Periodically Loaded Transmission Line with Effective Negative Refractive Index and Negative Group Velocity

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I. Introduction

Media with negative refractive index (NRI), as first proposed by Veselago [1], are expected to exhibit properties that are unusual compared to materials with a positive index of refraction. However, until recently, these properties were not experimentally observed since no NRI material occurs naturally. In recent studies, periodic structures with NRI have been constructed, first by a group at the University of California, San Diego and later at the University of Toronto [2-5]. The Toronto group created these artificial NRI materials by loading a cellular network of transmission lines with series capacitors and shunt inductors [4, 5]. These structures are completely planar and hence are suitable for RF/microwave integrated circuit applications.

This paper extends the work of the Toronto group by designing a medium that exhibits negative group velocity (NGV) in addition to the NRI. To achieve the NGV, a resonant circuit is embedded within each Loaded Transmission Line (LTL) unit cell. The resonance produces a region of anomalous dispersion in which the group delay, and thus the group velocity, is negative [6]. The NGV simply means that the peak of the output pulse emerges from the LTL prior to the peak of the input pulse, though much reduced in magnitude. It must be added that NGV is not a violation of relativistic causality since the front of the output pulse does not precede the front of the input pulse; that is, the output pulse front suffers the usual positive delay [7].

The proposed transmission line is fabricated using coplanar waveguide technology, and scattering matrix measurements are performed to verify the theoretical predictions.

II. Dispersion Analysis of Periodically Loaded Transmission Lines with NRI and NGV

A periodically loaded transmission line can exhibit both an effective NRI and a NGV, provided that the values of the loading elements are chosen properly. For this paper, the transmission line is loaded in series with a capacitor C_s and an $R_rL_rC_r$ resonator, and in shunt with an inductor L_{sh}, as shown in Fig. 1.

Using the *ABCD* matrix technique, the complex propagation constant γ of the periodic structure is given by

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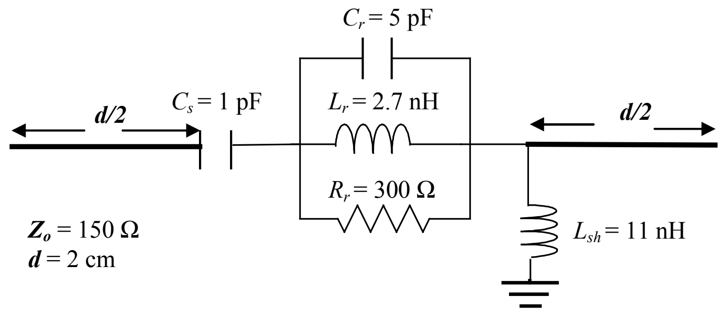


Fig. 1. Unit cell of the proposed loaded transmission line. Typical component values are shown.

$$\cosh \gamma d = \cos[(\alpha + j\beta)d] = \cos kd + j\frac{(Z_s + Y_{sh}Z_o^2)}{2Z_o}\sin kd + \frac{Z_s Y_{sh}}{2}\cos kd \qquad (1)$$

where α and β are the attenuation and phase constants of the periodically loaded medium, and k, Z_o, and d are the propagation constant, characteristic impedance, and length per unit cell of the unloaded line respectively. Z_s is the series impedance and Y_{sh} is the shunt admittance.

Figure 2 shows the dispersion diagram of the proposed periodic structure, obtained from Eq. (1). The component values used to produce the curves are indicated in Fig. 1. We are interested in identifying the region of both anomalous dispersion and NRI, that is, where both the group velocity and phase velocity are negative. Negative phase velocity corresponds to branches I and III in Fig. 2, where $\beta d < 0$. Group velocity is negative where the slope of the dispersion curve is negative; these regions are indicated on the figure. Thus it can be seen that for the frequency range $f_2 < f < f_3$ in branch I the phase and group velocities are both predicted to be negative.

