NEGATIVE-REFRACTIVE-INDEX TRANSMISSION-LINE METAMATERIALS AND ENABLING MICROWAVE DEVICES

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INTRODUCTION

Metamaterials are artificial periodic media with unusual electromagnetic properties. The size of their unit cells is much smaller than the incident electromagnetic wavelength, thus allowing the definition of effective material parameters such a permittivity ε, a permeability μ, and a refractive index n. In this paper, "left-handed" metamaterials are considered for which their permittivity and permeability are both negative. These metamaterials exhibit backward-wave propagation characteristics and therefore a negative index of refraction, as was predicted in the pioneering work of Victor Veselago [1]. Such "left-handed" or "Negative-Refractive-Index" (NRI) media were first implemented using bulk periodic arrays of thin wires to synthesize negative permittivity and split-ring resonators to synthesize a negative permeability [2]. A different approach for synthesizing 2-dimensional (2-D) NRI media in planar form was proposed in [3],[4]. The 2-D NRI structure presented in [3],[4] was realized by periodically loading a planar network of printed transmission lines (TL) with series capacitors and shunt inductors in a dual TL (high-pass) configuration, when compared to a conventional TL. The 2-D NRI medium was subsequently interfaced with a commensurate conventional dielectric, arguably leading to the first experimental demonstration of focusing from a NRI metamaterial [4],[5]. More recently, a 3-region lens arrangement was used to observe focusing beyond the diffraction limit [6]-[7] as was predicted by J.B. Pendry [8]. A similar TL approach was followed by Itoh and Caloz and led to interesting structures and useful circuits [9],[10]. Moreover, unique and intriguing anisotropic transmission-line metamaterials have been reported by Balmain in [11]. Some of the first FDTD simulations of NRI media have been reported in the pioneering work of R. Ziolkowski et al. in [12] and fascinating alternative focusing arrangements have been reported by Engheta et al. in [13].

This work presents various electromagnetic applications and devices that have been developed at the University of Toronto using Transmission-Line Negative-Refractive-Index (TL-NRI) metamaterials. A brief description of the effective-medium theory for TL-NRI media is presented first. This is followed by various TL-NRI applications and devices, including a super-resolving planar lens, a leaky backward-wave antenna radiating its fundamental spatial harmonic, compact broadband antenna feed-networks and a coupled-line coupler with co-directional phase but contra-directional power flow.

EFFECTIVE-MEDIUM THEORY OF NEGATIVE-REFRACTIVE-INDEX METAMATERIALS

A practical, periodic 2-D transmission-line based NRI metamaterial can be realized using an array of unit cells, each as depicted in Fig. **1.** A host transmission-line medium (e.g. microstrip) is periodically loaded using discrete series capacitors and shunt inductors [3]-[5]. When the unit cell dimension d is much smaller than a guided wavelength, the array can be regarded as a homogeneous, effective medium, and as such can be described by effective constitutive parameters $\mu_N(\omega)$ and $\varepsilon_N(\omega)$. The material parameters are determined through a rigorous periodic analysis to be of the form [5]:

$$\varepsilon_N(\omega) = \varepsilon_P - \frac{g}{\omega^2 L_0 d}, \qquad \mu_N(\omega) = \mu_P - \frac{1/g}{\omega^2 C_0 d} \tag{1}$$

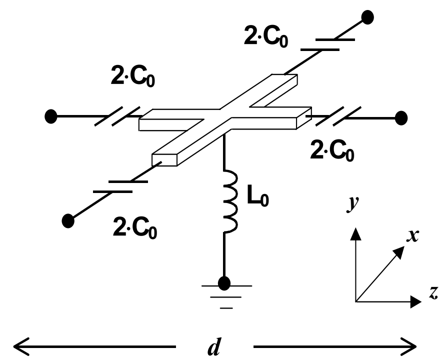

Figure 1: Unit cell for the 2-D TL-based NRI metamaterial

Here, ε_P and μ_P are positive constants describing the host transmission line medium, and it is clear that this particular arrangement of the inclusions L_0 and C_0 provides the desired negative contribution that diminishes with frequency ω. The geometrical factor g relates the characteristic impedance of the transmission line network to the wave impedance of the effective medium. When the parameters are simultaneously negative, these structures exhibit a negative effective refractive index and have experimentally demonstrated the predicted associated phenomena, including negative refraction, focusing, and focusing with sub-wavelength resolution [3]-[6].

In practical realizations, the sub-wavelength unit cell of Fig. 1 is repeated to synthesize artificial 2-D materials with overall dimensions that are larger than the incident electromagnetic wavelength. Therefore, the resulting structures are by definition distributed. However, the loading lumped elements could be realized either in chip [4,5] or in printed form [14],[15],[16],[17].

