UNIVERSITY OF TORONTO
Department of Electrical and Computer Engineering
Fields and Waves Laboratory
Courses ECE 320F and ECE 357S
III Year

RESONANT CAVITY

1. Object

The purpose of the experiment is to investigate properties of electromagnetic resonant cavity consisting of a section of coaxial transmission line.

2. References

(1) Your lecture notes.
(2) S. Ramo, J.R. Whinnery, T. VanDuzer, "Fields and Waves in Communication Electronics," Sections 5.13, 5.14.
(3) "Supplementary Instructions" for this experiment.

3. Resonant Cavity

The object of this experiment is to investigate properties of electromagnetic cavities in the vicinity of resonant frequencies, and to illustrate some general properties of resonant systems.

A dominant feature of a resonant system is its ability to store AC energy. In the case of an electromagnetic system the energy comprises electric and magnetic energies. Each of these components varies with time and to enable the system to store energy effectively it must contain components capable of storing electric and magnetic energies so that time variation of one type of energy can be accompanied by compensating variation of the other type. Thus in lumped component resonant circuit the energy of the system shuttles between the capacitor, electric energy storage, and the inductor magnetic energy storage elements respectively. If the compensating cooperation of the capacitor and inductor be effective the phases of the voltage, carrier of electric energy, and current, its magnetic counterpart must differ by 90°. This can be accomplished at a particular frequency only, the resonant frequency.

In resonant cavities the interplay between electric and magnetic energies depends on the spatial patterns of distributions of electric and magnetic fields. For the case of transmission line cavities the distribution of these fields can be described in terms of standing wave patterns of voltage and current, which are spatially displaced by quarter of wavelength and by 90° phase difference in time domain.

The cavity configuration which will be employed in the experiment is a transmission line cavity consisting of a short-circuited section of a transmission line partially isolated from the outside space (input line) by a high value susceptance jB approximating a short circuit as shown in Fig. 1.

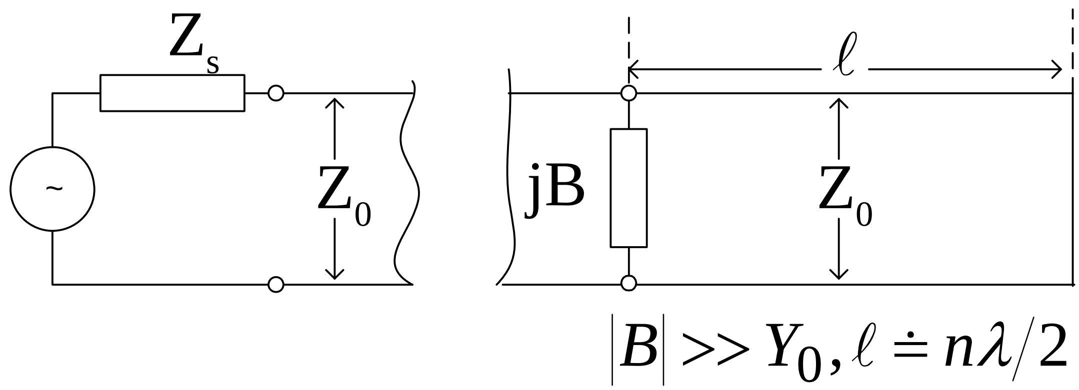


Figure 1. Transmission line cavity.

For the common configuration in which the value $|B|$ of the iris susceptance is much larger than the characteristic admittance Y_0 the effect of impedance Z connected in parallel to jB is negligible and the signal arriving from the source will effectively encounter only the high input (iris) susceptance approximating a short circuit and will be reflected back toward the generator.

If, however the length ℓ approximates integral multiple of half wavelength the impedance Z becomes small $\left(Z = jZ_0 \tan\beta\ell \doteq 0\right)$ for $\ell \doteq n\,\lambda/2$ and, being capable of changing sign may affect the value of the parallel combination of jB and itself, presenting thereby the incoming signal with impedance which will enable power to be delivered past the barrier formed by the input susceptance.

