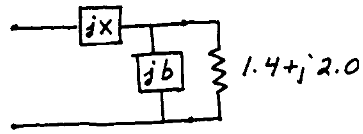
Chapter 5

5.1 (Smith chart solutions)

a) $z_L = 1.4 + j2.0$

inside $1+jx$ circle

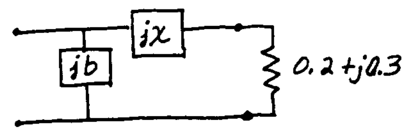


SOL'N #1: $b_1 = -0.10$ ✓
$x_1 = -1.7$ ✓

SOL'N #2: $b_2 = 0.78$ ✓
$x_2 = 1.7$ ✓

b) $z_L = 0.2 + j0.3$

outside $1+jx$ circle

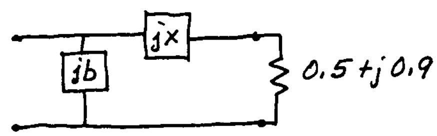


SOL'N #1: $x_1 = 0.10$ ✓
$b_1 = 2.0$ ✓

SOL'N #2: $x_2 = -0.70$ ✓
$b_2 = -2.0$ ✓

c) $z_L = 0.5 + j0.9$

outside $1+jx$ circle

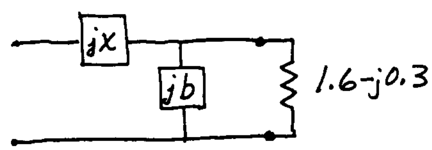


SOL'N #1: $x_1 = -0.40$ ✓
$b_1 = 0.96$ ✓

SOL'N #2: $x_2 = -1.4$ ✓
$b_2 = -0.96$ ✓

d) $z_L = 1.6 - j0.3$

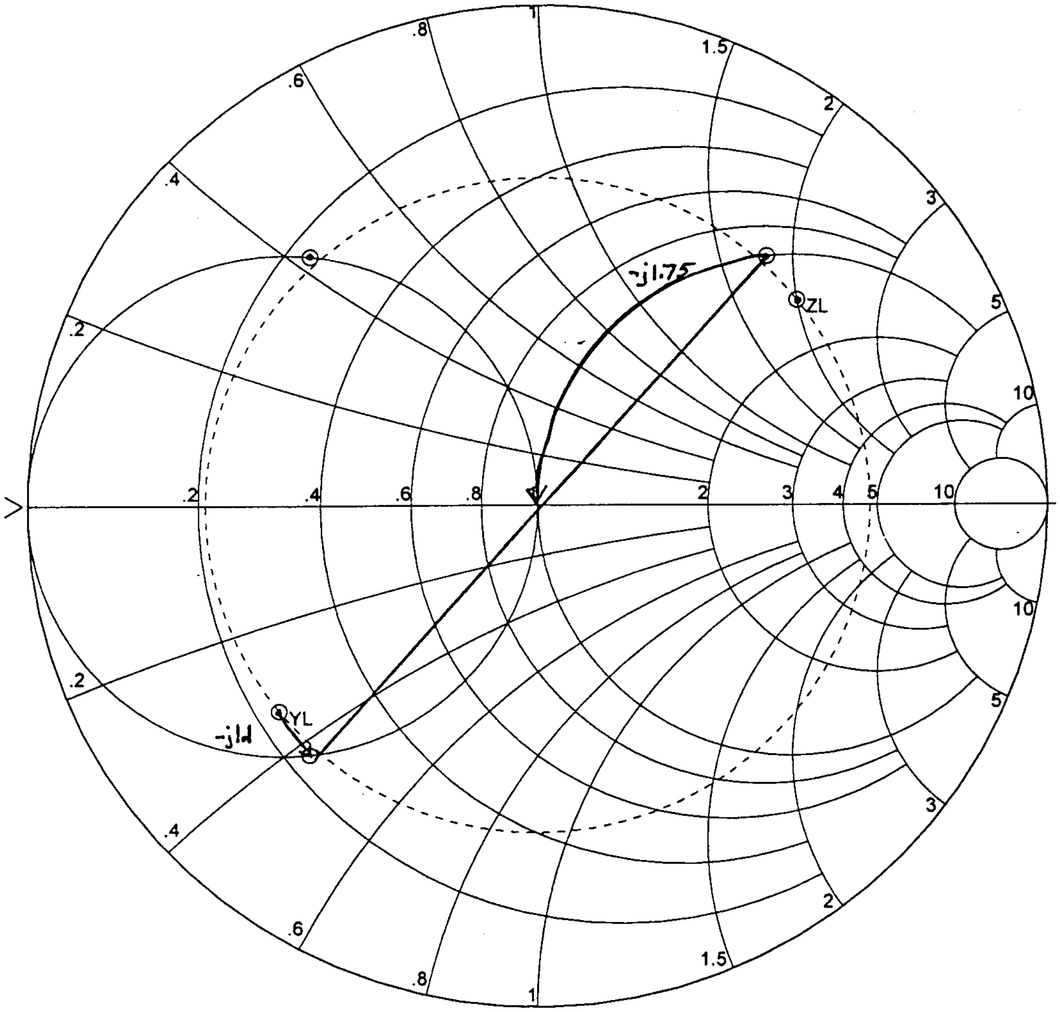
inside $1+jx$ circle



SOL'N #1: $b_1 = 0.38$ ✓
$x_1 = 0.80$ ✓

SOL'N #2: $b_2 = -0.62$ ✓
$x_2 = -0.80$ ✓

(The Smith chart for 5.1a is shown on the following page.)



Smith chart for Problem 5.1a

5.5 (Smith chart solutions)

The normalized load impedance is $z_L = 0.40 - j1.2$. To intersect the $1+jx$ circle, we must move back from the load either of the following distances:

