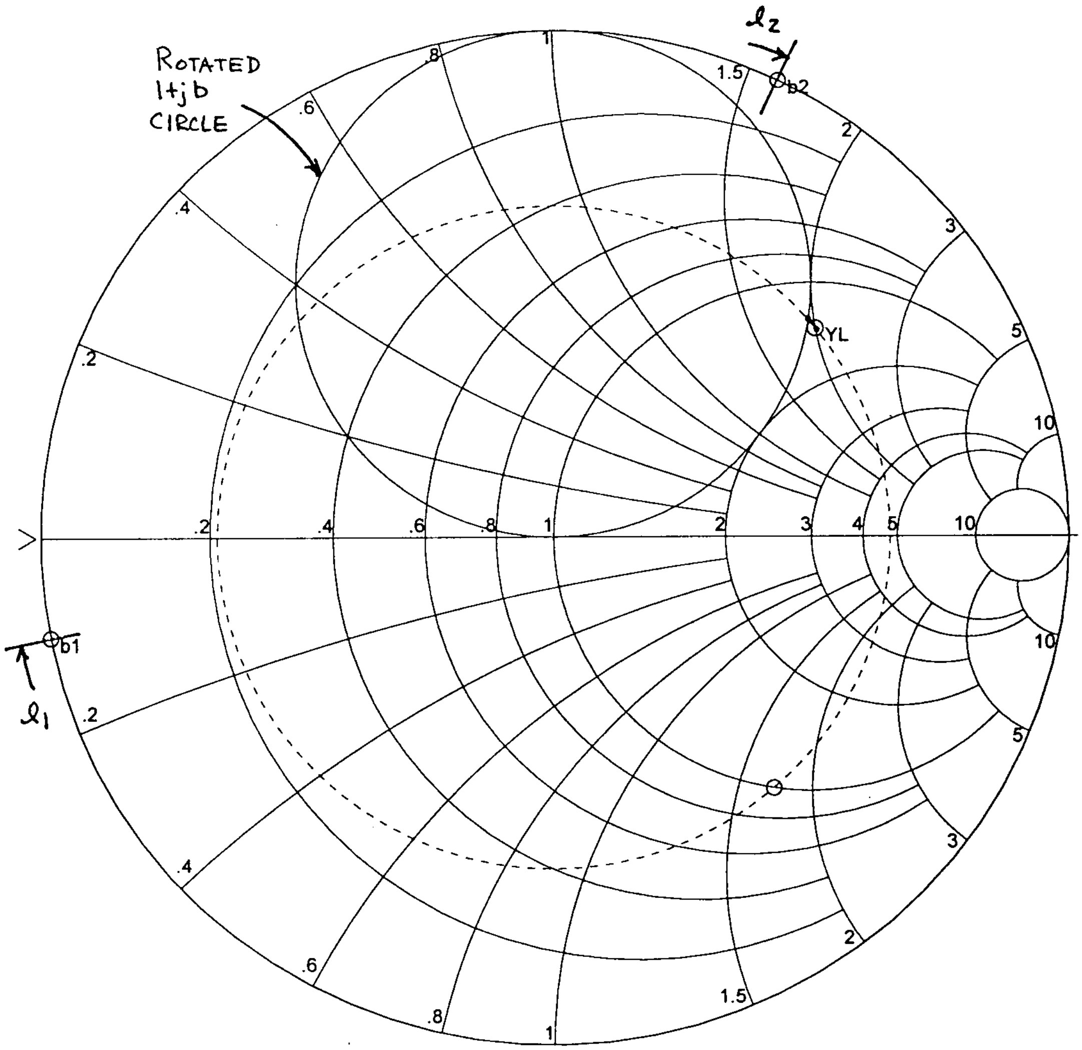
(Analytically, we obtain $l_1 = 0.487\lambda$, $l_2 = 0.163\lambda$)

The alternative matching solution has,

$$l_1' = 0.326\lambda \checkmark$$
$$l_2' = 0.053\lambda \checkmark$$

The Smith chart for the first solution is shown on the following page.





Smith chart for Problem 5.9