NEGATIVE REFRACTION

Veselago was the first to report in the open literature the feasibility of media characterized by simultaneously negative permittivity and permeability [1]. He concluded that such media are allowed by Maxwell's equations and plane waves propagating in them would have their electric field, $\overline{\boldsymbol{E}}$, magnetic field, $\overline{\boldsymbol{H}}$, and propagation constant , $\overline{\boldsymbol{k}}$, forming a left-handed triplet. Therefore he coined the term "left-handed" to describe these hypothetical media. Also Veselago realized that one has to choose the negative branch of the square root to properly define the corresponding refractive index, i.e. $\boldsymbol{n} = -\sqrt{\varepsilon\mu}$. Thus, such left-handed media support negative refraction of electromagnetic waves, something that was demonstrated experimentally more than three decades later by Shelby, Smith and Schultz [2]. Moreover, due to the fact that $\overline{\boldsymbol{E}}$, $\overline{\boldsymbol{H}}$, and , $\overline{\boldsymbol{k}}$, form a left-handed triplet whereas the $\overline{\boldsymbol{E}}$, $\overline{\boldsymbol{H}}$ vectors and the Poynting vector $\overline{\boldsymbol{S}}$ form a right-handed triplet, Veselago concluded that in left-handed media the propagation constant $\overline{\boldsymbol{k}}$ is anti-parallel to the Poynting vector $\overline{\boldsymbol{S}}$. In retrospect, what Veselago was describing were backward waves. Certainly, one-dimensional backward-wave lines are not new to the microwave community and there is an interesting connection to familiar concepts and structures [18]. However what is remarkable and surprising in Veselago's work is his realization that 2- or 3-dimensional isotropic and homogeneous media supporting backward waves should be characterized by a negative refractive index. Consequently when such media are interfaced with conventional dielectrics, Snell's law is reversed leading to the negative refraction of an incident electromagnetic plane wave. One way to understand negative refraction is through the notion of phase matching as is explained in Fig. 2. Since Snell's law is a manifestation of phase matching of the transverse propagation vector at the interface between two dielectrics, Fig. 2 readily suggests that the left-handed medium should be characterized by a negative refractive index.

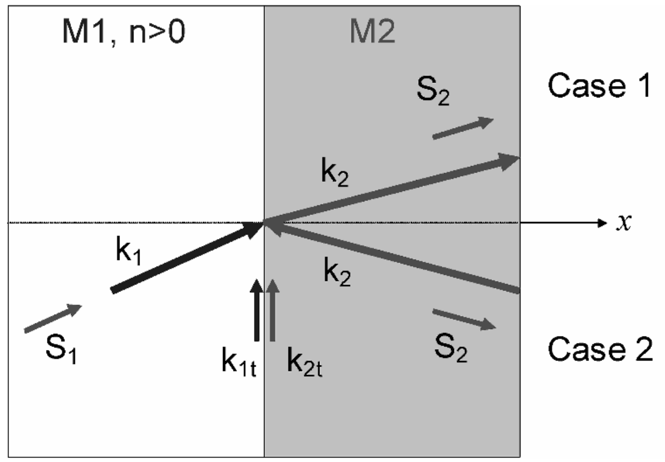

Figure 2: Negative refraction of a plane wave incident from a regular dielectric to another regular dielectric (Case 1) or a negative-refractive-index medium (Case 2). The arrows on the rays represent the propagation vectors; observe the underlying phase matching of the tangential components of these vectors in Case 2. Another implied principle is that the Poynting Vector S should point away from the interface in the second medium [4].

Harnessing the phenomenon of negative refraction, entirely new refractive devices can be envisioned such as a flat lens without an optical axis also proposed by Veselago as shown in Fig. 3.

