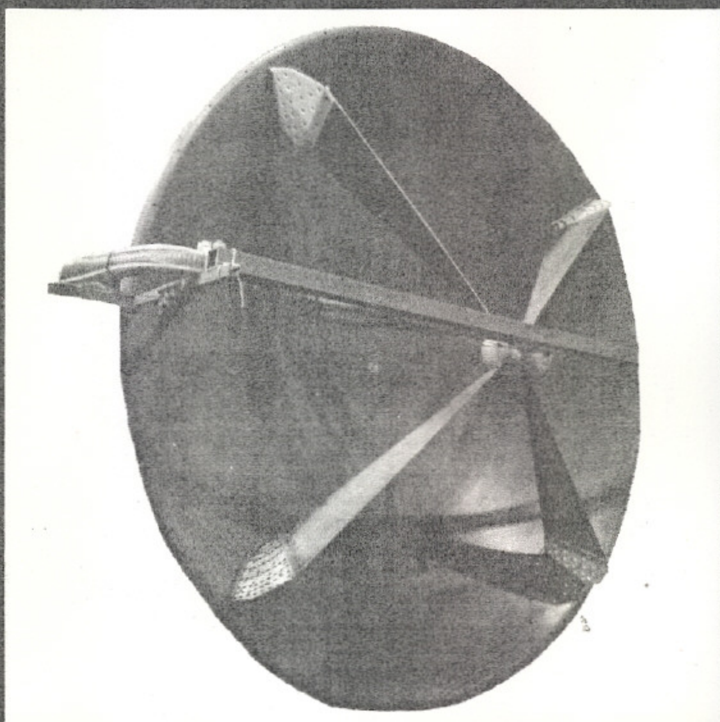
ULTRA-WIDEBAND, SHORT-PULSE ELECTROMAGNETICS 3



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THE TIME EVOLUTION OF PHOTONIC CRYSTAL BANDGAPS

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ABSTRACT

The concept of a scaled group delay time is applied to a finite one-dimensional periodic array of dielectrics as a means of obtaining a group velocity. The scaling factor is shown to be the physical distance and this derived group velocity is compared to the group velocity of an infinitely periodic structure. Joint time-frequency analysis is performed on the response of a one-dimensional structure and the time-to-formation of the pass bands is shown to be determined by the peak group velocity in a given band. These concepts are then extended to three-dimensional photonic crystals and shown to give good agreement.

INTRODUCTION

Photonic crystals (PCs) are three- or lower-dimensional periodic dielectric structures that exhibit pass- and stop-bands. The one-dimensional PC has a wide range of applications in the optical domain as reflectors, filters and anti-reflection coatings[1]. However, for lower frequency microwave/RF applications, conventional technology has limited the use of the one-dimensional PCs. On the other hand, the two- and three-dimensional PCs, such as frequency selective surfaces (two-dimensional) or photonic bandgap crystals (three-dimensional), have found some applications in the microwave domain such as substrates for narrowband antennas[2], filters[3], and frequency selective reflectors for high power microwave systems[4]. For ultra-wideband (UWB) systems, usage of PCs requires a better understanding of the time evolution of the pass- and stop-bands in the crystal. Fortunately, the ability to generate short electromagnetic pulses has made it possible to investigate the interaction of UWB signals with highly dispersive structures[5]. This paper addresses the issue of the band formation in PCs. Initially, one-dimensional structures are used to gain insight to the problem, and subsequently the ideas are extended into the experimental properties of a three-dimensional structure.

ONE-DIMENSIONAL PHOTONIC CRYSTALS

The analysis of one-dimensional PCs begins with the study of an infinitely periodic array of dielectric slabs. In order to study the evolution of the pass- and stop-bands, the group velocity of the system needs to be calculated. The group velocity is the inverse of the first derivative in the Taylor series expansion of the Bloch propagation constant (K) about a given frequency[6]. For this simple case, the required dispersion relation (ω vs. K) can be obtained analytically by applying periodic boundary conditions to the electric field[1].

