# SUPERLUMINAL BUT CAUSAL WAVE PROPAGATION

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**Abstract:** A series of experiments in recent years have shown that under carefully designed circumstances the group velocity, or even more surprisingly the energy velocity can exceed the speed of light in vacuum or become negative (abnormal velocities). These abnormal results have led some researchers to question the validity of special relativity, or at least cast doubt on the relevance of these principles to the aforementioned experiments. In this work series of experiments with single electromagnetic pulses measured in both time and frequency domain are described. It is seen that while these experiments verify the aforementioned abnormal velocities, they are not in contradiction with the principles of special relativity (Einstein causality). In this regard, the important concept of "front" or "Sommerfeld forerunner" is reintroduced, and it is argued that the only physical velocity required to obey the Einstein causality is the "front velocity."

## I. Introduction

The fact that the group velocity of an electromagnetic wave packet (pulse) can exceed the speed of light in vacuum (become superluminal) has been demonstrated in many experiments using single photons [1, 2], at optical frequencies [3], and using microwaves [4-10]. As a starting point, an interested reader may consult the review by Chiao and the references therein [11]. Despite one's initial impression, the superluminal group or even energy velocities (defined as the ratio of the Poynting vector to the stored electromagnetic energy) are not at odds with the requirements of relativistic causality (Einstein causality), and indeed it can be shown that they must exist as the natural consequence of the Kramers-Kronig relations, which in themselves are a statement of the system linearity and causality [12-15].

The point that in the regions of anomalous dispersion, group velocity can become superluminal was first considered by Sommerfeld and his student Brillouin [16]. In their studies, they examined a sinusoidally modulated step-function propagating through a medium with Lorentzian dispersion. They identified five different velocities: phase, group, energy, Sommerfeld forerunner ("front†") and Brillouin forerunner velocities‡. However, with the passage of time, and for reasons unknown to the authors, while the first three velocity terms have received much attention in both undergraduate and graduate books, the latter two have not enjoyed the same status. This is even more surprising since, among the above velocities, it is only the

† To be more rigorous the term "front" refers to the onset of Sommerfeld forerunner propagation.
‡ To be complete one has to add the term "signal velocity" defined as the velocity of the half maximum point to the list. However, by their own admission such a definition is arbitrary [16] and as discussed in Ref. (4) can become superluminal.

velocity of the "front" that must satisfy the requirements of Einstein causality under all circumstances. In other words, it is rather a naïve understanding of Einstein causality to equate the group velocity with the velocity of information transfer under all circumstances, particularly when one is concerned with the propagation of "attenuated traveling waves[§]."

Our objective here is to discuss the phenomenon of superluminal and negative group and energy velocities which generically is referred to as the abnormal velocities. In Sec. II a time-domain experiment used to detect the superluminal group velocities in the case of a one dimensional photonic crystal (1DPC) is described. Section III discusses a frequency-domain experiment used to demonstrate the same superluminal phenomenon. The case of superluminal or negative group and energy velocities for an inverted medium (medium with gain) or in the case of medium with negative index of refraction is considered in Sec. IV. Section V is intended to put the reader's mind at ease by providing some general arguments on why the abnormal velocities discussed in the previous sections are not in contradiction with the requirements of special relativity. Section VI is our condensed attempt is addressing the issue of superluminality in the limit of very weak light (very few photons). Our final remarks and a discussion for the general public can be found in Sec. VII.