Rigorous description of the phenomenon is based on the analysis of the behaviour of the cavity input impedance. In as much as the impedances in transmission line systems depend on the impedance terminals positions it is convenient to select a suitable pair of terminals in a manner shown below and discussed in detail in the “Electromagnetic Cavity” supplement.

The positions of suitable terminals and relationship between input and load impedances Z_{in} and Z_2 are defined in Fig. 2. Proof of the relationships is given in the Appendix of the above mentioned supplement.

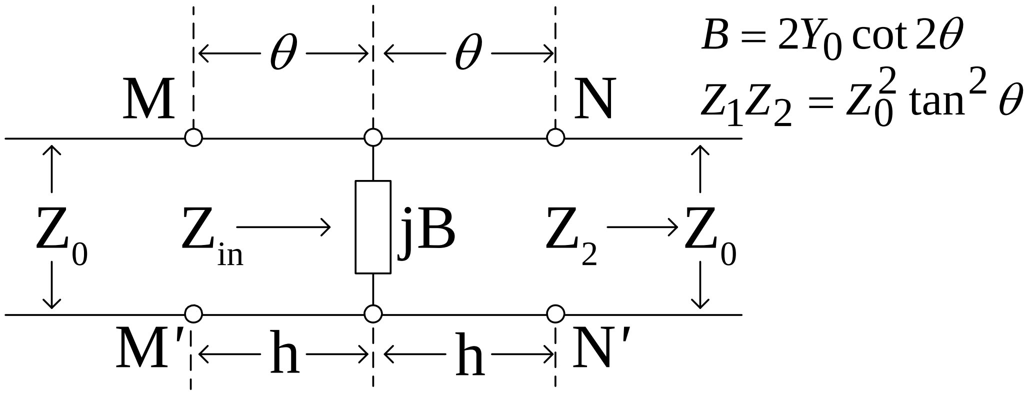


Figure 2. Inverter circuit

With the aid of inverter circuit the circuit of Fig. 1 representing a cavity can be reduced to the form shown in Fig. 3.

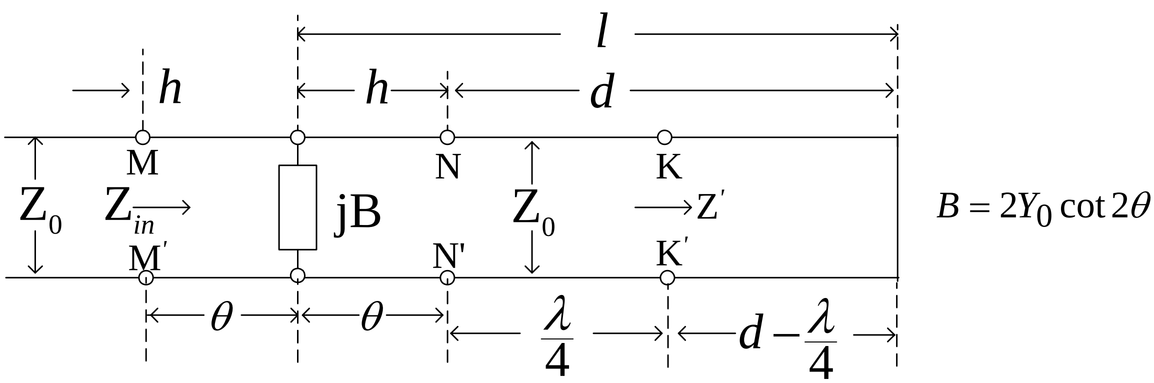


Figure 3. Transmission line cavity

It is convenient to designate the term $\cot\theta$ in Fig. 2 by letter *n*. The expression for the input impedance Z_{in} in Fig. 3 then becomes

$$Z_{in} = \frac{Z_0^2}{n^2} \frac{1}{Z} \tag{1}$$

An additional insight into properties of the circuit can be derived by considering the impedance Z′ at terminals K K′ located quarter wavelength away from terminal N N′ as shown in Fig. 3.

