Tunable Metallic Photonic Crystals with an Effective Negative Index of Refraction

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1. Introduction

The design of negative refractive index (NRI) metamaterials is an exciting field of research, even though the properties of these materials were predicted over 35 years ago [1]. One characteristic of waves propagating in these structures is that, in the pass-band, the group velocity and the wave vector are anti-parallel, which gives rise to a backward wave (BW) behaviour. Both dielectric and metallic photonic crystals (PCs) with two-dimensional square and triangular lattices of scattering rods have been considered as possible candidates for NRI metamaterials [2-4]. Although in general PCs can produce "negative refraction" with and without BW behavior, in the present study we mostly consider the negative refraction in the BW bands.

This paper presents a study of metallic photonic crystals with a 2D square lattice of cross shaped scatterers. It is found that even a small volume fraction of metal (when compared to the rods of Ref. [3]) creates more and wider bands with the required curvature for BW behaviour. Furthermore these bands can easily be tuned and isolated (in the spectrum) by changing the dimensional characteristics of the crosses. Our design process was simplified by first tuning the shapes of the bands with the metallic crosses, and then scaling the bands to the desired frequency by immersing the crosses in a dielectric background.

2. Negative Index of Refraction vs. Negative Angle of Refraction

The dispersion relation of a two dimensional periodic structure consists of many surfaces, $\omega(k_x, k_y)$, folded within the first Brillouin zone, where $\vec{k} = (k_x, k_y)$ is the crystal wave vector. Power flow within the PC is given by the Poynting vector ($\vec{S}$), which is collinear with the group velocity $\vec{v}_g = \nabla_{\vec{k}}\omega(k_x, k_y)$ in the pass-band. This direction can be found by examining the equi-frequency contours of the band structure [4]. Wave vector solutions are then chosen such that energy continues to propagate forward through the crystal. In references [2, 3] it has been shown that the refracted waves in the PC exhibit a negative refraction angle

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due to the convex equi-frequency contours centered on the M point. This effect is observable in many PCs (including this work), but it is not an effective negative index of refraction. An effective negative index of refraction and BW behaviour exist simultaneously when the convex contours are centered on the Γ point [4].

3. Numerical Method

The finite-difference time-domain (FDTD) technique was used to calculate the band structures of our metallic PC. The unit cell of the 2D square lattice was represented by a grid of 50 by 50 cells and periodic boundary conditions were applied [5]. The metal was modeled as a perfect conductor, and only the *H* polarization (*H* perpendicular to the periodicity) was considered. The cell was excited by applying a temporal Gaussian pulse to the *H* component. The time response was recorded by sampling the magnetic field at each time step.

The eigen-frequencies for each $\vec{k}$ can in general be found by simply applying an FFT to the times series. Since the calculations are repeated for many different wave vectors, and the resulting spectral resolution is inversely proportional to the number of points in the time data, an estimation of the eigen-frequencies using a signal processing technique would drastically decrease the calculation time. This is even more crucial for the case of 3D simulations. Hence, we have used the Matrix Pencil method [6] to estimate the eigen-frequencies from a short time series for each $\vec{k}$, also resulting in a finer frequency resolution than a direct FFT.

4. PCs with Metallic Crosses

Metallic photonic crystals are suitable candidates for NRI metamaterials because of the richness of their band structures and their better focusing abilities and polarization dependencies as compared with dielectric PCs [3]. For various geometries of the metallic scatterers the band structures can be severely distorted from a folded light cone. In order to have more flexibility in designing NRI metamaterials, and also reduce the conduction losses, a two dimensional square lattice PCs with metallic crosses were studied. Figure 1 shows one unit cell of such a structure, where *l* is the length of the arms, *t* is the thickness of the wires, *f* is the length of the flares, and *a* is the lattice constant.

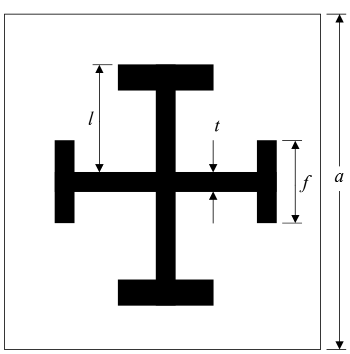
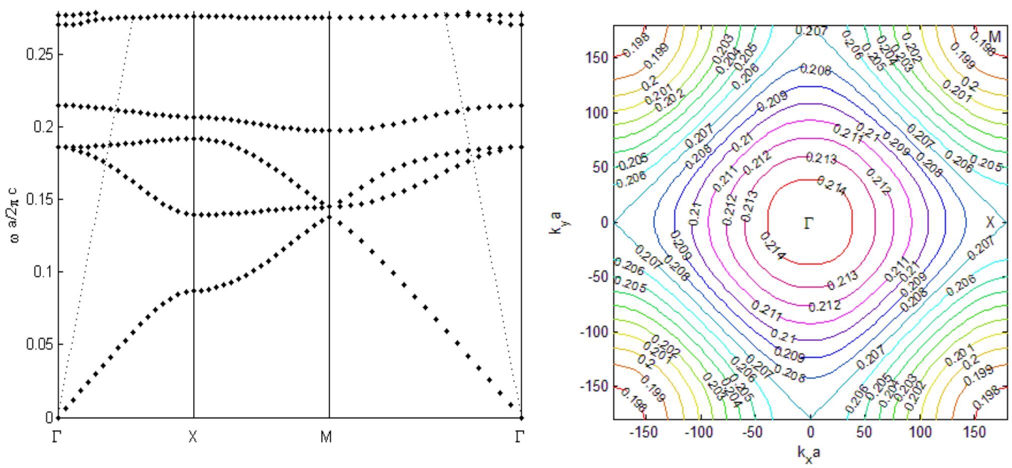


Figure 1: Metallic basis.

