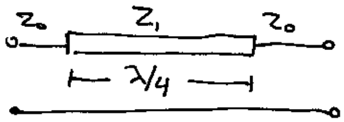
ECE424F - 2001: Homework #8

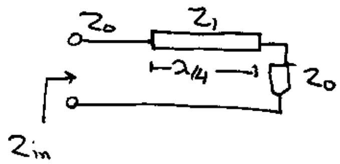
Question #1

Step #1:



$\rightarrow$ Find the S-parameters for this line.

S_{11}



$$Z_{in} = \frac{Z_1^2}{Z_o} \Rightarrow S_{11} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = \frac{Z_1^2/Z_o - Z_o}{Z_1^2/Z_o + Z_o} = \frac{Z_1^2 - Z_o^2}{Z_1^2 + Z_o^2}$$

$$\therefore \boxed{S_{11} = S_{22} = \frac{Z_1^2 - Z_o^2}{Z_1^2 + Z_o^2}}$$

S_{12}

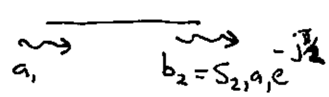
The line is lossless $\Rightarrow$ $\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* \\ S_{21}^* & S_{22}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\rightarrow S_{11} S_{12}^* + S_{12} S_{22}^* = 0$ But S_{11} is real $\Rightarrow$ $S_{12} + S_{12}^* = 0$

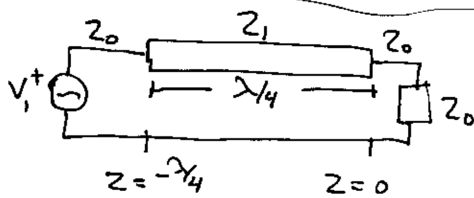
$\therefore S_{12} = S_{21}$ is imaginary

Also, $|S_{21}|^2 = 1 - |S_{11}|^2 = \dfrac{4 Z_1^2 Z_o^2}{(Z_o^2 + Z_1^2)^2}$ from the lossless condition

So $S_{21} = \pm j \dfrac{2 Z_o Z_1}{Z_o^2 + Z_1^2}$, choose the $-j$ branch so that when $Z_o = Z_1$ $S_{21} = -j = e^{-j\pi/2}$



OR: **S_{12}**



Voltage along Line $\ni$ $V(z) = V^+(e^{-j\beta z} + \Gamma_L e^{j\beta z})$

where $\Gamma_L = \dfrac{Z_o - Z_1}{Z_o + Z_1}$

① $V(z=0) = V^+(1 + \Gamma_L) = V_2^+ + V_2^- = V_2^-$ (Since $V_2^+ = 0$ due to matched port #2)

② $V(z = -\lambda/4) = V^+(j - j\Gamma_L) = V_1^+ + V_1^- = V_1^+(1 + S_{11})$ (since $S_{11} = \dfrac{V_1^-}{V_1^+}$)

Using ① + ② to solve for $S_{21} = \dfrac{V_2^-}{V_1^+} = (1 + S_{11}) \dfrac{(1 + \Gamma_L)}{j(1 - \Gamma_L)} = \left(\dfrac{2 Z_1^2}{Z_1^2 + Z_o^2}\right)\left(\dfrac{2 Z_o}{\cancel{Z_o + Z_1}}\right)\left(\dfrac{\cancel{Z_o + Z_1}}{2 Z_1}\right)\dfrac{1}{j}$

$$\therefore \boxed{S_{21} = \frac{-j 2 Z_1 Z_o}{Z_1^2 + Z_o^2}}$$

Steps #2/3:

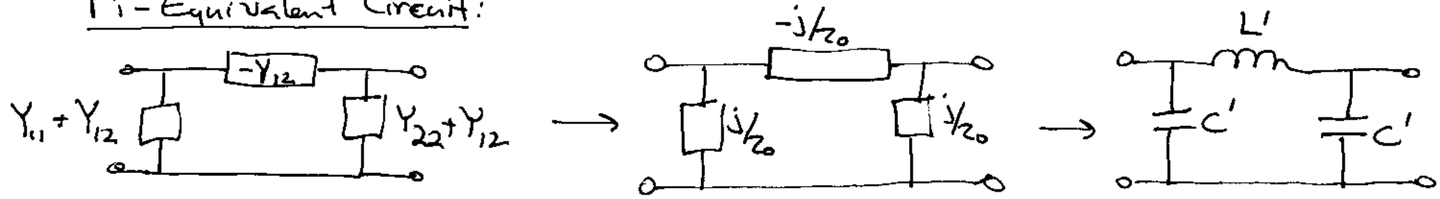
i) Convert S-matrix to Y-matrix for $Z_1 = Z_0$.

$$S = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} 0 & j/Z_0 \\ j/Z_0 & 0 \end{bmatrix}$$

$L' = Z_0/\omega$

$C' = \frac{1}{\omega Z_0}$ $\omega = 2\pi f$.