If one applies quarter wave transformer relation $Z = Z_0^2 / Z'$ to Equation 1 the expression for the input impedance Z_{in} (at the chosen terminals M M′) becomes

$$Z_{in} = \frac{1}{n^2} Z' \tag{2}$$

simulating the relationship between the primary and secondary impedances of an ideal transformer. The behaviour of resonant circuit of Fig. 3 can be simulated by the behaviour of an equivalent circuit shown in Fig. 4.

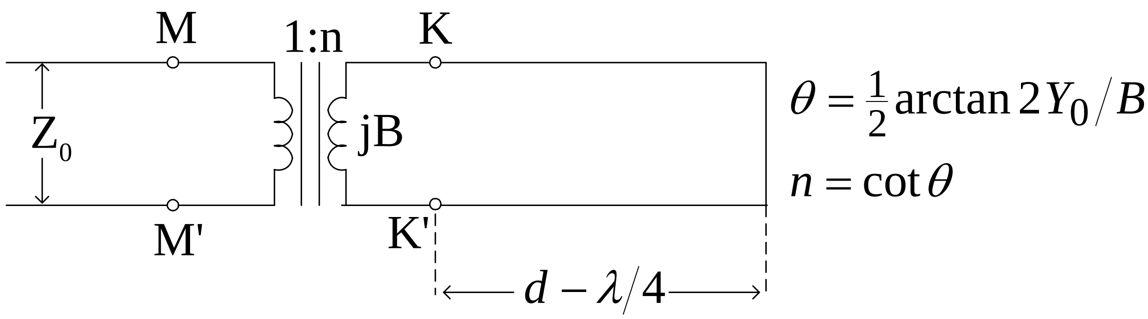


Figure 4. Equivalent circuit of a resonant cavity.

It was mentioned earlier that the susceptance B was chosen approximately to simulate a short circuit and its absolute values therefore is larger than Y_0. It is evident from the definition of θ ($2\cot 2\theta = B/Y_0$) that θ is a small number. As a consequence the turns ratio n ($n = \cot\theta$) is large. The input impedance Z_{in} is therefore the impedance of the $d-\lambda/4$ stub (see Fig. 4) seen through a step down transformer and unless the impedance of the stub is large the input impedance is small, simulating approximately a short circuit. The input impedance will acquire significant values only when the transformer secondary impedance Z' is large.

High value of impedance Z' occurs when the length of the stub is equal to, or approximates $\lambda/4$. Referring back to the circuit of Fig. 2 it is apparent that this condition obtains when the length d is equal or close to an integral multiple of half wavelength $\lambda/2$.

The observation that resonance frequency is governed by effective length d of the cavity rather than by the physical length ℓ of the cavity is due to the fact that iris susceptance B is not exactly a short circuit. The effect is commonly designated by the expression “frequency pulling” by the iris reactance.

A typical distribution of voltage in the cavity at resonant frequency is shown in Fig. 5.