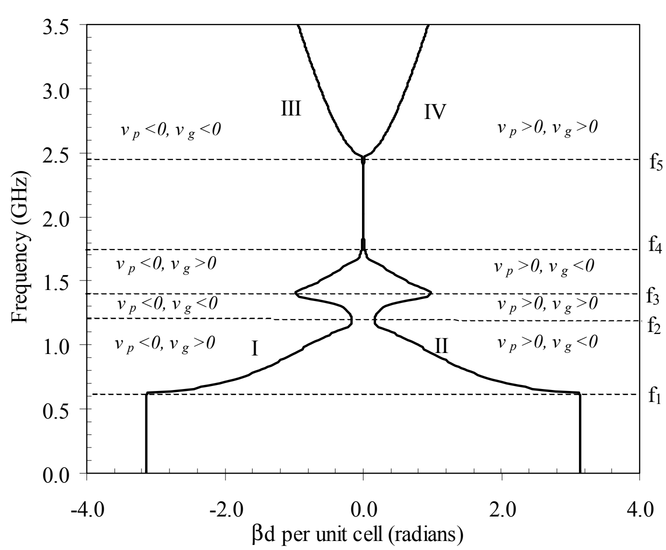
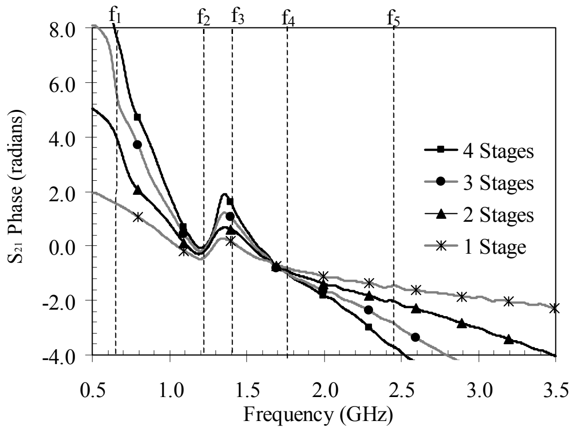


Fig. 2. Dispersion diagram of the proposed transmission line medium exhibiting simultaneous NRI and NGV in the first pass-band (branch I, $f_2 < f < f_3$).



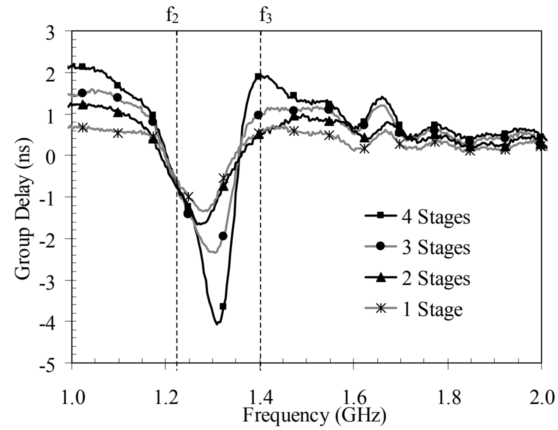


Fig. 3: (a) Measured unwrapped S_{21} phase and (b) measured group delay of the periodically loaded transmission lines with one, two, three, and four unit cells.

III. Experimental Results

To verify our theoretical predictions, a coplanar waveguide (CPW) was designed and printed on a Rogers 5880 substrate with a dielectric constant of 2.2 and thickness of 15 mils. The CPW line was periodically loaded with surface-mounted chips of 1.5 mm by 0.5 mm, such that one unit cell was approximately 2 cm long. LTLs with 1, 2, 3, and 4 cascaded unit cells were then fabricated. The measured transmission phase for these periodically LTLs is shown in Fig. 3(a).

Neglecting the mismatches at the boundaries, the difference in insertion phase ($\phi = -\omega\, n\, d/c$) between a LTL with N stages and length d_N, and a LTL with M stages and length d_M, is given by

$$\Delta\phi = \phi_M - \phi_N = -\frac{\omega n(\omega)}{c}\left(d_M - d_N\right) \tag{2}$$

Note that for $d_M > d_N$ and normal media ($n > 0$), the difference in the insertion phase calculated from Equation (2) is negative ($\Delta\phi < 0$), whereas in the frequency band $f_1 < f < f_4$, $\Delta\phi$ is positive, indicating an equivalent negative refractive index [see Fig. 3(a)].

The measured group delay is shown in Fig. 3(b). From these curves, the frequency range of negative group delay and hence NGV is seen to be $f_2 < f < f_3$ for all four LTLs, corresponding to the region of anomalous dispersion. Note that the negative group delay is accompanied by strong absorption, which can be seen in the S_{21} magnitude plot (not shown here).

Thus, the experimental results confirm the theoretical predictions that the LTL circuit of Fig. 1 shows NRI for frequencies in the range $f_1 < f < f_4$, and both NRI and NGV in the range $f_2 < f < f_3$ (note that in Fig. 3(b) the measured frequency range is slightly shifted with respect to the predicted range due to non-ideal component values used in LTLs).

IV. Conclusion

We present the design and implementation of a periodically LTL that simultaneously exhibits a NRI and a negative group delay. The proposed transmission line structure is loaded in series with capacitors and resonant *RLC* circuits and in shunt with inductors. The series-C and shunt-L configuration produces backward propagating waves (positive group velocity and negative phase velocity) leading to a NRI [4, 5], while the resonant *RLC* circuit provides the region of anomalous dispersion and hence the NGV and negative group delay.

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