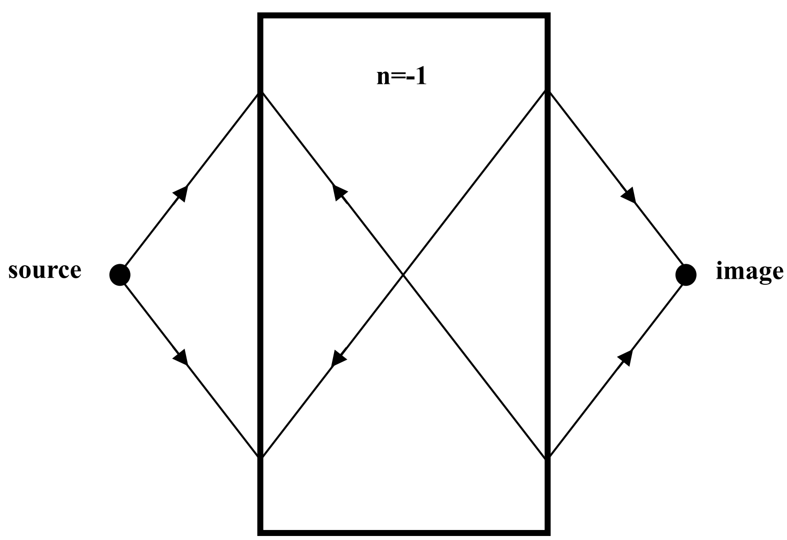

Figure 3: Veselago's lens. As shown, negative refraction is utilized in order to focus a point to a point. This leads to a lens with flat surfaces and no optical axis. The rays converge to the same point when n=-1 thus leading to aberration free focusing and no reflections from the lens surfaces. The thickness of the lens is half the distance from the source to the image.

A SUPER-RESOLVING NEGATIVE-REFRACTIVE-INDEX TRANSMISSION-LINE LENS

A picture of a planar version of Veselago's lens is shown in 4. The NRI lens is a slab consisting of a 5X19 grid of printed microstrip strips, loaded with series capacitors (C_0) and shunt inductors (L_0). This NRI slab is sandwiched between two unloaded printed grids that act as homogeneous media with a positive index of refraction. The first unloaded grid is excited with a monopole (point source), which is imaged by the NRI lens to the second unloaded grid. The vertical electric field over the entire structure is measured using a detecting probe (for details, see [6]).

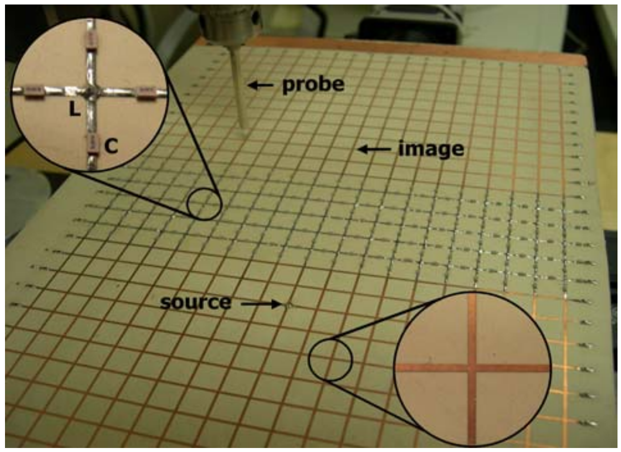

Figure 4: Photograph of a planar super-resolving lens [6]

The measured half-power beamwidth of the point-source image at 1.057 GHz is 0.21 effective wavelengths, which is appreciably narrower than that of the diffraction-limited image corresponding to 0.36 wavelengths (see Figure 5). The physical mechanism behind this sub-wavelength focusing relies on the enhancement of evanescent waves predicted by J.B. Pendry [8] and demonstrated for the specific structure under consideration in [19]. Figure 5 shows the measured vertical electric field above the central row of the lens, which verifies the exponential growth of the fields inside the NRI medium [6]. Since there is some controversy regarding losses in NRI metamaterials, we hereby report that the loss tangent of the NRI medium at 1.05 GHz is estimated to be $\tan(\delta) = 0.062$ which attests to the low-loss nature of the NRI transmission-line lens. Moreover, the super-resolving imaging properties of the structure shown in Fig 5 have been theoretically investigated by means of a rigorous periodic Green's function analysis in [6],[7]. The results of Figure 5 imply that evanescent waves corresponding to transverse wavenumbers upto 3Ko participate in the formation of the image. Therefore the lens of Figure 4 offers three times higher resolution than a commensurate conventional lens [6].