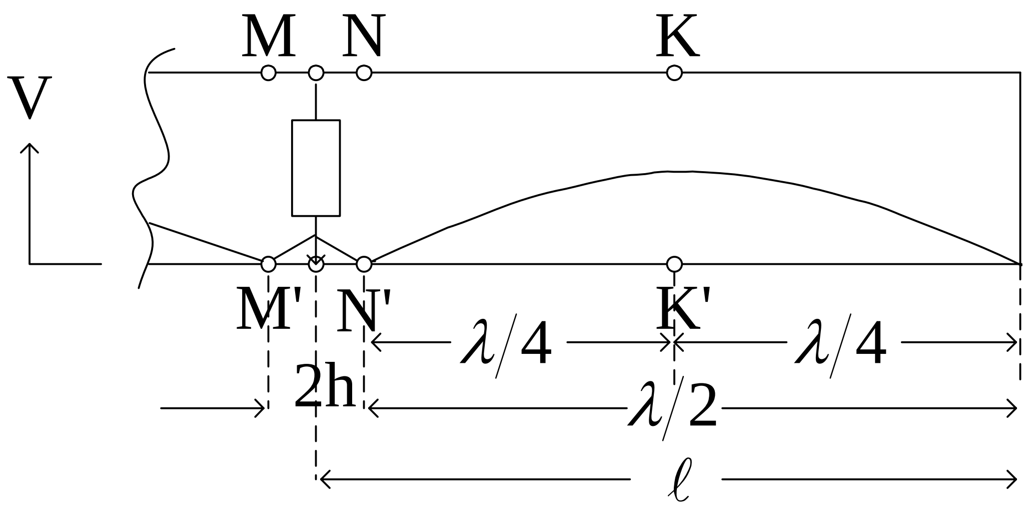


Figure 5. Voltage pattern in a cavity at resonance.

The expression for the input impedance Z_{in} can be derived by utilizing cavity equivalent circuit shown in Fig. 5 and by invoking impedance transformation relationship of ideal transformer as applied to impedance of a short-circuited line section. Thus

$$Z_{in} = Z'/n^2 = j\frac{Z_0}{n^2}\tan\beta(d-\lambda/4) = -j\frac{Z_0}{n^2}\beta d, \tag{3}$$

with $\beta = 2\pi/\lambda = \omega/u$.

At resonant frequency ω_0 for which $d = \lambda_0/2$ the value of $\omega_0\, d/u$ is π.

Thus for frequency ω,

$$\tan(\omega d/u) = \tan\left[\left(\frac{\omega-\omega_0}{u}\right)d + \pi\right] = \tan\left(\frac{\omega-\omega_0}{u}\right)d \tag{4}$$

For frequencies ω in the vicinity of ω_0 the dominant term in the series expansion of $\cot \beta d$ is $1\Big/\left(\frac{\omega-\omega_0}{u}\right)d$ so that the expression for Z_{in} in Equation 3 becomes

$$Z_{in} = -j\frac{Z_0}{n^2}\Big/\frac{(\omega-\omega_0)}{u}d = \frac{2}{\pi n^2}\Big/\left[Y_0 2\frac{(\omega-\omega_0)}{\omega_0}j\right]. \tag{5}$$

One observes that the term $1/Y_0 \dfrac{2(\omega-\omega_0)}{\omega_0} j$ resembles input impedance $1\Big/\left[\omega_0 C \dfrac{2(\omega-\omega_0)}{\omega_0} j\right]$ of an L, C parallel resonant circuit as discussed in the "Resonance" supplement.

The above discussion allows one to introduce a lumped components equivalent circuit for the resonant cavity valid, albeit only for frequencies close to resonance, as shown in Fig. 6.

$$1:\left(\frac{\pi n^2}{2}\right)^{1/2}$$

$$n = \cot\theta$$

$$\omega_0^2 = 1/LC$$

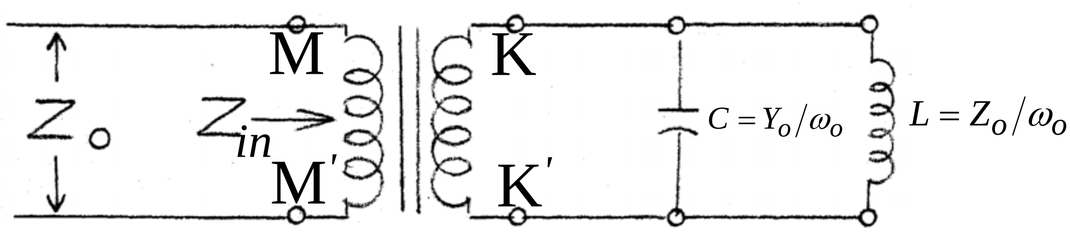


Figure 6. Lumped components equivalent circuit of resonant cavity.

The results obtained can be summarized by observing that in the vicinity of resonance cavity behaviour can be simulated by that of a parallel resonant circuit coupled to the input line by an ideal transformer the turns ratio of which depends on the iris susceptance B.