To determine the evolution times in a finite periodic structure, a group velocity needs to be defined which should approach the group velocity of an infinitely periodic crystal in the

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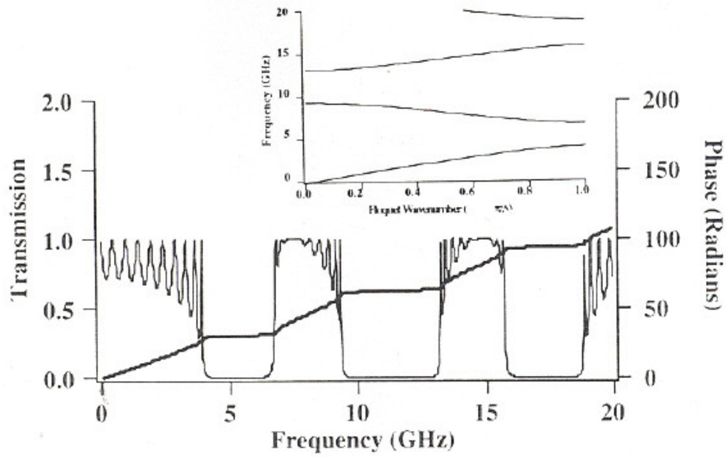


Figure 1. Magnitude (thin line) and unwrapped phase (thick line) of the transmission response of a 10 period multi-layer dielectric structure. $d_1=d_2=0.635$ cm, $n_1=3.162$, $n_2=1$. The inset is the corresponding dispersion curve obtained from the eigenvalue equation for the infinite structure.

limiting case. In order to discuss group velocity, the concept of group delay, which is simply the derivative of the phase of the transfer function with respect to frequency[7], is utilized. If the group delay is scaled by a length, the result is the desired group velocity.

In order to obtain the phase of the transfer function, a transmission line model is used. In this model, one period of the dielectric multi-layer is represented by two transmission lines with characteristic impedance Z_i, length d_i, and propagation constant k_i, for i=1,2, such that the overall ABCD matrix can be obtained[6]. The one period matrix is raised to the power of N, where N is the number of periods in the structure, and hence the transmission coefficient can be determined from the resultant matrix[8]. Figure 1 shows the transmission magnitude and unwrapped phase through the structure with the corresponding dispersion curve for the infinitely periodic structure shown as an inset.

To determine the scaling factor, consider an infinitely periodic structure. The relation between any field point and a field point NΛ away is given by Bloch's transformation theorem:

$$E(x+N\Lambda,K)=E(x,K)e^{iKN\Lambda}$$

where N is the number of periods and Λ is the physical length ($\Lambda=d_1+d_2$) of one period. The ratio of the two fields leads to a transfer function whose magnitude is 1 and whose phase, Φ, is KNΛ. The derivative of the phase with respect to frequency, which is the group delay, is given by

$$\frac{\partial\Phi}{\partial\omega}=\frac{N\Lambda}{v_g}$$

From the above it is clear that the scaling factor is the physical distance of the structure as opposed to the optical path length ($\Lambda=d_1+d_2$ vs. $n_1d_1+n_2d_2$, where n_i is the index of refraction). Figure 2 shows the comparison of the group velocity of the infinite structure (markers), calculated from the derivative of the dispersion curve, with a 10 period multi-layer (solid line), calculated from the scaled group delay. Away from the transition regions between the stop bands and the pass bands (i.e. band edges), the infinitely periodic result is approximately the average value of the finite structure. Near the band edges there is an insufficient number of periods to approximate the group velocity to any reasonable accuracy. However, the work here will be relying on the peak group velocity which occurs well away from the band edges.

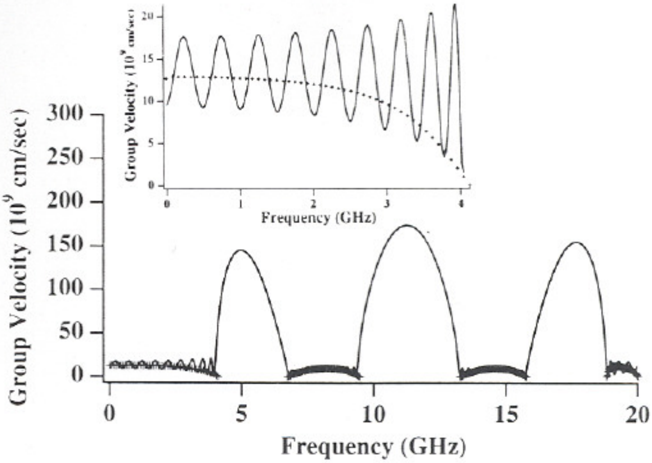


Figure 2. Group velocity calculated for the infinitely periodic array of dielectrics (markers) and a 10 period finite periodic structure (solid line). The inset is an expanded view of the first band.

