

# Dispersion Engineering: Using Negative Phase and Group Indices to Compensate Dispersive Effects

M. Mojahedi, G.V. Eleftheriades, O. Siddiqui, and S. Erickson

Department of Electrical and Computer Engineering, University of Toronto  
10 King's College Road, Toronto, Ontario, M5S 3G4 Canada  
Fax: 416-971-2286, email: mojahedi@waves.utoronto.ca

## Abstract

We have designed and tested negative index of refraction media which display negative group velocities in addition to negative phase velocities. These hybrid systems for which the two concepts of negative phase delay (phase velocity) and negative group delay (group velocity) are combined, will provide the possibility of controlling and engineering various “dispersive effects” associated with electromagnetic pulse propagation. Theoretical foundations for these anomalous behaviors and experimental results verifying the theoretical predictions are presented.

## 1. Introduction

As electromagnetic (EM) wave packets (pulses) travel through a medium, they suffer from so-called “dispersive effects.” These effects can generally be attributed to the dependence of the propagation vector on the frequency,  $k(\omega)$ , which by using photon dispersion relation,  $k(\omega) = \omega n_p(\omega)/c$ , it can also be expressed in terms of the frequency-dependent phase index  $n_p(\omega)$  (customarily referred to as the index of refraction.) These dispersive effects are commonly manifested in terms of delays associated with the underlying harmonics (phase velocity), delays associated with the wave packet envelope (group velocity), broadening of the EM pulse (group velocity dispersion), and so on.

Two recent discoveries have given the researcher more flexibility in controlling these dispersive effects in general and their associated sign (positive or negative) in particular. These discoveries are related to the possibility of constructing media with a negative index of refraction (NIR), also known as the left handed media (LHM) [1], and the ability to propagate EM wave packets with abnormal group velocities for which the group velocity can be negative or in excess of the speed of light in vacuum (superluminal) without violating the fundamental requirements of special relativity (Einstein's causality) [2].

In this paper we present our results in constructing and testing media that exhibit negative or positive group velocities (group index) in addition to exhibiting negative or positive phase velocities (phase index), depending on the frequency of operation. These media can be used to dynamically control two of the aforementioned “dispersive effects”, i.e. the phase and group indices. In the next section we briefly discuss the dynamics of wave propagation in a medium with NIR capable of supporting negative phase velocities, and the physical meaning and correct interpretation of superluminal or negative group velocities. We then present a periodically loaded transmission line (PLTL) that combines both effects, hence providing the designer with the ability to choose a combination of

positive or negative phase and group velocities at will. Section 3 describes our experimental results, and Section 4 contains our final remarks.

## 2. Theory

As stated in the previous section the dispersive effects associated with a propagating EM pulse can be described by the frequency-dependent wave vector,  $k(\omega)$ , which can be expanded using the Taylor's theorem according to

$$k(\omega) = v_p^{-1} \omega_0 + v_g^{-1} (\omega - \omega_0) + \dots = \frac{1}{c} \left\{ \omega_0 n_p \Big|_{\omega_0} + n_g \Big|_{\omega_0} (\omega - \omega_0) + \dots \right\} \quad (1)$$

In Eq. (1) the phase velocity, group velocity, and group index are related by

$$v_p = c/n_p; \quad v_g = \frac{c}{n_g(\omega)} = \frac{c}{n_p + \omega dn_p/d\omega}. \quad (2)$$

Considering a medium of length  $L$ , a more inclusive description of the “dispersive effects” equally applicable to both spatially extended systems [ $k(\omega)L \gg 1$ ] and lumped element circuits [ $k(\omega)L \ll 1$ ] can be obtained by considering the insertion phase function  $\phi(\omega)$ . In the case of spatially extended systems, the insertion phase is related to the propagation effects [ $\phi(\omega) = -k(\omega)L$ ], whereas in the case of lumped element circuits it can be attributed to the phase of the system impulse response. Once again, using the Taylor's expansion the phase function can be written as

$$\phi(\omega) = \tau_p \omega_0 + \tau_g (\omega - \omega_0) + \dots \quad (3)$$

In the case of spatially-extended system [ $k(\omega)L \gg 1$ ] the relations between the phase and group velocities and the phase and group delays ( $\tau_p, \tau_g$ ) are given by

$$v_p = L/\tau_p; \quad v_g = L/\tau_g. \quad (4)$$

Whereas, in the case of lumped element circuits, the phase and group delays are the consequence of various time constants associated with the circuit elements. The possibility of constructing media with NIR and structures supporting abnormal group velocities now will allow us to combine these effects such that the hybrid system can demonstrate a combination of positive or negative phase and group delays and hence positive or negative phase and group velocities.