## II. Time-domain Experiment

Consider the problem of electromagnetic wave propagation through a periodic structure. Figure 1 shows an experimental setup used to detect the superluminal group velocity for a microwave wave packet tunneling through a 1DPC. A backward wave oscillator (BWO) was used to generate the pulse, and a mode converter (MC) was used to convert the $TM_{01}$ mode of the BWO to a $TE_{11}$. The pulse was then radiated via a conical horn antenna (CHA). The frequency output of the source was tuned to the mid-gap frequency of the 1DPC at 9.68 GHz (FWHM of 100 MHz) and was detected by two HP 8470-B, Schottky diode detector (provided in pairs). The CHA radiation intensity was sampled along two distinct directions (paths), referred to as "side" and "center". A series of microwave pulses were fired in order to measure and then remove the time difference between the two paths due to the differences in cable lengths, internal detection of the oscilloscopes (Tektronix SCD 5000) and other incompatibilities. This measured time difference was electronically compensated such that the peaks corresponding to the pulses traveling through the "center" path and "side" path in the absence of the 1DPC arrived at the same time. At this point, the 1DPC was inserted along the "center" path and series of single pulse were fired. Figure 2 shows the result. It is seen that the peak of the wave packet propagating along the "center" path and tunneling through the 1DPC arrives $(440 \pm 20)$ ps earlier than the accompanying pulse propagating through free-space along the "side" path. This

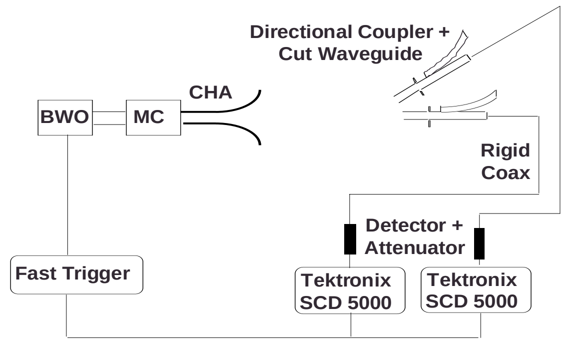


**Fig. 1:** Time-domain experimental setup

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§ We have used the term "attenuated traveling waves" in the same sense as in Ref. (17), although, sometimes the term evanescent is used to signify the same thing.

advancement in time for the tunneling pulse can easily be translated to a measure of the pulse group velocity, indicating that the tunneling wave packet propagated with a

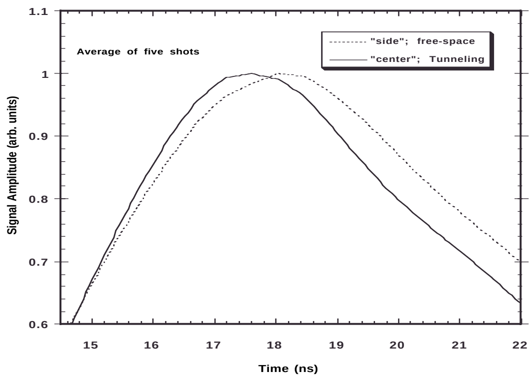


**Fig. 2:** Superluminal propagation for the tunneling pulse

group velocity $(2.38 \pm 0.15)\, c$.

Furthermore, the traditional view of pulse propagation through a region with high attenuation (regions of anomalous dispersion) held that the extreme

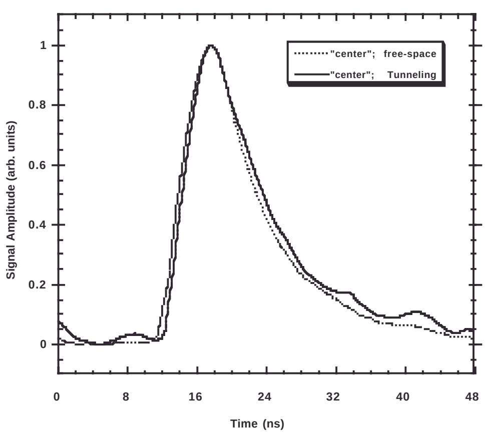


**Fig. 3:** A measure of the pulse broadening due to tunneling through the 1DPC. The two pulses have propagated along the same path ("center"); one in the free-space and the other through the 1DPC.

attenuation, coupled with the dispersion, would distort the signal to such an extent that the originally well defined wave packet and its peak would not be recognizable upon the emergence from such a medium [17 16]. Figure 3 shows that in contrast to this common belief, the tunneling wave packet of Fig. (2) suffered minimal dispersion such that the FWHM of the pulse after tunneling was only increased by 1.5%. In obtaining this figure the tunneling wave packet was manually moved to later times as to make the comparison between the two pulses easier. A full description of the above experiment can be found in Ref. (4).