4. <u>Effect of losses</u>

In discussion above it was assumed that circuits investigated were lossless, which is an acceptable idealization for the purpose of developing basic understanding of the resonance phenomenon. The presence of losses introduces often minor but not necessarily negligible perturbations of idealized behaviour.

The effect of losses in a lumped components parallel resonant circuit can be simulated by introducing a resistance R connected in parallel with the reactive elements C and L as shown in Fig. 7.

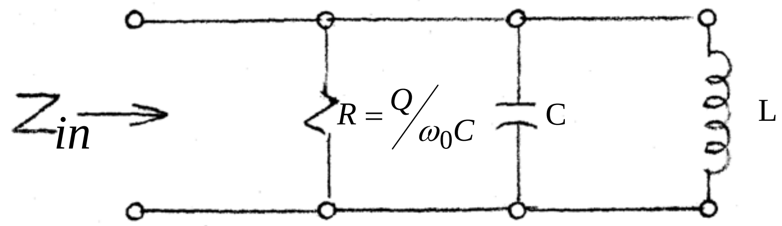


Figure 7. Lossy parallel resonant circuit.

It is convenient in the discussion of lossy resonant circuits to introduce a dimensionless parameter Q, called the quality factor of the circuit and defined by the relation

$$R = Q/\omega_0 C = QL\omega_0 \tag{6}$$

The impedance of a lossy parallel resonant circuit expressed in terms of Q is

$$Z = \frac{Q}{\omega_0 C}\frac{1}{j2Q\frac{(\omega-\omega_0)}{\omega_0}+1}. \tag{7}$$

When these relations are applied to lossy resonant cavity the equivalent circuit thereof is obtained by modification of the equivalent circuit of Fig. 6 which becomes

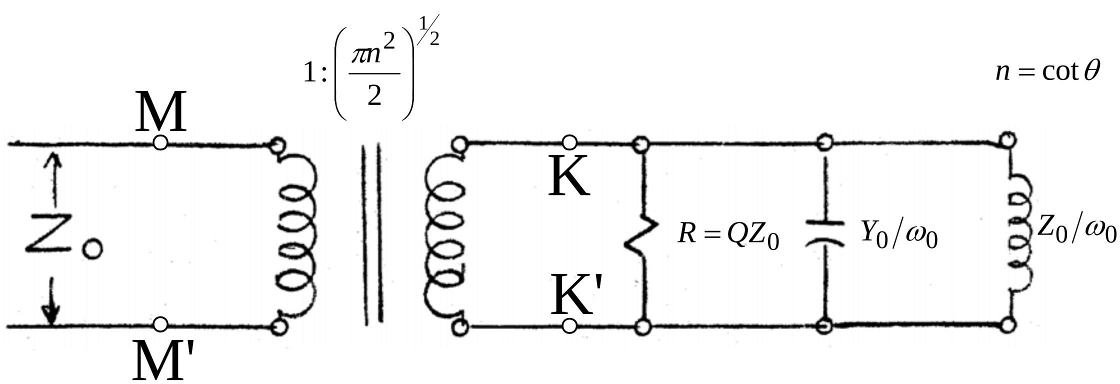


Figure 8. Lumped components equivalent circuit of lossy resonant cavity.

The expression for input impedance Z_{in} can be obtained by suitably modifying its expression as given in Equation 5 and becomes

$$Z_{in} = Z_0\frac{2}{\pi n^2}Q\bigg/\left[2jQ\frac{\omega-\omega_0}{\omega_0}+1\right]. \tag{8a}$$

It is useful to introduce a symbol Q_e for the expression $\pi n^2/2$ in the above equation. It is called the external Q of the cavity and is a measure of the degree of coupling of the cavity to the input transmission line. The final expression for the input impedance Z_{in} becomes then

$$Z_{in}/Z_0 = Q/Q_e \frac{1}{\left[2jQ\left(\frac{\omega_0 - \omega_0}{\omega_0}\right) + 1\right]} \tag{8b}$$