An effective NIR can be obtained in various structures such as the split ring resonator (SRR) and strip wire (SW) medium, photonic crystals, and PLTL [1, 3, 4]. While in the present paper we focus on the PLTL approach, the same conclusions are also applicable to the other structures and results regarding SRR and SW will be presented at the conference.

An effective NIR can be obtained for a distributed network of transmission lines in which the position of the customarily series inductor and shunt capacitor are exchanged. Such system is capable of supporting the backward waves for which the group velocity (or more precisely the Poynting vector) and the propagation vector are anti-parallel. For example, such a transmission line excited at one end by a source, will produce a pulse envelope that moves away from the source (positive group velocity), while the underlying elementary excitations move toward the source (negative phase velocity).

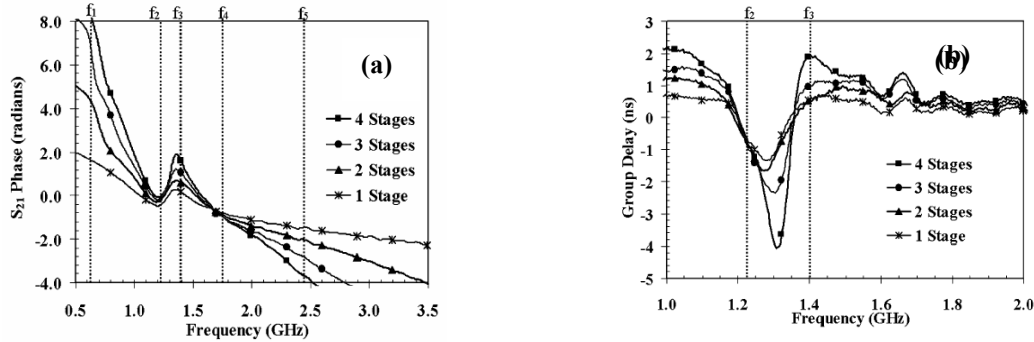
On the other hand, a transmission line capable of supporting negative group delay and hence negative group velocity can exhibit a more striking effect. This transmission line, excited at one end by a smoothly-varying EM pulse such as Gaussian or Sinc input, and connected at the other end to a detector, will generate a Gaussian or Sinc function at the output which precedes the input. This shift to earlier times of the output peak (and output envelope) is the physical meaning of negative group delay and group velocity. There is a point worth emphasizing here. While this shift to earlier times of the output peak compared to the input peak is counterintuitive, it does not present any difficulty with the requirements of Einstein's causality, since from a purely theoretical point of view the information

and group velocity are not the same under all circumstances. In other words, one may decide to practically identify the arrival of an EM wave packet by detecting its peak, half maximum, or any other point on the envelope; however, from a fundamental point of view the genuine information carried by such a pulse must be attributed to its earliest transient behavior, where the pulse is not analytical. These transients are commonly referred to as the pulse front and precursors. Here, for the sake of brevity, it suffices to mention that such points of non-analyticity are present for any physically realizable signal, and while due to their high frequency content and small amplitude may not be the most suitable candidates for routine detection of the signal, nevertheless they exist and travel with exactly luminal speed under all circumstances.

Figure 1 shows the unit cell of a PLTL that simultaneously exhibits negative phase and group delays, i.e. negative phase and group velocities. The dispersion for such a PLTL with (solid line) and without (dashed line) the  $R_r, L_r, C_r$  resonator is also shown. The branches (I) and (IV) describe the dispersive behavior of the one dimensional system in its fundamental and next higher mode. The figure also displays the regions for which the phase and group velocity are anti-parallel ( $v_p < 0, v_g > 0$ ) and where both are negative ( $v_p < 0, v_g < 0$ ).

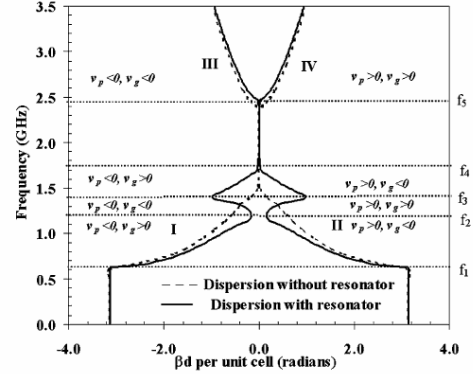
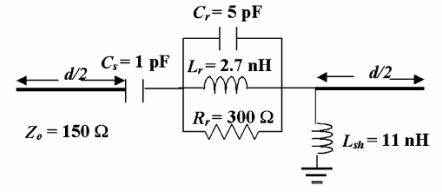
### 3. Practice