## III. Frequency-Domain Experiment

In the last section the feasibility of measuring superluminal group velocities directly in time-domain was discussed. This abnormal behavior can also be demonstrated in frequency-domain. Figure 4 shows the

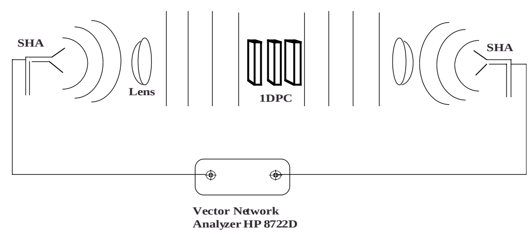


**Fig. 4:** Frequency-domain experimental setup

free-space setup used to detect the superluminal group velocities in frequency-domain. The setup consists of two K-band standard horn antennas (SHA) configured a transmitter and receiver and connected to ports 1 and 2 of an HP 8722D vector network analyzer (VNA). The radiated quasi continuous waves are collimated using two microwave lenses and the setup is enclosed in a anechoic chamber to reduce stray signals.

The essence of the approach is to measure *accurately* and *reliably* the transmission phase associated with the 1DPC. Once this quantity is measured, the group delay ($\tau_g$) and group velocity ($V_g$) can be calculated according to

$$\tau_g = -\partial \phi / \partial \omega, \qquad (1)$$

$$\frac{v_g}{c} = \frac{L_{pc}}{c\, \tau_g} = \frac{-L_{pc}}{c\, (\partial\phi/\partial\omega)}, \qquad (2)$$

where $\phi$ is the transmission phase, and $L_{pc}$ is the physical length of the 1DPC.

Fortunately, recent advances in non-coaxial (free-space) calibration techniques for VNA such as the "Thru-Line-Reflect" (TRL), [18, 19] make it possible to measured the transmission coefficient accurately and reliably. After calibrating the system (without the 1DPC), a reference plane of unit magnitude for transmission magnitude and zero phase for $\phi$ is established midway between the two SHAs. At this point, the 1DPC is inserted and the receiver horn is moved back exactly by a length equal to the thickness of the 1DPC ( $L_{pc}$).

Figure 5 is the calculated (solid line)

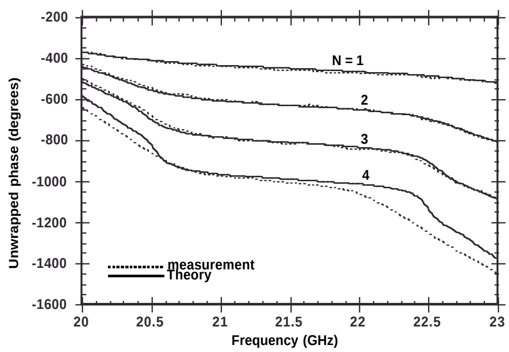


**Fig 5:** The unwrapped transmission phase for the 1DPC with various number of Eccostock® dielectric slabs.

and measured (dotted line) unwrapped phase for a 1DPC with four, three, two and one dielectric slabs (the spacer is always air). The theoretical calculations are based on the diagonalization of one period matrix, and is presented in Ref. (5).

According to Eqs. (1) and (2), to ascertain the group delay and group velocity the data presented in Fig. 5 must be differentiated. However, differentiating a noisy data amplifies the noise and may lead to spurious effects. To avoid the arbitrariness associated with smoothing, a best nonlinear least square fit of the experimental phase data is obtained. The parameters used in fitting the experimental data all match the actual variables very well and for the sake of brevity are not given here, but can be found in Ref. (5).