It was found that as the arms of the thin crosses were made longer, the band structure became more and more perturbed from the folded light cone. Furthermore, the bands, in general, were lowered in frequency and decreased in bandwidth. This produced large band gaps, and hence isolated the BW bands that would otherwise have been unusable for single beam propagation. Figures 2 and 3 show the band diagrams and the equi-frequency contours for two different cases of our

2D PCs. The fourth band in Fig. 2(a) has BW behaviour for most choices of cross dimensions. This band has the widest bandwidth and becomes spectrally isolated for *l*=0.32*a*, *t*=0.04*a*, and *f*=0 (no flares). A background dielectric of ε=13 was then added to scale the band structure down to a frequency range where "isotropic" behavior is found when an interface with air is considered. The band structure around the irreducible Brillouin zone is shown in Fig. 2(a), and the equi-frequency contours for the fourth band are shown in Fig. 2(b). This band has "isotropic" BW behavior for $0.208 < \omega a / 2\pi c < 0.212$ (2% bandwidth). Flaring the ends of the arms only reduced the bandwidth of the fourth band, and thicker wires narrowed the bandwidth too much.

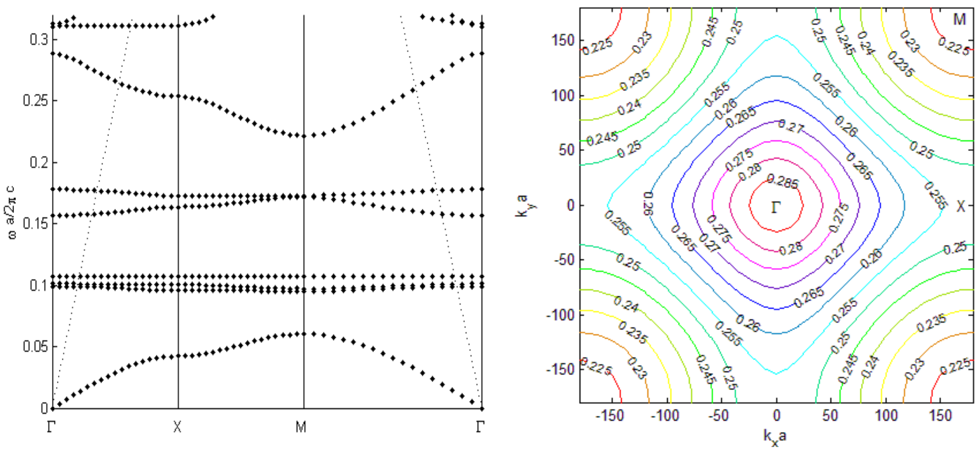


(a) (b)

Figure 2: (a) Band structure and (b) equi-frequency contours for *l*=0.32*a*, *t*=0.04*a*, *f*=0, and ε=13.

Figures 3(a) and 3(b) show the band structure for our metallic PCs with *l*=0.4*a*, *t*=0.04*a*, *f*=0.52*a*, and the equi-frequency surfaces for the band 7. As the figure indicates, the seventh band is convex around Γ (a BW band) and becomes spectrally isolated when flares are added. It was observed that increasing the flare (*f*) moved the band to lower frequencies while increasing its bandwidth. However, if the bandwidth grows too large, the equi-frequency contours become more anisotropic, and hence less desirable. We also studied the effect of increasing the cross arm length (*l*) and observed that it had an outcome similar to increasing the flare, but to a lesser degree. Interestingly, when *l* is too large, the higher bands move into the same frequency region, hence making the coupling to a single mode more difficult. We found that the values in Fig. 3(a) are a good compromise among many possibilities. Once again, by immersing the structure in a host dielectric of ε=13, we were able to lower the frequency of the BW band, so that the frequency range for isotropic response when interfaced with free space is approximately $0.255 < \omega a / 2\pi c < 0.265$ (3.8% bandwidth).

While a 2D metallic PC, as discussed above, can be used in a free-space setup for near field imaging, a 3D NRI metamaterial can also be constructed by sandwiching layers of thin metallic crosses between thin layers of dielectric. Our studies indicate that if the layer thickness is much smaller than *a*, the band



(a) (b)

Figure 3: (a) Band structure and (b) equi-frequency contours for *l*=0.4*a*, *t*=0.04*a*, *f*=0.52*a*, and *ε*=13.

structure is approximately the same for all wave vectors in the *x-y* plane. This layered structure then can be used in integration with microwave networks. Furthermore, the decrease in metal volume fraction will allow the use of these structures at much higher frequencies.

5. Conclusion

An alternative type of metal basis element for 2D PCs that is suitable as a NRI metamaterial has been proposed. More flexibility in the design of the band structure is provided when both metal and dielectric are used. Two bands with an effective negative index of refraction were found and optimized with a thin cross element. In this study the contours of the bands were determined by shaping the metal inclusions, and then the response was scaled to the desired frequency range by immersing the structure in a uniform dielectric background. The proposed metallic elements are suitable for planar fabrication and high frequency operation.

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