The significance of external Q_e becomes evident if one considers power P delivered to resonant cavity by a source of EMF E matched to the transmission line. The power P is given by the expression

$$P = \frac{E^2}{|Z_0 + Z_{in}|^2} \mathrm{Re}(Z_{in}) = \frac{E^2}{Z_0} \frac{1}{QQ_e} \frac{1}{\left[\left(\frac{1}{Q} + \frac{1}{Q_e}\right)^2 + \left(2\frac{\omega - \omega_0}{\omega_0}\right)^2\right]}. \tag{9}$$

It is apparent from the above equation that effect on power delivery to the cavity of the internal loss mechanism characterized by Q, and that of external, characterized by Q_e, are indistinguishable. It also follows from the equation that the driver frequency deviation ω-ω_0 from resonant frequency at which the power delivered drops to one half of its value at resonance also depends symmetrically on Q and Q_e. If one introduces the parameter effective Q defined by the relation $1/Q_{eff} = 1/Q_e + 1/Q$, the 3 dB power reduction frequency duration can be expressed in the form

$$\omega - \omega_0 = \pm \omega_0 / Q_{eff} \tag{10}$$

simulating corresponding relationship in lumped circuits resonance analysis (as outlined in “Resonance” supplement).

The expression for Z_{in}/Z_0 given in Equation 8b provides means of determining experimentally the loss and coupling parameters Q and Q_e.

A convenient quantity to observe is standing wave ratio $S = (1+|\rho|)/(1-1|\rho|)$ which at resonance is the larger of the two values of $(Q/Q_e)^{\pm 1}$. The unambiguous quantity (Q/Q_e) is the normalized input impedance at resonance.

At frequencies away from resonance the resistive part of the input impedance and the phase of the impedance provide means of determining the internal Q as will be shown below.

From Equation 8b,

$$\frac{Z_{in}}{Z_0} = \frac{Q}{Q_e} \frac{1}{\left[1 + 2jQ\dfrac{\omega - \omega_0}{\omega_0}\right]} \tag{11}$$

It is apparent from the above equation that for frequencies for which $2Q(\omega-\omega_0)/\omega_0$ is equal to ± 1, the absolute value of the input impedance has dropped to $1/\sqrt{2}\,(= 0.707)$ of its value at resonance, and also the phase of the impedance is ±45°. These observations provide a convenient method of determining the value of Q.

5. Experiment

The sketch of the configuration of the resonant cavity employed in the experiment is shown in Fig. 9.

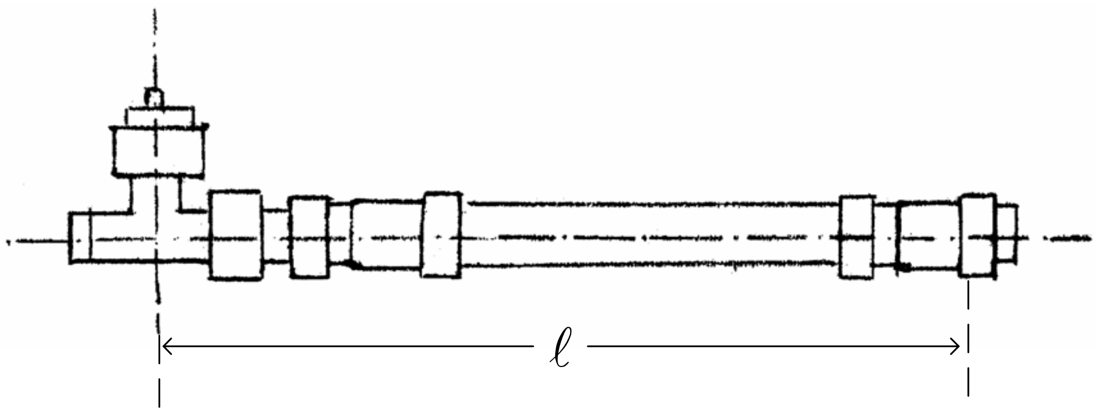


Figure 9. Transmission line resonant cavity.