Figure 6 shows the result of the least square fit to the phase data of Fig. 5 together

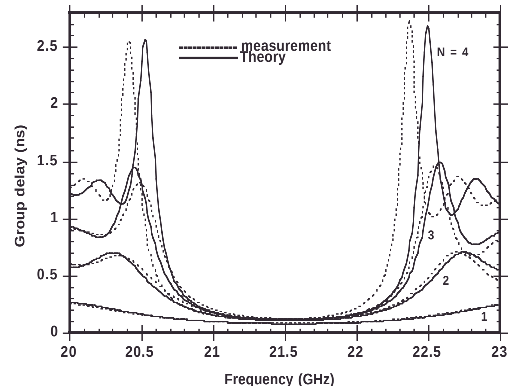


**Fig. 6:** Measured and calculated group delay for the 1DPC. The parameters used to obtain the fitted curves (measurement) and the calculated curves (theory) are given in Ref (5).

with Eq. (1), in order to determine the group delay in a 1DPC with one, two, three, and four dielectric slabs. Consequently, the normalized group velocity given by Eq. (2)

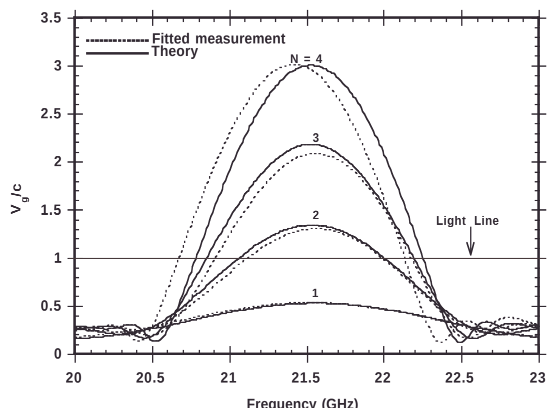


**Fig. 7.** Normalized group velocity for the 1DPC. The dotted curves are the measured results obtained from the fitted curves in Fig. 6 and Eq. 2. The solid curves are theoretically calculated.

can be obtained from the Fig. 6 and is shown in Fig. 7. Along with the velocities derived from the fit (dotted curves), the theoretical group velocities calculated from the measured values of thicknesses and indices are also shown (solid curves). As Fig. 7 indicates, in the case of *N=4* and *N=3*, a maximum superluminal group velocities 3 and 2.1 times $c$ is observed. The results depicted in this figure cab be interpreted as the following. If one is to construct a pulse entirely composed of the frequency components for which the superluminal behavior is predicted, then the pulse is expected to propagate with group velocity exceeding $c$, similar to the situation discussed in the Sec. II.

## IV. Abnormal Velocities in Inverted Medium and Medium with Negative Index

The circumstances under which the group or even energy velocity are abnormal are not limited to the evanescent wave propagation discussed so far. In this section three situations are described which exhibit the aforementioned abnormal behavior.

First, for an inverted medium (medium with gain) described by a Lorentz-Lorenz dispersion, the index of refraction is given by

$$n(\omega)=\left(1-\frac{\omega_p^2}{\omega_0^2-\omega^2-i\gamma\omega}\right)^{1/2} \qquad (3)$$

where $\omega_0$ is the resonance frequency, $\gamma$ is a small damping factor, and $\omega_p$ is the effective plasma frequency. Note that a negative sign precedes the second term under the square root due to the population inversion of the medium. A plot of the index of refraction for both the inverted and non-inverted medium is shown in Fig. 8. From the figure it is clear that for an inverted medium, in the limit of very low frequencies, the index is less than one implying that the phase velocity is superluminal. More importantly, at the low frequencies, the group velocity given by

$$V_g(0)=\frac{c}{\left[n(\omega)+\omega\, dn/d\omega\right]_{\omega\to 0}}= \frac{c}{n(0)}=V_p(0)>c \qquad (4)$$

is also superluminal. Under these circumstances the energy velocity ($V_e$), given by the ratio of Poynting vector ($S$), to the stored energy density ($u$), is also equal to the phase and group velocity and exceeds the speed of light in vacuum.

$$V_e=\frac{S}{u}=\frac{c}{n(0)}=V_p(0)>c \qquad (5)$$

The equivalence of the above three velocities is merely a statement of the fact that in the limit of low frequencies the medium is transparent and dispersionless [13].