The joint time-frequency analysis (JTFA) response can be obtained from the transmission response in Figure 1. Figure 3 shows the spectrogram using an adaptive, short-time fourier transform and Gabor algorithms[9], where in all cases, the vertical axis is time, the horizontal axis is frequency and the relative intensities are shown as the spectrogram. Independent of algorithm, the time-to-formation of the pass-bands, which is the start of the pulse to where the first wave appears, is governed by the peak group velocity. In other words, the first wave to appear is the undiffracted wave that is traveling at the peak group velocity for a given pass band. For all bands, there is good agreement between the delay time obtained from the JTFA spectrogram, the scaled group delay obtained from the phase of the transfer function and the derivative of the dispersion curve for the infinitely periodic structure. A summary of the results is given in Table 1.

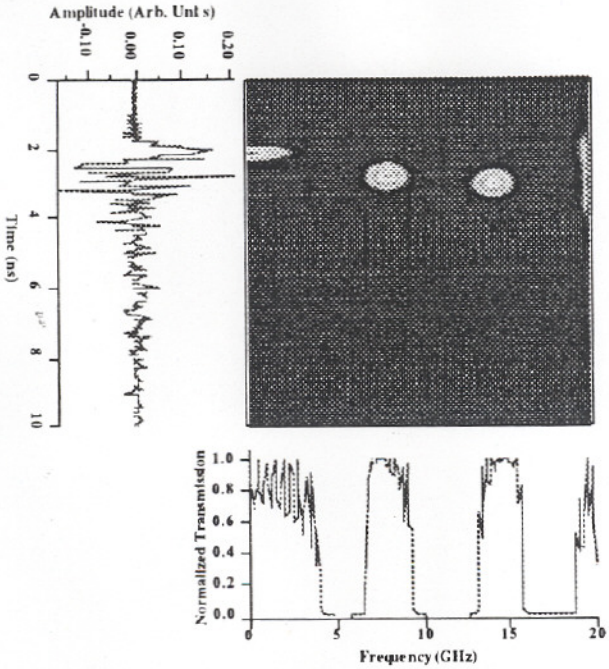
Table 1. Summary of the group velocities obtained by scaling the JTFA delay time, scaling the group delay and the derivative of the dispersion curve for the infinitely periodic structure.

Band Number	JTFA (cm/s)	Group Delay (cm/s)	Infinite (cm/s)
1	1.41×10^{10}	1.29×10^{10}	1.308×10^{10}
2	1.12×10^{10}	0.97×10^{10}	0.99×10^{10}
3	1.01×10^{10}	0.94×10^{10}	0.97×10^{10}

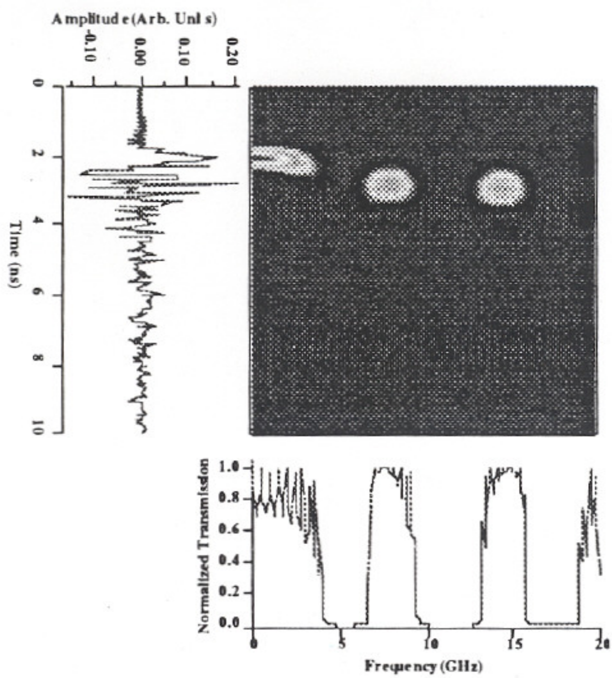
On the other hand, the completion of the band, which is defined as the time from the start of the pass band to the end of the pass band, is difficult to deduce from the JTFA due to the algorithm dependence of the spectrograms. Hence it is difficult to differentiate between the real features and the extraneous ones. In other words, the decomposition of the time signal, to obtain the JTFA spectrogram, is dependent on the basis of the decomposition. This basis dependence creates cross-terms in the spectrogram which may be mistaken for real features. Hence, to avoid this dependence, the focus will be the formation time.