Pi-Equivalent Circuit:

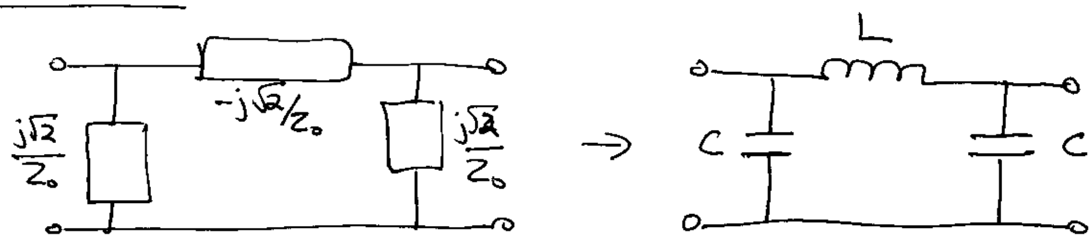


ii) Convert S-matrix to Y-matrix for $Z_1 = Z_0/\sqrt{2}$

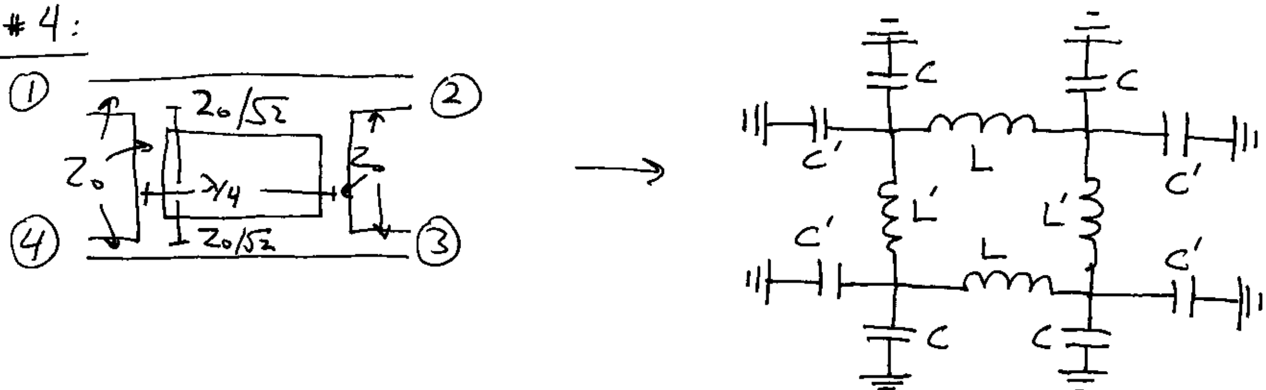
$$S = \begin{bmatrix} -1/3 & -j2\sqrt{2}/3 \\ -j2\sqrt{2}/3 & -1/3 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} 0 & j\sqrt{2}/Z_0 \\ j\sqrt{2}/Z_0 & 0 \end{bmatrix}$$

$L = \frac{Z_0}{\omega\sqrt{2}}$ $C = \frac{\sqrt{2}}{Z_0 \omega}$

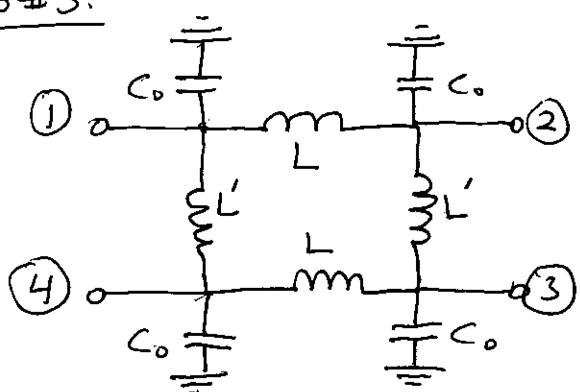
Pi-Equivalent Circuit



Step #4:



Step #5:



$C_0 = C + C'$

At $f = 1\ GHz$, $Z_0 = 50\ \Omega$

$L = Z_0/\sqrt{2}\omega_0 = 5.63\ nH$

$C = \frac{\sqrt{2}}{Z_0 \omega_0} = 4.5\ pF$

$L' = Z_0/\omega_0 = 7.96\ nH$

$C' = \frac{1}{\omega_0 Z_0} = 3.18\ pF$

7.18 :

$b = 0.32\,cm$, $\varepsilon_r = 2.2$, $Z_{oe} = 70\,\Omega$, $Z_{oo} = 40\,\Omega$

Hence $\sqrt{\varepsilon_r}\, Z_{oe} = 104\,\Omega$, $\sqrt{\varepsilon_r}\, Z_{oo} = 59\,\Omega$

From Figure 7.29

$$S/b = 0.075 \Rightarrow S = 0.24\,mm$$

$$W/b = 0.67 \Rightarrow W = 2.1\,mm$$

Problem 7.21 :

$C = 10^{-19.1/20} = 0.1109$, $f = 8\,GHz$, $Z_0 = 60\,\Omega$

$$Z_{oe} = Z_0 \sqrt{\frac{1+C}{1-C}} = 67.1\,\Omega, \quad Z_{oo} = Z_0 \sqrt{\frac{1-C}{1+C}} = 53.7\,\Omega$$

For a stripline having $\varepsilon_r = 2.2$, $b = 0.32\,cm$,

$\sqrt{\varepsilon_r}\, Z_{oe} = 99.5\,\Omega$ and $\sqrt{\varepsilon_r}\, Z_{oo} = 79.7\,\Omega$

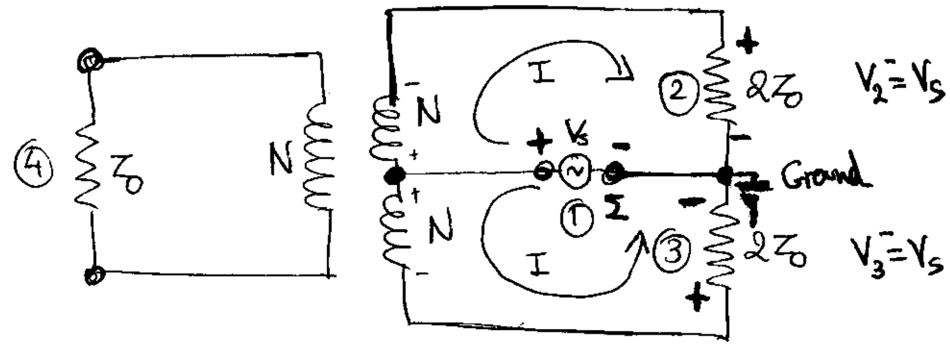
From Figure 7.29

$$S/b = 0.36 \Rightarrow S = 1.15\,mm$$

$$W/b = 0.60 \Rightarrow W = 1.92\,mm$$

Line length : $l = \frac{\lambda_g}{4} = \frac{c}{4\sqrt{\varepsilon_r} f} = 6.32\,mm$

Problem 7.28 :



1.) Assume exciting the input port ① while matching the rest ports to their characteristic impedance.

* Due to symmetry, the primary windings of the transformer are excited out-of-phase. Hence, the induced voltage on the secondary is zero and $V_4^- = 0$, i.e port ④ is isolated and $S_{41} = S_{14} = 0$.

* Since $I = V_s/2Z_0$, the voltages across ports ② and ③ are $V_2^- = V_s$ and $V_3^- = +V_s$ respectively.

* V_s sees the parallel combination of $2Z_0 // 2Z_0$, i.e $Z_{in}^{①} = Z_0$.
Hence $V_1^+ = V_s$, $V_1^- = 0$ and $S_{11} = 0$ (matched).