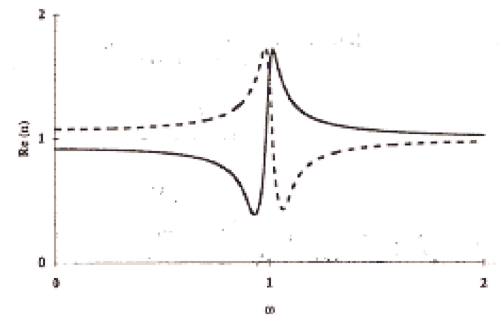


**Fig. 8:** The real part of the index of refraction for an inverted medium (solid curve) and non-inverted medium (dashed curve.)

Second, it is also possible to observe negative group velocities for electromagnetic (EM) pulses tuned slightly away from a gain line of an inverted medium. [14, 20-22]. The physical meaning of a negative group velocity can be explained as the following. Consider two cells of physical length $L$ containing an inverted medium and vacuum as shown in Fig. 9.

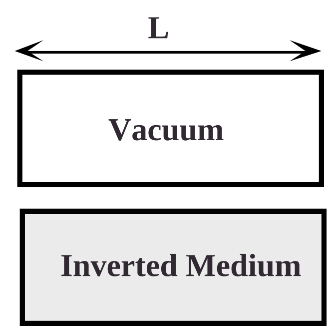


**Fig. 9:** Two cells of equal length containing inverted medium and vacuum.

The time difference between two well behaved identical EM pulses propagating through the lower (inverted) and the upper cell (vacuum) is given by

$$\Delta t = \tau_g - t_{vac} = \frac{L}{V_g} - \frac{L}{c} = \frac{L}{c}\left(n_g - 1\right) \quad (6)$$

where $n_g$ is the group index. From the above equation it is clear that if the group index is zero the time difference between the two pulses is given by the negative of $L/c$ In other words, when one of the EM pulses is at the exit face of the lower cell the other pulse is about to enter the upper vacuum cell. Stating this point differently, if one only consider a single cell containing the inverted medium, for a negative group index, the peak of the transmitted wave packet leaves the cell prior to the peak of the incoming wave packet entering the medium. It must be pointed out that as shown by Landauer [23-25] it is naïve to regard the peak of the outgoing pulse as the causal response of the medium to the peak of the incident pulse. The theoretical prediction by one of us [22] regarding the feasibility of detecting negative group velocity was recently verified in an experiment by Wang [26] in which the inverted medium was a cell containing Cesium vapor.

Finally, let us consider a situation for which the medium effective index is a negative value. A point worth emphasizing is the fact that for these media it is the effective index and not the actual material index which is negative. In other words, the wavelength of the incident wave is many times larger than the physical size of the components comprising the media, allowing one to characterize the over all response of the media in terms of an effective index.

The first theoretical work in this area was done by Veselago [27, 28], and the more recent interest in the subject was reignited by the work of Pendry [29, 30] and Smith et. al. [31, 32], which demonstrated the possibility of manufacturing these media at microwave frequencies. Figure 10, shows the dispersion relation for a negative index

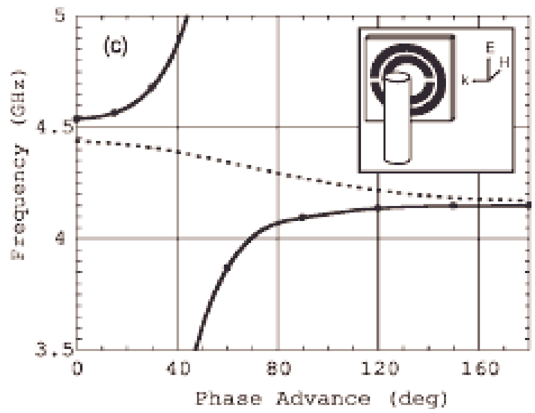


**Fig. 10:** The dispersion curves for a medium with negative effective index

medium, borrowed from Ref. (32). From the figure it is clear that for a certain frequency range, the derivative of the curve depicted in Fig. 10, (i.e. the group velocity) is negative. Even more surprising is the fact that the energy velocity, given by Eq. (7), is also negative, since in these media both permittivity and permeability are negative parameters.