$$d_1 = 0.19 + (0.5 - 0.355) = 0.335\lambda \checkmark$$

or,

$$d_2 = 0.31 + (0.5 - 0.355) = 0.455\lambda \checkmark$$

The reactances necessary for matching are then,

$$x_{s1} = -2.1$$

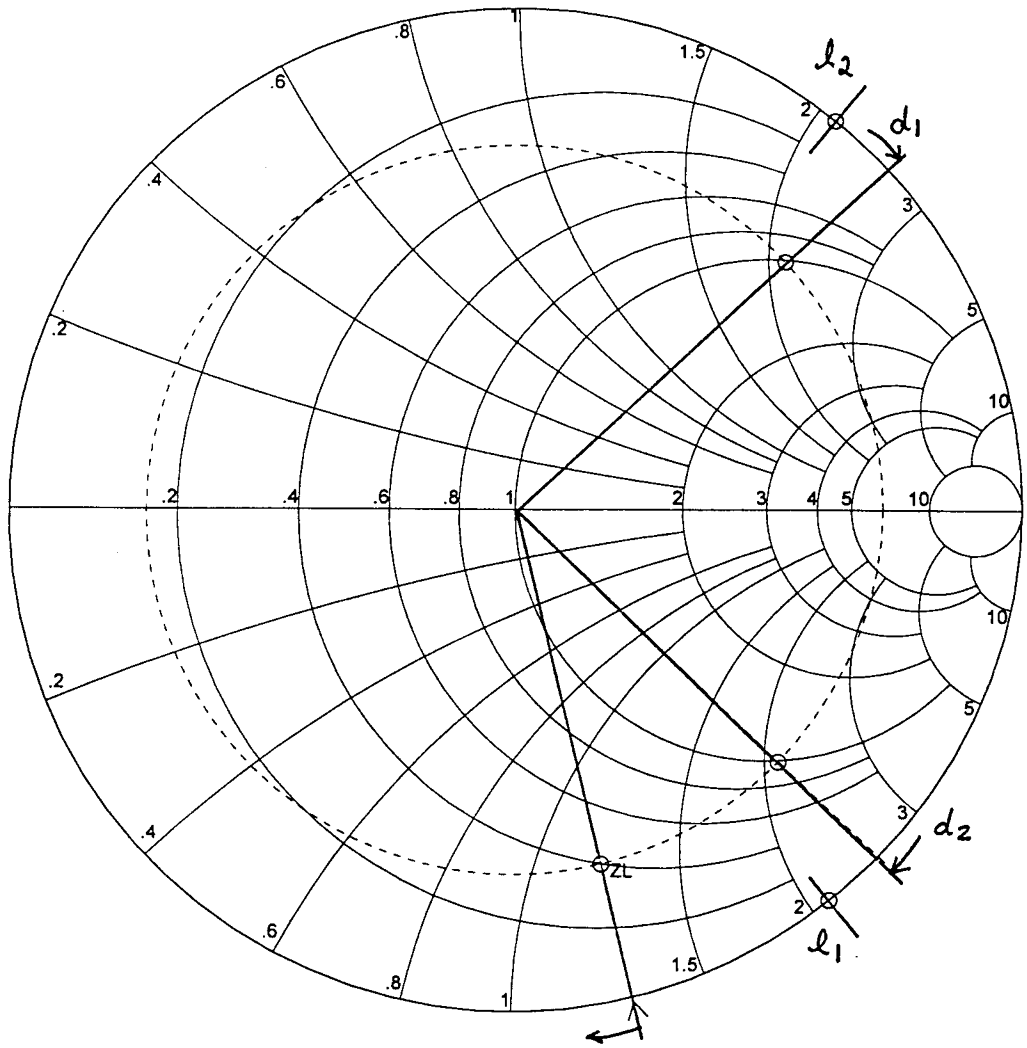
$$x_{s2} = 2.1$$

Then the open-circuited stub lengths are,

$$\ell_1 = 0.32 - 0.25 = 0.07\lambda \checkmark$$

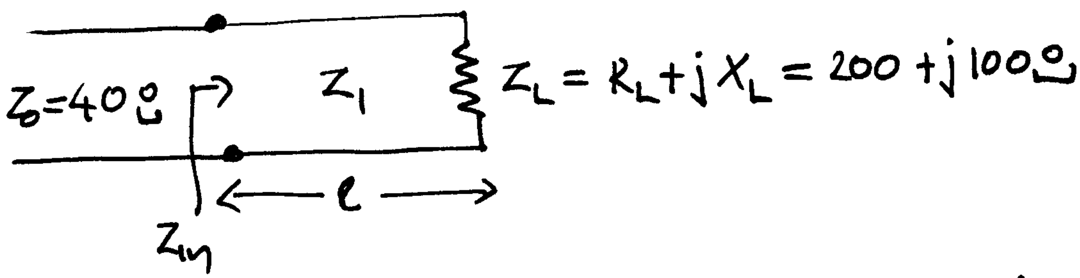
$$\ell_2 = 0.25 + 0.18 = 0.43\lambda \checkmark$$

(The Smith chart for this problem is shown on the following page.)



Smith chart for Problem 5.5

Problem 5.7.



To achieve matching $\quad Z_{in} = Z_0 = Z_1 \dfrac{Z_L + jZ_1 t}{Z_1 + jZ_L t}$, $t = \tan\beta l$

Hence $\quad Z_0 Z_1 + jZ_0 Z_L t = Z_1 Z_L + jZ_1^2 t$

Equating real parts: $\quad Z_0 Z_1 - Z_0 X_L t = Z_1 R_L \quad \cdots \quad (1)$

— // — Imaginary parts: $\quad Z_0 R_L t = Z_1 X_L + Z_1^2 t \quad \cdots \quad (2)$

Let $\quad u = -\dfrac{Z_0 X_L}{R_L - Z_0}$ then from (1) $\quad Z_1 = ut \quad \cdots \quad (3)$

From (2),(3) : $\quad Z_1^2 + uX_L - Z_0 R_L = 0$

Hence $$Z_1 = \sqrt{Z_0}\sqrt{R_L + \frac{X_L^2}{R_L - Z_0}} \qquad (4)$$

and $$t = \tan\beta l = \frac{Z_1}{u} = -Z_1 \frac{R_L - Z_0}{Z_0 X_L} \qquad (5)$$

In the example: $Z_0 = 40$, $R_L = 200$, $X_L = 100$.

Hence,

$$\boxed{\begin{aligned} Z_1 &= 102.5\,\Omega \\ l &= 0.288\,\lambda \end{aligned}}$$ and $\tan\beta l = -4.10$

i.e.