* In order to determine S_{21}, S_{31} we should be careful and take into account that ports ① and ②,③ are normalized with a different characteristic impedance. Thus, we should use generalized scattering parameters:

$$S_{21} = \frac{b_2}{a_1} = \frac{V_2^-/\sqrt{2Z_0}}{V_1^+/\sqrt{Z_0}} = \frac{V_2^-}{V_1^+\sqrt{2}}$$

Similarly, $S_{31} = \frac{V_3^-}{V_1^+}\frac{1}{\sqrt{2}}$

I.e $S_{21} = \dfrac{V_2^-}{V_1^+\sqrt{2}} = \dfrac{V_s}{V_s\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

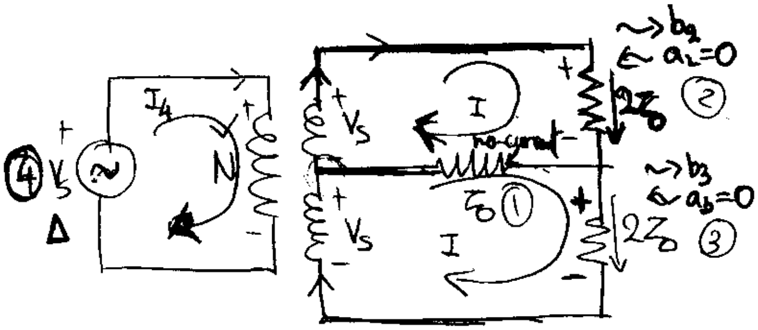
and $S_{31} = \dfrac{V_3^-}{V_1^+\sqrt{2}} = \dfrac{-V_s}{V_s\sqrt{2}} = +\,1/\sqrt{2}$

Finally, $S_{21} = S_{12} = 1/\sqrt{2}$

$S_{31} = S_{13} = +1/\sqrt{2}$ ✔

Note that in general, passive RLC circuits are **reciprocal**.

2) Assume now exciting port #4 while matching the rest ports:



Reference of V_3: $-$ / $+$

* In this case, port ① is excited in an **odd** fashion and <u>no current passes through Z_0!</u> Thus, $S_{14} = 0$ as expected.

* Again V_s sees the parallel combination of $2Z_0 // 2Z_0$ and therefore $Z_{in}^{④} = Z_0$, i.e $V_4^+ = V_s$, $V_4^- = 0$ and $S_{44} = 0$.

* $\left.\begin{aligned} V_2^- &= V_s = V_4^+ \\ V_3^- &= -V_s = -V_4^+ \end{aligned}\right\} \Rightarrow \begin{aligned} S_{24} &= 1/\sqrt{2} \\ S_{34} &= -1/\sqrt{2} \end{aligned}$

<u>**Also**</u>, $I_4 = 2I$ but $I = V_s/2Z_0 \Rightarrow$

$I_4 = \dfrac{2V_s}{2Z_0} = V_s/Z_0 \Rightarrow Z_{in}^{④} = Z_0$ and $S_{44} = 0$

$S_{24} = \dfrac{b_2}{a_4} = \dfrac{V_2^-/\sqrt{Z_0}}{V_4^+/\sqrt{Z_0}} = \dfrac{V_s}{V_s} \cdot \dfrac{1}{\sqrt{2}} = 1/\sqrt{2}$

Thus port ④ acts as the difference port, while port ① acts as the sum port.

For the remaining S-parameters we can use even and odd mode analysis for exciting ports ② and ③. However, we can just use the fact that our circuit is lossless:

$$|S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 + |S_{42}|^2 = 1$$

But $|S_{12}|^2 = |S_{42}|^2 = 1/2 \implies S_{22} = S_{32} = 0$

Similarly

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 + |S_{43}|^2 = 1$$

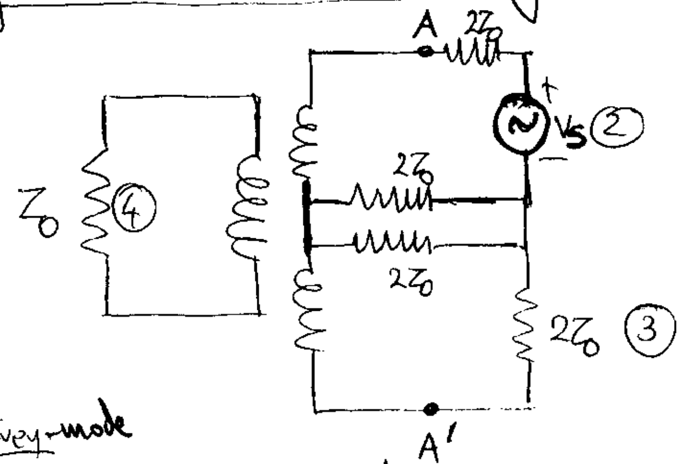
with $|S_{13}|^2 = |S_{43}|^2 = 1/2 \implies S_{23} = S_{33} = 0$

The conclusion is that the indicated network behaves like a 3dB, 180° Hybrid Coupler.