THREE-DIMENSIONAL PHOTONIC CRYSTALS

The concepts developed in the one-dimensional case are extended here. For the three-dimensional PC, a four-period face-centered-cubic structure is used. A detailed description of

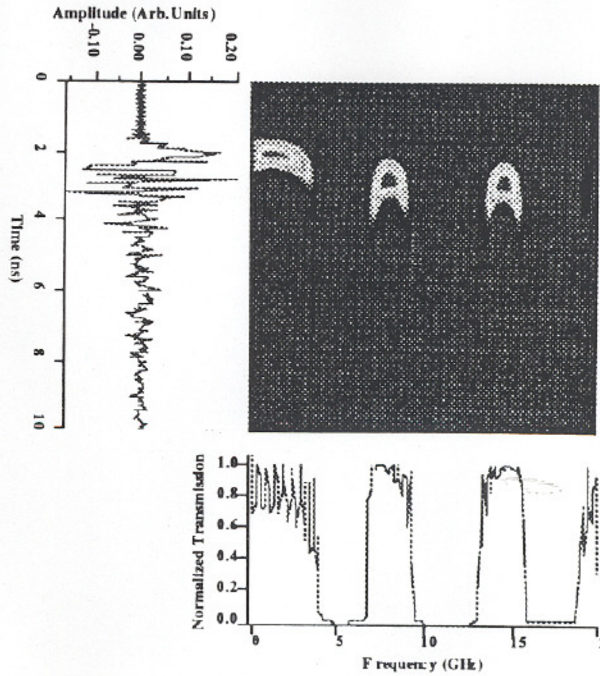

Amplitude (Arb. Units)
-0.10
0.00
0.10
0.20
0
2
4
6
8
10
Time (ns)
Normalized Transmission
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Frequency (GHz)

(a)


Amplitude (Arb. Units)
-0.10
0.00
0.10
0.20
0
2
4
6
8
10
Time (ns)
Normalized Transmission
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Frequency (GHz)

(b)

Figure 3.



(c)

Figure 3. Joint time-frequency analysis using (a). adaptive algorithm, (b). short-time fourier transform and (c). Gabor transform. The time-to-formation of each pass band is determined by the undiffracted wave that is traveling at the peak group velocity for the particular band.

the structure can be found in the paper by Brown, et al[10]. The transmission response (S_{21}) of the PC is experimentally obtained using a vector network analyzer (HP 8510) from 15 to 25 GHz. Here the phase information is preserved, hence the group delay can be calculated. Figure 4 shows the magnitude and unwrapped phase for the frequency response of the crystal at normal incidence (L-point), obtained in the experiment. The points shown in Figure 4 are a linear curve fit to the data. This facilitates the determination of the slope and hence the group delay for the structure. In order to obtain the group delay, the slope of the phase curve is divided by 2π to scale the frequency into radian frequency correctly. Since there are only two pass bands that exist in the frequency range of the network analyzer, the calculations of the group delays will be limited to these two bands.

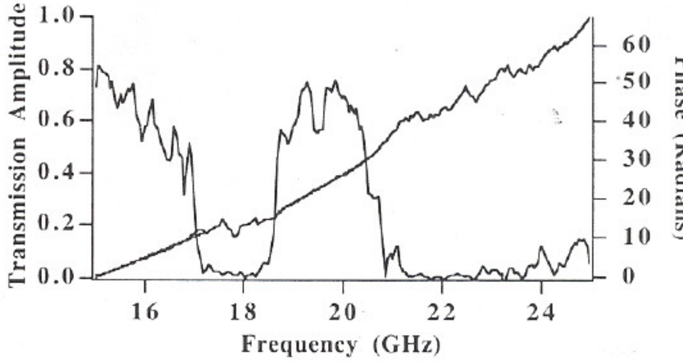


Figure 4. Magnitude and unwrapped phase of the experimental transmission response of a four-period, three-dimensional, face-centered-cubic photonic crystal. The points are a linear fit to the phase data which facilitate in the determination of the slope to obtain the group delay.