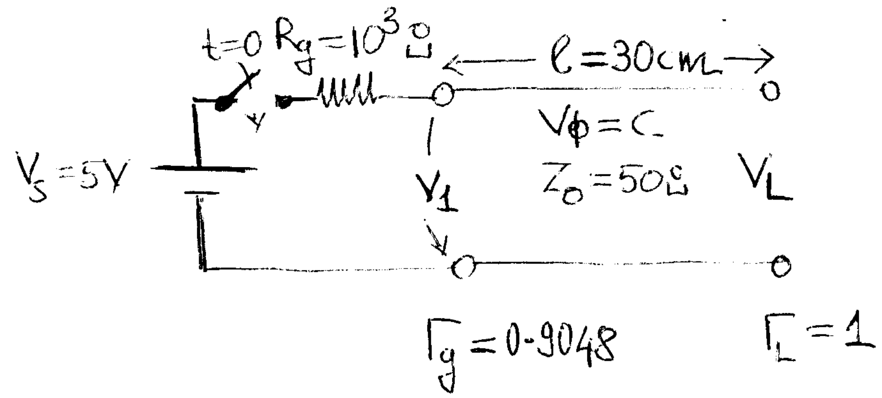
In general to achieve matching Z_1 must be real.

Hence from (4)

i) $R_L > Z_0$ then Z_1 is always real.

ii) $R_L < Z_0$ then $R_L > \dfrac{X_L^2}{Z_0 - R_L}$ i.e.

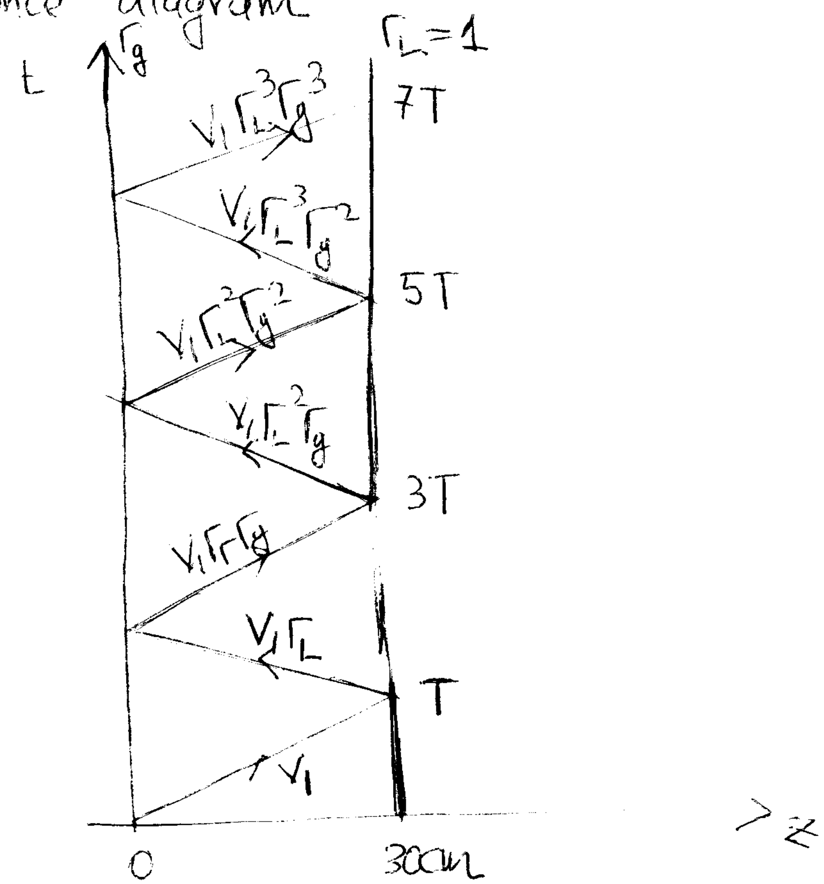
$$X_L^2 < R_L (Z_0 - R_L)$$



a) Transit time, $T = \dfrac{\ell}{c} = 1\,ns$

b) At $t = 0$, $V_1 = \dfrac{V_s Z_0}{R_g + Z_0} = 0.2381\,V$

c) Bounce diagram



d) Noting that $\Gamma_L = 1$, a general expression for the load voltage is

$$V_L = 2V_1 + 2V_1\Gamma_g + 2V_1\Gamma_g^2 + 2V_1\Gamma_g^3 + \cdots$$

Hence, $V_L = 2V_1 (1 + \Gamma_g + \Gamma_g^2 + \Gamma_g^3 + \cdots)$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ T & 3T & 5T \end{array} \cdots\cdots$$