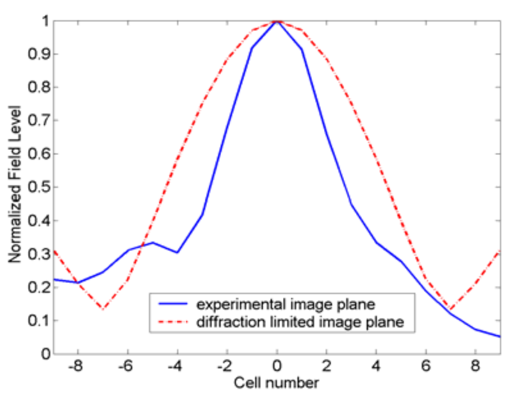

Figure 5: Experimental verification of sub-wavelength focusing

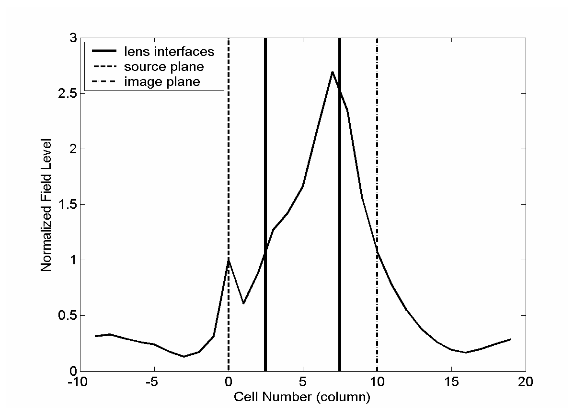

Figure 6: Experimental verification of growing evanescent waves in a super-resolving TL-NRI lens

A LEAKY-WAVE BACKWARD ANTENNA RADIATING ITS FUNDAMENTAL SPATIAL HARMONIC

The transmission-line (TL) approach to synthesizing NRI metamaterials has led to the development of a new kind of leaky-wave antenna (LWA). By appropriately choosing the circuit parameters of the dual TL model, a fast-wave structure can be designed that supports a fundamental spatial harmonic which radiates toward the backward direction [21].

The Co-Planar Waveguide (CPW) implementation of the leaky-wave antenna is shown in Fig. 7. The gaps in the CPW feedline serve as the series capacitors of the dual TL model, while the narrow lines connecting the centre conductor to the coplanar ground planes serve as the shunt inductors. The capacitive gaps are the radiating elements in this leaky-wave antenna, and excite a radiating Transverse Magnetic (TM) wave. Due to the anti-parallel currents flowing on each pair of narrow inductive lines, the shunt inductors remain non-radiating. Simulated and experimental results for this bi-directional leaky-wave antenna were reported in [15],[21]. Simulation results for a unidirectional LWA design were also presented in [21]. The unidirectional design is simply the leaky-wave antenna described in [15] backed by a long metallic trough as shown in Fig. 8. Since the LWA's transverse dimension is electrically small, the backing trough can be narrow (below resonance). The trough used is a quarter wavelength in height and width and covers the entire length of the antenna on the conductor side of the substrate. It acts as a waveguide below cut-off and recovers the back radiation, resulting in unidirectional far-field patterns.

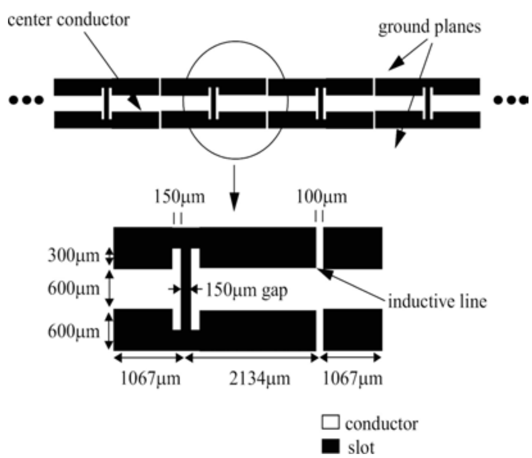

Figure 7: Backward leaky-wave antenna based on the dual TL model [15],[21]

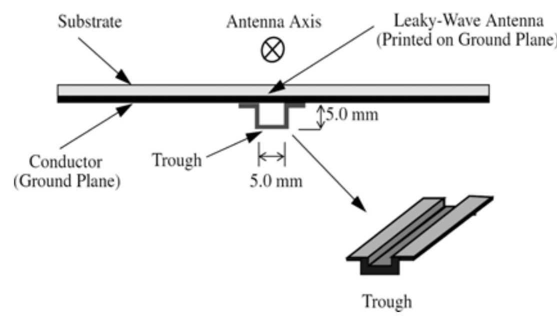

Figure 8: Unidirectional backward leaky-wave antenna design

A complementary forward unidirectional leaky-wave antenna, also radiating its fundamental spatial harmonic, is described in [22]. Here, we report experimental results for the unidirectional design proposed in [21]. As noted in [21], a frequency shift of 3%, or 400 MHz, was observed in the experiments compared to the method of moments simulations of the LWA using Agilent's Advanced Design System. As a result, the experimental unidirectional radiation patterns are shown at 14.6 GHz while the simulation patterns are shown at 15 GHz. The E-plane and H-plane patterns are shown in Fig. 9 and Fig. 10 respectively. A gain improvement of 2.8 dB was observed for the unidirectional design over the bi-directional design, indicating that effectively all of the back radiation is recovered with the trough.