The cavity consists of Type N TEE terminated in a short circuit, a section of 50 Ohm GR air line and a GR short circuit.

The experiment will consist of the following steps:

(i) determine the resonant frequency of the cavity provided and relate to dimensions,

(ii) determine the Q and Q_e factors and relate to iris susceptance B and offset parameters θ, and h,

(iii) observe the relationship between the frequencies of the –3 dB response points and the Q factors and,

(iv) determine the resonant frequency of modified length cavity.

6. Experimental procedure

6.1 Attach the GR874-WN short circuit termination to the air line. Employ the network analyzer and the instructions in Section 6.1 of the "Network Analyzer" handout to determine the frequency at which the cavity produces the lowest value of SWR, which is the characteristic of the resonance condition. Record the minimum SWR and the resonant frequency. The SWR observed is the larger of the two values of $(Q/Q_e)^{\pm 1}$.

Measure the physical length of the cavity ℓ. Use the attached data sheets for GR874 standard terminations to determine the positions of the effective short circuits. Calculate the effective length ℓ_0 using $\ell_0 = \ell + 8mm$, which accounts for the effect of the dielectric inserts in the coupling iris tee junction. The difference between the effective length ℓ_0 of the cavity and half resonant wavelength $\lambda_0/2$ is the offset length *h*, i.e. $h = (\ell_0 - \lambda_0/2)$.

Calculate the value of iris susceptance B from the relationship $B/Y_0 = 2 \cot 2\theta$ with $\theta = \beta h = \beta(\ell_0 - \lambda_0/2)$. Specify the inductive or capacitive character of B.

6.2 Determine the value of the cavity input impedance using the network analyzer and the instructions in Section 6.2 of the "Network Analyzer" handout. The process involves displaying the absolute value of the input impedance and adjusting the location of the observation terminals by adjusting the phase delay until the resonant frequency marker is moved to the peak of the display. Record the value of cavity input impedance and compare it with the expected value of $|Z_{in}|_0 = (Q/Q_e)Z_0$ as obtained from the SWR measurements.

6.3 Determine the value of the cavity Q using the network analyzer and the instructions in Section 6.3 of the "Network Analyzer" handout. The process involves moving the frequency markers to the frequencies at which the absolute value of impedance is reduced to $1/\sqrt{2}$ of its resonance (maximum value). This is the 3dB bandwidth, Δf_{3dB}. Record the 3dB bandwidth and the Q, as shown on the top RHS of the analyzer display.

Calculate the value of the internal Q from the relation $2Q(\omega - \omega_0)/\omega_0 = 1$, i.e. $Q = f_0 / \Delta f_{3dB}$. From the known values of SWR=Q/Q_e and Q, calculate Q_e and compare this to the value of Q_e obtained from $|Z_{in}|_0 = (Q/Q_e)Z_0$. Determine the value of the offset phase $\theta = \arctan(\pi/2Q_e)^{1/2}$ (Eq. 8a and 8b). Compare it with the value obtained from the measured offset distance h using $\theta = \beta h = \beta(\ell_0 - \lambda_0/2)$.

6.4 Replace the GR874-WN short circuit termination with the GR874-WN3 short circuit termination and repeat steps 6.1 - 6.3 using the network analyzer and the instructions in Section 6.4 of the "Network Analyzer" handout.

6.5 In your report compare the results of your measurements with theoretical predictions and comment on possible causes of any discrepancies observed. Note that the values of Q_e, offset distance h and phase θ should be the same for both cavity lengths. Relate the difference in frequencies for the two cavity lengths to the difference in lengths ℓ.

GR874® STANDARD TERMINATIONS

These terminations are used to establish known impedance at a specific location on coaxial lines.

Resistive terminations are useful in slotted-line measurements and for checking accuracy of directional couplers, bridges, and admittance meters. The known location of a purely resistive termination permits the production of many known complex impedances through the addition of sections of 874-L Air Line, fixed or adjustable.