Summing the geometric progress the load voltage at

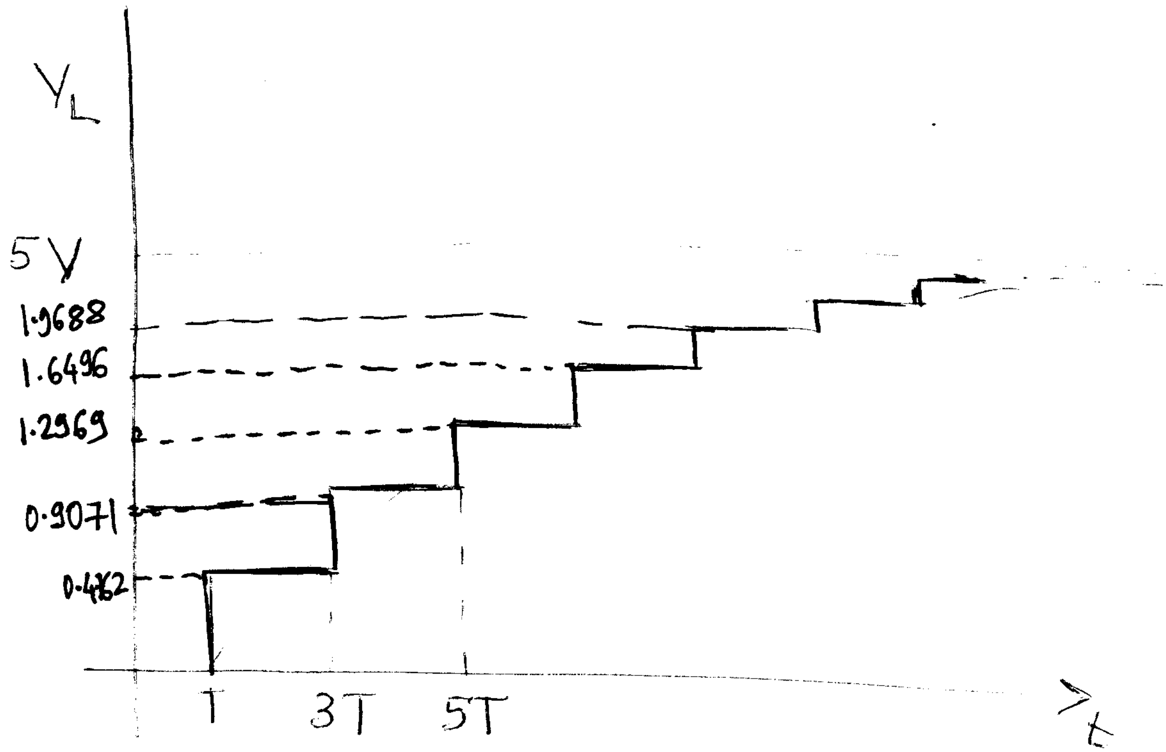
$t = T + 2T(n-1)$ is $V_L = 2V_1 \dfrac{1 - \Gamma_g^n}{1 - \Gamma_g}$, $n = 1, 2, 3, \cdots$

e) As $t \to \infty$ then $n \to \infty$ and

$$V_L = 2V_1 \frac{1}{1 - \Gamma_g} = 0.2381 \frac{2}{(1 - 0.9048)} = 5V = V_s$$

as expected

f) Using part (d) the step-response at the load is



Therefore, the gate is triggered at 5V with a delay which approaches an exponential variation with time.

(g) From (d) Let $V_L = 0.9 V_s$, hence

$$0.9 V_s = 2V_1 \frac{1-\Gamma_g^{\,n}}{1-\Gamma_g} \Rightarrow \frac{1-\Gamma_g^{\,n}}{1-\Gamma_g} = K$$

where $K = \frac{0.9 V_s}{2V_1}$. Therefore, with $K = 9.45$

$$n = \frac{\log[1-K(1-\Gamma_g)]}{\log(\Gamma_g)} = 23$$

I.e. $n = 23$ and $t = T + 2T(22) = T = 45\,T = 45\,nS$!

Your fast logic becomes quite slow!