$$V_e \propto \frac{\vec{E} \times \vec{H}}{\varepsilon \left|\vec{E}\right|^2 + \mu \left|\vec{H}\right|^2} \quad (7)$$

The presence of negative group and energy velocities for the above media can be understood in the following manner. The

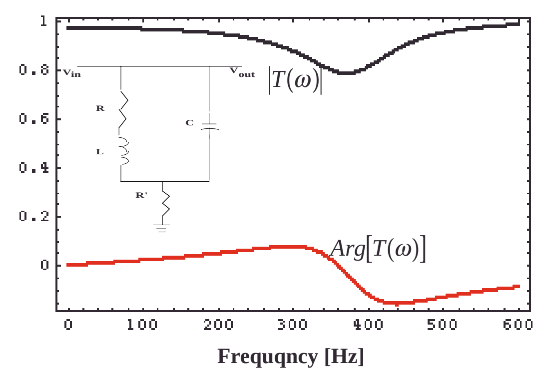


**Fig. 11:** Transmission magnitude and phase for the LRC circuit shown in the inset.

negative index medium, considered by Smith et. al. [31], is in essence a distributed

LRC transmission line that its response can be approximated by a lumped LRC circuit. The inset in Fig. 11 is a typical LRC circuit that exhibits negative group delay in the region of maximum attenuation[**]. Once again, if one is to construct an EM pulse mostly composed of frequency components having negative group delays, it is expected that the group and energy velocities for this EM pulse to be negative. We currently are pursuing the detection of the aforementioned abnormal velocities in the negative index media. We end this section by pointing out that in addition to negative velocities, the negative index medium has many other interesting properties such as inverted Doppler shift, Cherenkov radiation, and Snell's law.

## V. Superluminal Velocities and Einstein Causality

In so far we have discussed situations for which the phase, group, and energy velocities are abnormal (superluminal or negative). The reader may began to wonder whether or not these abnormal velocities are in contradiction with the requirements of relativistic causality. The short answer to this question is that under no circumstances the so called "front velocity" may exceed the speed of light in vacuum, and in fact under all circumstances the "front velocity" is exactly luminal. In other words, the requirement of Einstein causality that no "signal" (information) can be transmitted superluminally is satisfied in all cases, since the "front velocity' is always luminal. This means that the presence of the genuine information should not be associated with the pulse maximum, half maximum, or the envelope, but indeed is contained within the singularities (points of non-analyticity) of the pulse. Because of the important role played by the "front" in satisfying the requirements of special relativity, let us briefly discuss some of the most general ideas associated with this concept. The interested reader may consult the Ref. (4) for more detailed analysis.

The essential point to remember is the fact that any *physically realizable* signal is restrictively time-limited. In other words, any electromagnetic signal created and later propagated through free-space or a 1DPC must be generated at a point in time and space. One can then always point to a time prior to which the signal did not exist. This point in time, or more precisely the transient "turn on times," are points of non-analyticity for which the amplitude of the pulse or its first or higher derivative are discontinuous.

The importance of these points of non-analyticity becomes clear when considering the following. While the future behavior of a truly analytical signal such as a Gaussian wave packet can be completely predicted by means such as a Taylor expansion (or a Laurent expansion for functions that are holomorphic in an annular region), the presence or arrival of the singularities do not yield themselves to such an extrapolation. Moreover, as discussed in the above, no *physically realizable* signal can be presented by an *entire function*[††] hence, any communication of information must involve the transmission of the "front". To summarize, there is no more information in a pulse peak or envelope that is not already contained within the earliest parts of the signal.