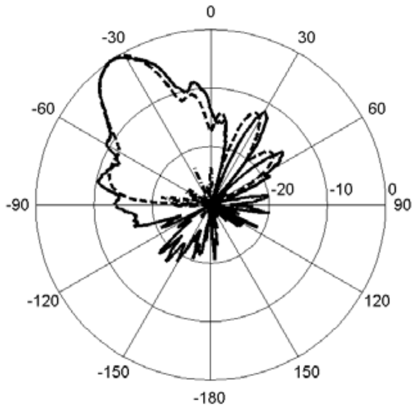

Figure 9: E-Plane Pattern for the Unidirectional Leaky-Wave Antenna at 15 GHz. (–) Experimental co-polarization, (-··-) Experimental cross-polarization, (---) Simulated co-polarization using Agilent ADS

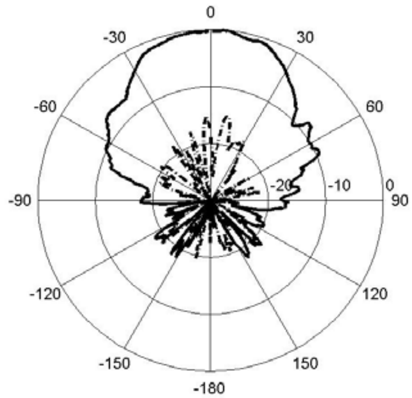

Figure 10: H-Plane Pattern for the Unidirectional Leaky-Wave Antenna at 15 GHz. (–) Experimental co-polarization, (-··-) Experimental cross-polarization, (---) Simulated co-polarization using Agilent ADS

COMPACT & BROADBAND METAMATERIAL ANTENNA FEED-NETWORKS

The 1-Dimensional (1-D) metamaterial phase-shifting lines presented in [23] can be used to develop compact and broadband, non-radiating, metamaterial feed-networks for antenna arrays. These can be used to replace conventional TL-based feed-networks, which can be bulky and narrowband. For series-fed arrays, the proposed metamaterial feed-networks have the advantage of being compact in size, therefore eliminating the need for conventional TL meander lines. In addition, the metamaterial feed-networks are more broadband when compared to conventional TL feed-networks, which enables series-fed broadside arrays to experience less beam squint when operated away from the design frequency.

In a typical series-fed linear array designed to radiate at broadside, the antenna elements must be fed in phase. In addition, an inter-element spacing d_E of less than a half a free-space wavelength ($d_E < \lambda_o/2$) is necessary to avoid capturing grating lobes in the visible region of the array pattern. In order to achieve these design constraints, traditional designs employing TL-based feed-networks have resorted to a meander-line approach, as shown in Fig. 11. This allows the antenna elements to be physically separated by a distance of $d_E = \lambda_o/2$, while still being fed in phase with a one guided-wavelength λ_g long meandered line that incurs a phase of -2π rad. Because the phase incurred by the TLs is frequency dependent, a change in the operating frequency will cause the emerging beam to squint from broadside, which is generally an undesirable phenomenon. In addition, the fact that the lines are meandered causes the radiation pattern to experience high cross-polarization levels, particularly in Co-Planar Waveguide (CPW) implementations, as a result of parasitic radiation due to scattering from the corners of the meandered lines [24]. The proposed feed-networks employ non-radiating metamaterial phase-shifting lines within a series-fed linear array (see Fig. 12) to mitigate some of the problems encountered with conventional TL-based feed-networks.

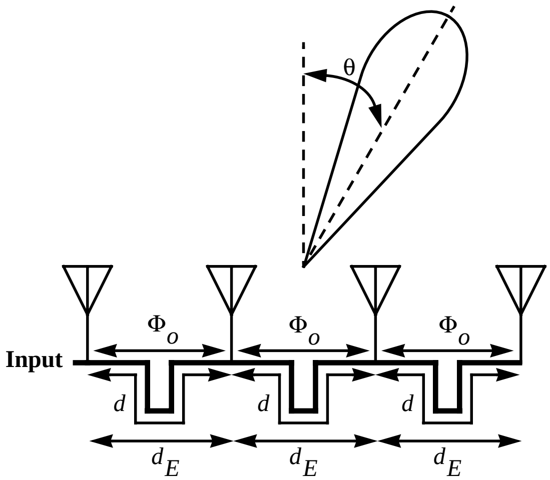

Figure 11: Series-fed linear array using conventional -2π transmission line meandered feed lines