Open- and short-circuit terminations are useful in establishing initial coaxial line-length conditions for impedance measurements.

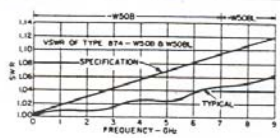
Catalog Number

RESISTIVE TERMINATIONS

50-OHM

Available with either locking or nonlocking connectors, these terminations consist of a 50-ohm cylindrical resistor mounted in a specially tapered coaxial holder. They are useful up to 9 GHz for impedance matching, establishment of reference conditions, and as terminations.

SWR: $<1.005 + 0.013f_{GHz}$.
DC Resistance: 50 Ω ±0.5%.
Max Power: 2 W continuous.

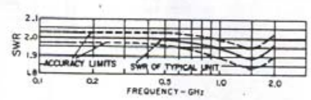


874-W50B

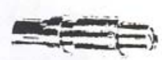
874-W50BL

874-W50B 50-ohm Termination 0874-9954
874-W50BL 50-ohm Termination, locking 0874-9955

100-OHM

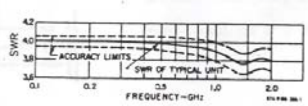


DC Resistance: 100 Ω ±1%.
Max Power: ⅓ W continuous.



874-W100 100-ohm Coaxial Standard 0874-9956

200-OHM



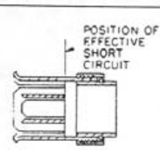
DC Resistance: 200 Ω ±1%.
Max Power: ¼ W continuous.

874-W200 200-ohm Coaxial Standard 0874-9958

SHORT-CIRCUIT TERMINATIONS

Type 874-WN

A fixed short circuit mounted in a GR874 connector for establishing reference conditions on coaxial lines.
Position of Short-Circuit Plane: 0 to 0.07 cm toward load, dc to 7 GHz.



874-WN

874-WN Short-Circuit Termination 0874-9970

GR874® STANDARD TERMINATIONS

(continued)

Catalog Number

Type 874-WNL

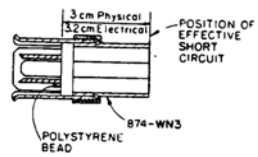
Similar to 874-WN but with locking GR874® connector.

Position of Short-Circuit Plane: 0 to 0.07 cm toward load, dc to 9 GHz.

874-WNL Short-Circuit Termination, locking — 0874-9971

Type 874-WN3

Same as 874-WN except that short circuit is 3.2 cm (electrical distance) beyond face of bead in GR874 connector. This distance corresponds to the bead-to-reference-plane distance in 874-ML Component Mount and 874-UBL Balun.



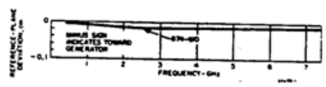
874-WN3 Short-Circuit Termination — 0874-9972

OPEN-CIRCUIT TERMINATIONS

Type 874-WO

Produces an open circuit at the same point that the 874-WN produces a short. Acts as a shielding cap for open-circuited lines.

Position of Open-Circuit Plane: 0 to 0.05 cm toward load, dc to 7 GHz.



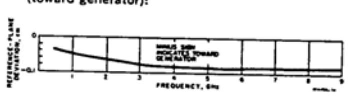
874-WO Open-Circuit Termination — 0874-9980

Type 874-WOL

Produces an open circuit at the same point the 874-WNL produces a short.

Position of Open-Circuit Plane: 0 to 0.10 cm toward load, dc to 9 GHz.

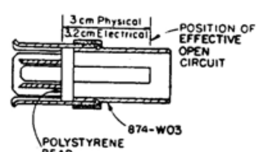
Typical Reference-Plane Deviation (toward generator):



874-WOL Open-Circuit Termination, locking — 0874-9981

Type 874-WO3

Same as 874-WO except that open circuit is at a position corresponding to that of short circuit in 874-WN3 and to the bead-to-reference-plane distance in the 874-ML Component Mount and 874-UBL Balun.



874-WO3 Open-Circuit Termination — 0874-9982