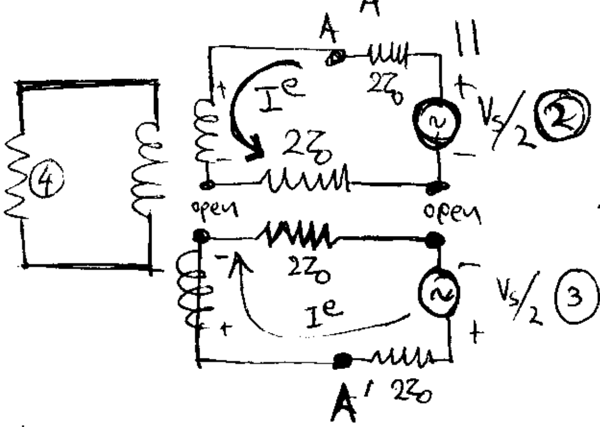
$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & +1 \\ 1 & 0 & 0 & -1 \\ 0 & +1 & -1 & 0 \end{bmatrix}$$

APPENDIX I

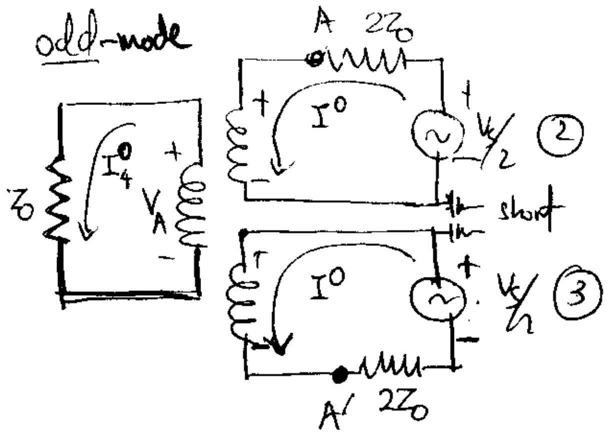
Using Even and Odd mode Analysis: (To determine S_{22} and S_{32})



Even-mode



odd-mode



Even Excitation:

(a) Port ④ is not excited as voltages on trans. primaries are out-of-phase.

(b) $I^e = \frac{V_s/2}{2(2Z_0)} = \frac{V_s}{8Z_0}$, $V_A^e = \frac{V_s/2}{2} = V_s/4$, also $V_A^e = V_s/4$

(c) Note that $Z_{in_A}^e = 2Z_0 \Rightarrow \Gamma_e = 0$ (matched)

Odd Excitation:

(a) $I_4^o = 2I^o$ and $V_A = I_4^o Z_0 = 2I^o Z_0 \Rightarrow I^o = \frac{V_A^o}{2Z_0}$

(b) $Z_{in_A}^o = \frac{V_A}{I^o} = 2Z_0 \Rightarrow \Gamma_o = 0$ (matched)

(c) Since $Z_{in_A}^o = 2Z_0 \Rightarrow$ from the voltage divider at ② and ③

$$V_A^o = V_s/4,\quad V_{A'}^o = -V_s/4 \Rightarrow I^o = \frac{V_s/4}{2Z_0} = \frac{V_s}{8Z_0}$$

Finally: ① $S_{22} = \frac{1}{2}(\Gamma_e+\Gamma_o) = 0$ or $Z_{in}^A = \frac{V_A^e + V_A^o}{I_e + I_o} = \frac{V_s/4 + V_s/4}{\frac{V_s}{8Z_0} + \frac{V_s}{8Z_0}} = \frac{V_s/2}{V_s/4Z_0} = 2Z_0$

$\Rightarrow S_{22} = \frac{Z_{in}^A - 2Z_0}{Z_{in}^A + 2Z_0} = 0 \Rightarrow \boxed{S_{22}=0}$

② $S_{32} = \frac{V_3^-}{V_2^+}$ But $V_3^- = V_{A'}^e + V_{A'}^o = \frac{V_s}{4} - \frac{V_s}{4} = 0 \Rightarrow \boxed{S_{32}=0}$