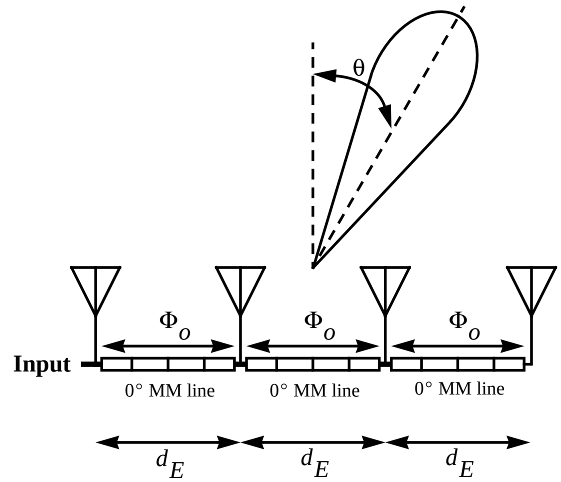

Figure 12: Series-fed linear array using 0^o metamaterial (MM) feed lines

The phase-shifting lines presented in [23] and whose unit cell is shown in Fig. 13 [4], can incur an arbitrary insertion phase, are compact in size and exhibit a linear, flatter phase response with frequency compared to conventional TL delay lines. In order to ensure that the phase-shifting lines do not radiate, they can be operated in the NRI backward-wave region, while simultaneously ensuring that the propagation constant of the line exceeds that of free space. This will effectively produce a slow-wave structure with a positive insertion phase, Φ_{MM} . By cascading this with a conventional TL, that inherently incurs a negative insertion phase, Φ_{TL2} , this results in a combined slow-wave metamaterial phase-shifting line, as shown in Fig. 14.

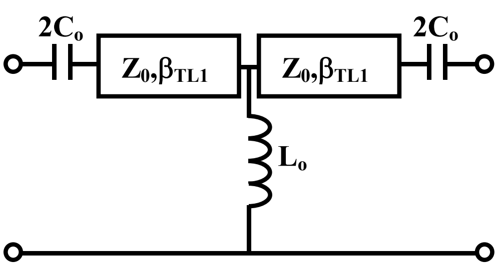

Figure 13: 1-D metamaterial phase-shifting unit cell

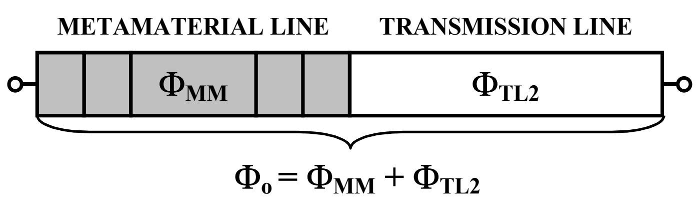

Figure 14: A combined slow-wave metamaterial phase-shifting line

If Φ_{MM} and Φ_{TL2} are equal but opposite in value, then the structure will incur a zero insertion phase, given by $\Phi_o = \Phi_{MM} + \Phi_{TL2} = 0$. The metamaterial phase-shifting lines can incur a positive, negative or zero insertion phase, by adjusting the values of the loading elements C_o and L_o. Thus, for a given section of TL with intrinsic phase shift $\Phi_{TL1} = \omega\sqrt{LC}d_{TL1}$ and characteristic impedance Z_o, the phase shift for an n-stage metamaterial line is given by (2), subject to the impedance matching condition of (3):

$$\Phi_{MM} = n\left(\omega\sqrt{LC}d_{TL1} + \frac{-1}{\omega\sqrt{L_oC_o}}\right) \qquad (2)$$

$$Z_o = \sqrt{\frac{L_o}{C_o}} = \sqrt{\frac{L}{C}} \qquad (3)$$

Assuming that the same type of TL sections are used for $TL1$ and $TL2$, then Z_o, L and C will be the same for both lines. Therefore, Φ_{TL2} is given by $\Phi_{TL2} = \omega\sqrt{LC}d_{TL2}$. Correspondingly, for a transmission line of length λ_g, the phase as a function of frequency is given by $\Phi_{\lambda_g} = \omega\sqrt{LC}\lambda_g$. The scan angle for each of the metamaterial-based and TL-based linear arrays with an inter-element phase shift Φ_o can therefore be written as:

$$\theta_{SCAN,MM} = \sin^{-1}\left(-\frac{\Phi_o}{k_o d_E}\right) = \sin^{-1}\left(-\frac{\Phi_{MM} + \Phi_{TL2}}{k_o d_E}\right) \quad (4) \qquad \theta_{SCAN,TL} = \sin^{-1}\left(-\frac{\Phi_o}{k_o d_E}\right) = \sin^{-1}\left(-\frac{\Phi_{\lambda_g}}{k_o d_E}\right) \quad (5)$$