One-dimensional PLTLs with 1, 2, 3, and 4 unit cells were constructed and the devices were characterized in both the frequency and time domains. For the sake of brevity, here we only show



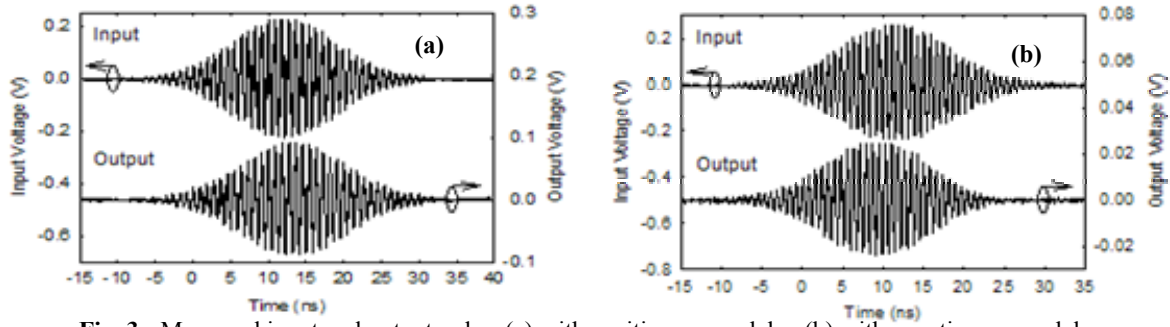
**Fig. 2:** (a) Measured transmission phases, (b) measured group delays for the PLTLs with 1, 2, 3, 4, unit cells

some of the most relevant results, leaving the full discussion to the actual presentation. Figures 2 (a) and (b) show the measured transmission phases and group delays for these devices. Figure 2(a) shows that for frequencies less than  $f_4$  the insertion phase of a longer line lies above that of a shorter line, as expected for a medium with NIR, while the order is reversed for frequencies greater than  $f_4$ . In other words, while for frequencies less than  $f_4$  the transmission line operates within the NIR band marked (I) in Fig. 1, for frequencies larger than  $f_4$  the positive index of refraction band marked (IV) properly describes the propagating mode. More importantly, note that in the frequency range  $f_2 < f < f_3$  the slope of the transmission phase is reversed hence implying a reversal of the sign associated with the group delay ( $\tau_g = -\partial\phi/\partial\omega$ ). Figure 2(b) shows the measured group delays from which the reversal of the sign associated with the group delay and hence group velocity is evident.



**Fig. 1:** Transmission line model and the dispersion relation for the PLTL exhibiting simultaneous negative phase and group velocity.

The effect of negative group delay can also be measured directly in the time domain. Figure 3 shows



**Fig. 3:** Measured input and output pulse, (a) with positive group delay (b) with negative group delay.

the result of exciting a 3-stage PLTL (with a slightly different value of  $R_r$  than indicated in Fig.1) with a 40 ns wide Gaussian pulse. Figures 3(a) and (b) show the relations between the input and output pulses when the central frequency of the Gaussian wave packet is within the positive and negative group delay regions respectively. In both cases the frequency content of the Gaussian is within the NIR band. While Fig. 3(a) demonstrates negative phase but positive group velocities (backward waves), Fig. 3(b) is an example of both negative phase and negative group velocities. Note that while in Fig. 3(a) the peak of the output is delayed by approximately 1.5 ns, in Fig. 3(b) the output peak is advanced by approximately 2.32 ns ( $\tau_g = -2.32$  ns). This advance in time is precisely the physical meaning of negative group velocity. Three additional points are worth mentioning. First, with appropriate design and excitation of the transmission line one can ensure that the overall shape of the output pulse closely resembles the input, minimizing the pulse-broadening effects. Second, while Fig. 3(b) shows a reduction in the amplitude of the transmitted pulse for our passive PLTL, this pulse attenuation is not a universal requirement of negative group delay, and in principle it is possible to design systems without attenuation. Third, similar behavior has also been observed in NIR consisted of SRR and SW, the results of which will be discussed during the presentation.

#### 4. Conclusion

We have designed, constructed, and tested PLTLs which exhibit negative or positive group index in addition to negative or positive phase index. This will allow engineers to design systems that exhibit positive or negative phase and group velocities depending on the frequency of operation.

#### Acknowledgement

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