The mathematical proof that no signal (information) may be detected sooner than $t_0 = x/c$ can be seen via contour integration of an expression such as Eq. (8). Equation (8) describes the field at the position $x$ and time $t$ for a wave packet

[**] In obtaining Fig. 11, in contrast to the curves depicted in Fig. 5, a time dependency of $e^{-i\omega t}$ in place of $e^{j\omega t}$ was used. In other word, the group delay has the opposite sign of that shown in Eq. (1).

[††] An *entire function* is the one that is analytical everywhere in the complex domain.

impinging at normal incident on a medium characterized by an index of refraction, $n$, [33].

$$u(x,t) = \int_{-\infty}^{+\infty} \frac{2}{1+n(\omega)} A(\omega)\, e^{i k(\omega) x - i \omega t}\, d\omega, \qquad (8)$$

$$A(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} u(x=0,t)\; e^{i\omega t}\, d\omega. \qquad (9)$$

Transforming the integral in Eq. (8) into the complex domain and closing the contour over the upper-half-plan, along with requiring that the medium characterized by $n$ to be causal, and that the incident wave packet has a "front," are sufficient conditions to show that the value of the integral is identically zero for $t \le t_0 = x/c$ or equally for velocities, $V = x/t > c$. The condition that the medium characterized by $n$ is causal, means that for this medium the effect can not proceed the cause. Mathematically this is expressed as $G(\tau) = 0$ for $\tau < 0$, where $G(\tau)$ is the susceptibility kernel given by

$$G(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[\varepsilon(\omega)/\varepsilon_0 - 1\right] e^{-i\omega\tau}\, d\omega. \qquad (10)$$

For times immediately after $t_0$, ($t >\approx t_0$) the earliest part of the signal known as the Sommerfeld forerunner or precursor can be detected. The frequency of oscillation and the field amplitude for the Sommerfeld forerunner are discussed by Mojahedi et. al. [4, 34]. To summarize those results, the frequency of oscillation is given by

$$\omega_s = \sqrt{G'(0)} \Big/ \sqrt{2\left(\frac{t}{t_0} - 1\right)}, \qquad (11)$$

where $G'(0)$ is the time derivative of the susceptibility kernel [33] evaluated at $t = 0$. Furthermore, for the incident wave packet proportional to $t^m$ ($m$ is an integer) the Sommerfeld forerunner is described by a Bessel function of order $m$ according to

$$u(x,t) \approx a\left(\frac{t-t_0}{\gamma}\right)^{m/2} J_m\left(2\sqrt{\gamma(t-t_0)}\right); \qquad (12)$$

$$\gamma = \frac{G'(0)\, t_0}{2}; \qquad for\ t > t_0.$$

From the above discussion it is clear that, for a given medium if the quantity $G'(0)$ is known, the calculation of the Sommerfeld forerunner frequency of

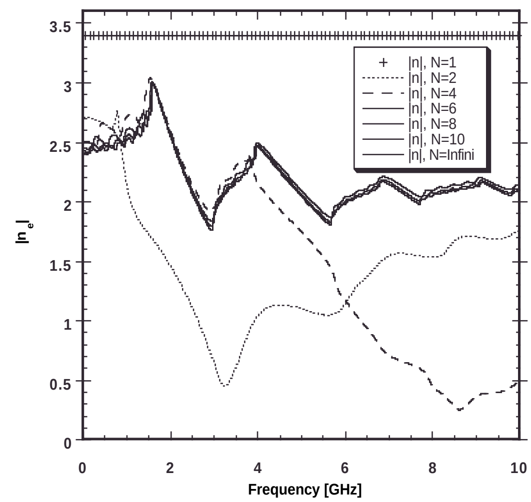


**Fig. 12:** Effective index for a 1DPC. The structure parameters are: $d_i = 1.76$ cm, $d_j = 1.33$ cm, $n_i = 1$, $n_j = 3.40$

oscillation and functional form is relatively straight forward. In other words, if one is capable of calculating $\varepsilon(\omega)/\varepsilon_0 - 1 = n^2 - 1$ for a 1DPC, undersized waveguide, or any other photonic barrier used in the superluminal experiments, then one can perform the inverse Fourier transform and the differentiation operation to obtain $G'(0)$. For example, let us consider the case of 1DPC used in the experiments discussed in Sections. II and III.