The metamaterial-based and TL-based feed networks were evaluated in CPW technology at a design frequency of 2 GHz. Two designs were considered: an array with an inter-element spacing of $d_E = \lambda_o/2$ and an array with a spacing of $d_E = \lambda_o/4$. The corresponding scan-angle characteristics for the metamaterial-based and TL-based linear arrays with $d_E = \lambda_o/2$ are shown in Figure 15. It can be observed that the scan angle for the TL-fed array exhibits its full scanning range from +90° to -90° within a bandwidth of 2.67 GHz, while the corresponding scanning bandwidth for the metamaterial-fed array is 4.27 GHz. Thus, the metamaterial-fed array offers a more broadband scan angle characteristic, while simultaneously eliminating the need for meander lines. Also shown in Fig. 15 is the scan-angle characteristic for a low-pass loaded TL also of length $\lambda_o/2$. It can be observed that the performance of this line is identical to that of the TL feed line (the tow curves are on top of each other in Fig. 15). Thus, although the loaded line can eliminate the need for meander lines, it does not provide the advantage of an increased scan-angle bandwidth that the metamaterial feed lines offer.

The scan-angle characteristics for the $\lambda_o/4$ feed-network are shown in Fig. 16. It can be observed that the bandwidth of the scanning angle for the TL-fed array and the loaded TL array decreases to 1.07 GHz, while the corresponding scanning bandwidth for the metamaterial-fed array remains at 4.27 GHz. Thus, as the spacing between the antenna elements decreases, the scan-angle characteristic for a metamaterial-fed array remains constant, while the corresponding scan-angle characteristic for the TL-fed array becomes more narrowband.

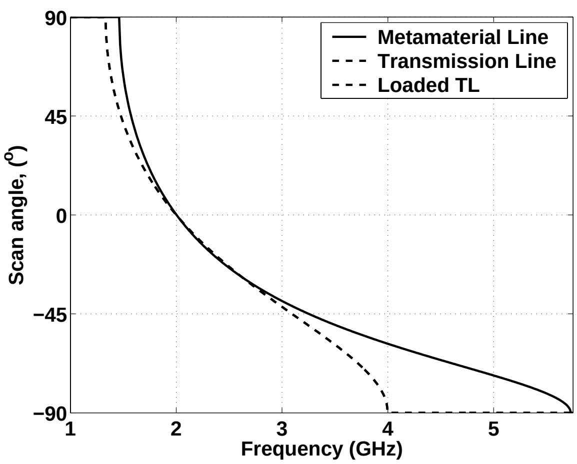

Figure 15: Scan angle performance of a series-fed linear array with $d_E=\lambda_o/2$ using different feeding techniques

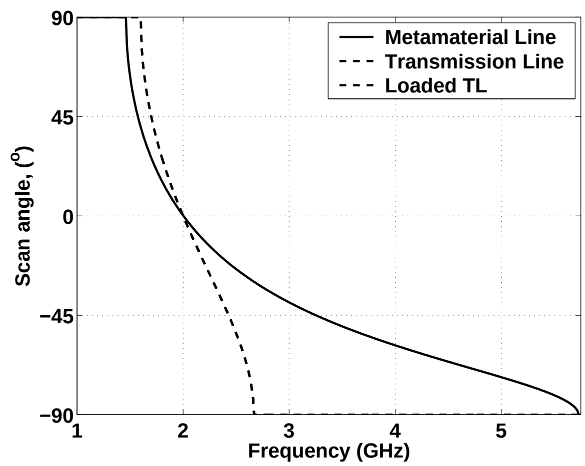

Figure 16: Scan angle performance of a series-fed linear array with $d_E=\lambda_o/4$ using different feeding techniques

A PECULIAR NRI/MICROSTRIP COUPLED-LINE COUPLER HAVING CO-DIRECTIONAL PHASE BUT CONTRADIRECTIONAL POWER FLOW

A peculiar coupled-line coupler (see Fig. 17) can be realized using a regular microstrip (MS) line that is edge-coupled to a negative-refractive-index (NRI) line [25]. Such a coupler exhibits co-directional phase but contra-directional Poynting vectors on the lines, thus leading to backward power coupling.