Table 2. Comparison of group delay and JTFA delay for various angles of incidence for the three-dimensional PC.

Crystal Direction	Band Number	Group Delay (ns)	JTFA Delay (ns)
L-point (0°)	1	0.844	0.9
	2	1.250	1.3
K-point (35.26°)	1	0.994	1.0
	2	2.020	1.9
W-point (39.2°)	1	0.943	1.0
	2	1.910	1.8

As in the one-dimensional case, the transmission response is inverse fourier transformed and the JTFA spectrogram is obtained. The adaptive algorithm is used to determine the spectrogram and is shown in Figure 5. Once again, the time-to-formation is determined from the delay in the spectrogram and calculated from the derivative of the phase. Here, the group delays can be compared directly using the two methods since there is no scaling factor in calculating delay times. For the three-dimensional PC, the transmission response for various high-symmetry directions (incident angles) are measured and the corresponding delays are calculated. The results are shown in Table 2. Good agreement is obtained with the two methods in determining the time-to-formation of the pass bands.

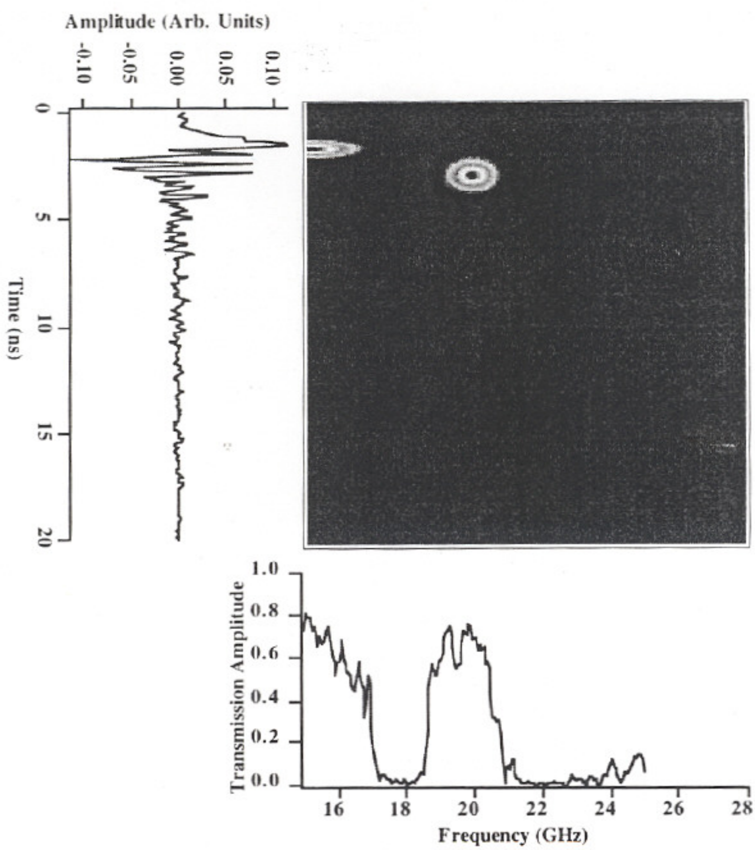


Figure 5. Joint time-frequency response, using the adaptive algorithm, of a four-period, three-dimensional face-centered-cubic photonic crystal at normal incidence. The delay shown is in good agreement with the group delay calculated by taking the derivative of the phase.

In summary, a group delay (velocity) is derived for a finite periodic structure. For the one-dimensional PCs, the group velocity is compared to the group velocity obtained by taking the derivative of the dispersion curve for an infinitely periodic structure. It is determined that the first wave to appear in all cases is the one travelling at the peak group velocity for each band. For the three-dimensional structure, the complex transmission is experimentally obtained and the phase is used, as in the one-dimensional case, to determine the group delay in the bands. The group delay is then compared to the results obtained using JTFA, where, the JTFA algorithm provides a method of pictorially obtaining a delay. Good agreement is obtained using the group delay and the JTFA algorithm for both the one-dimensional and three-dimensional structures.

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