At the normal incidence the dispersion relation ($K\ vs.\ \omega$) can be obtained from

$$\cos(K\,\Lambda) = \cos\left(\frac{\omega\, n_i\, d_i}{c}\right)\cos\left(\frac{\omega\, n_j\, d_j}{c}\right) - \frac{1}{2}\left(R + R^{-1}\right)\sin\left(\frac{\omega\, n_i\, d_i}{c}\right)\sin\left(\frac{\omega\, n_j\, d_j}{c}\right), \quad (13)$$

where $R$ is the ratio of the indices given by $R = n_i/n_j$, $\Lambda$ is the one-period length $\Lambda = d_i + d_j$, and $K$ is the Bloch wave vector. The above equation can be used to solve for the real and imaginary parts of the Bloch wave vector, and Eqs. (14)-(16), below, can in turn be used to transform the photonic crystal spatial dispersion [Eq. (13)] to a more manageable temporal dispersion

$$\mathrm{Re}(n_e) = n'_e = \frac{c}{\omega}\,\mathrm{Re}[K(\omega)], \quad (14)$$

$$\mathrm{Im}(n_e) = n''_e = \frac{c}{\omega}\,\mathrm{Im}[K(\omega)], \quad (15)$$

$$|n_e| = \left[(n'_e)^2 + (n''_e)^2\right]^{1/2} = \frac{c}{\omega}\,|K(\omega)|. \quad (16)$$

The results are shown in Fig. (12) which displays our first attempt in obtaining an effective index for a 1DPC, with 1, 2, …, and infinite number of dielectric slabs. The next step is to perform the Fourier transform indicated by Eq. (10), followed by the differential operation evaluated at time equal to zero. Having obtained the quantity $G'(0)$, the frequency of oscillation and the functional form of the Sommerfeld forerunner in a 1DPC can be arrived at with the help of Eqs. (11) and (12).

## VI. Superluminal Propagation and Quantum Noise in the Limit of Very Weak Pulse.

The question of superluminality in the limit of very weak pulse (one or few photons) was considered in a recent work [35]. For the sake of brevity, we refer the interested reader to Ref. (36) for a complete and detailed analysis of the situation. Here, we suffice to mention that according to Aharonov et. al. [36] in the limit of few photons, signal must be exponentially large in order to distinguish it from the quantum noise. In other words, the signal-to-noise ratio becomes vanishingly small. In Ref. (36) this assertion is investigated and it is seen that if the condition stated by Aharonov et. al. is replaced by a weaker condition, the signal-to-noise ratio can exceed unity even for one photon pulse. It is worth mentioning that the original experiment by Chiao and Steinberg, [2] although involved the detection of single photon, but the results were interpreted in terms of statistics of many photons.

## VII. Concluding Remarks: A Discussion for General Public

A simple yet interesting description of superluminal propagation can be found at web link http://www.abqjournal.com/scitech/180964scitech11-19-00.htm.

This article written by John Fleck, the science writer for Albuquerque Journal, tries to explain our newly published paper in *Physical Review E* to the general public. To use John's analogy consider two dragsters competing against each other, driving the same exact cars and traveling the same exact distances. However, whereas one of the dragsters travels through air (vacuum if you like) with the maximum allowable speed, the other driver travels through a series of barriers normally thought to slow his car. The question is then the following: What does the referee at the end line observe? The answer depends on the referee detection equipment. If the referee is well equipped with the most sensitive and expensive detection systems he or she will observe that the front bumpers of the two cars arrive at the finishing line at exactly the same instance. The referee will also observe that the bulk (the main body, the cockpit and the driver) of the dragster's car who tunneled through the barriers reaches the end line sooner than his challenger. Interestingly, if the race is decided by arrival of the cars main body or if the referee is not equipped with the most sensitive detection apparatus, he or she will invariably call the race for the tunneling dragster.

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