Using coupled-mode theory, it can be shown that coupled-modes with complex propagation constant are excited in this coupler at the frequency where the propagation constants of the two isolated lines become equal [26]. For a sufficiently long coupler operated at this frequency, the exponentially increasing modes can be discarded and line voltage/current expressions take the following form:

$$\begin{pmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ j & -j \\ 1/Z & 1/(-Z^*) \\ j/(-Z) & -j/Z^* \end{pmatrix} \begin{pmatrix} V_c^+ e^{-\gamma_c z} \\ V_\pi^+ e^{-\gamma_\pi z} \end{pmatrix} \tag{6}$$

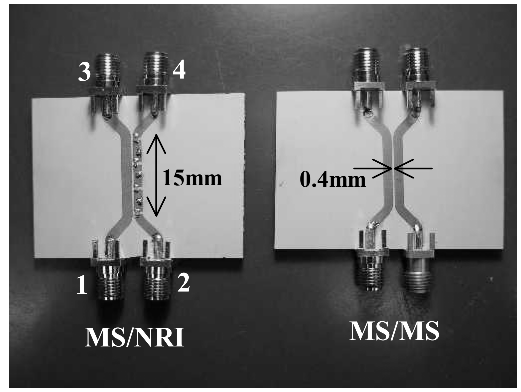

Figure 17: MS/NRI and MS/MS (λ/4) couplers of equal length, line spacing and propagation constants designed for operation at 2.8 GHz

Here, γ_c and γ_π are the complex-conjugate eigenmodes, $\gamma_c=\alpha+j\beta$ and Z is the impedance of the symmetric π-mode on the MS line. If port 1 (see Fig. 17) is excited, from (6) it can be shown that:

$$\frac{1}{2}\mathrm{Re}(V_1 I_1^*\big|_{z=0}) = \frac{\mathrm{Re}(Z)}{2|Z|^2}\left(|V_c|^2 - |V_\pi|^2\right) = -\frac{1}{2}\mathrm{Re}(V_2 I_2^*\big|_{z=0}) \tag{7}$$

Equation (7) demonstrates that there is complete backward transfer of power from port 1 to port 2 (see fig. 17). In order to compare the performance of a MS/NRI coupler to its ordinary counterpart of equal length, line spacing and propagation constant, a benchmark coupler was designed and is shown in Fig. 17. This benchmark MS/NRI coupler was constructed with unit cells 5mm long and loading elements of 2.7 nH shunt inductors and 0.9 pF series capacitors for the NRI line. The line widths are 2.45 mm (MS) and 2 mm (NRI) and the transverse line separation is 0.4 mm. The performance of this 3-unit-cell MS/NRI coupler is compared to that of an ordinary quarter-wavelength MS/MS coupler at 2.8 GHz when the isolated propagation constants of the two lines are similar (see fig. 17). From Fig. 18 and Fig. 19 it is evident that compared to its ordinary counterpart of the same length and line spacing, this new MS/NRI coupler exhibits better performance in terms of coupled power (higher coupling), return loss and isolation without any bandwidth degradation or significant change in insertion loss. A 3dB MS/NRI coupler has been presented in [26]. Moreover, the theory of a waveguide version of this coupler is reported in [27]. On the other hand, a different backward-wave coupler comprising two negative-refractive-index lines has been described in [28].

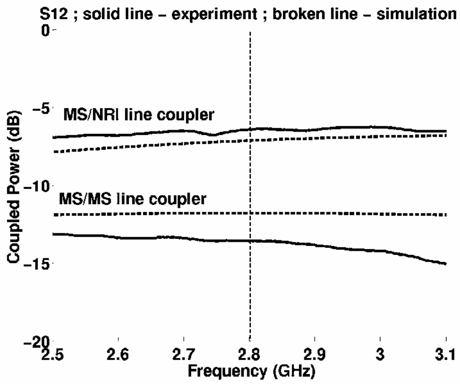

Figure 18: Comparison of coupled power levels for the MS/NRI and the MS/MS couplers

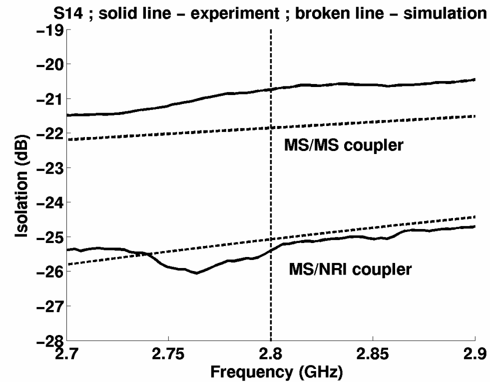

Figure 19: Comparison of isolation for the MS/NRI and the MS/MS couplers

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