Time and Frequency Evolution of Precursor Fields in Dispersive Media using FDTD and Joint Time-Frequency Analysis

Reza Safian, Costas D. Sarris*, Mohammad Mojahedi
The Edward S. Rogers Sr. Department of Electrical and Computer Engineering
University of Toronto, Toronto, ON, M5S 3G4, Canada *

Abstract

In this paper, the Finite Difference Time Domain (FDTD) technique is combined with time-frequency analysis, to provide a versatile approach for the modeling of precursor fields in dispersive media. Previous analytical studies, based on asymptotic analysis, are restricted to simple media, while the proposed approach can address spatially complex and spatially dispersive media, including artificial dielectrics. In addition, the time-frequency analysis provides an insightful description of broadband pulse propagation in dispersive media, including the temporal evolution of pulse harmonics.

1 Introduction

Wave propagation in dispersive media is a subject of profound theoretical and practical interest. It has its origin in the classic analysis of Sommerfeld and Brillouin [1], who used the asymptotic method of steepest descent to describe the propagation of a unit step modulated signal of constant carrier frequency in a single resonance Lorentz medium. The purpose of their analysis was partly to obtain the physically proper velocity at which the "signal" propagated in the medium that led to the discovery of the first (or Sommerfeld) and second (or Brillouin) precursor fields whose dynamical evolution preceded the evolution of the main signal. The Sommerfeld and Brillouin definition of the concept of wave velocity into such terms as phase, group, energy, and forerunner (both Sommerfeld and Brillouin forerunners) continues to be the standard description today. It is well known that all these velocities except the forerunner's velocity can exceed c, being "superluminal" in special circumstances [2, 3]. Despite this counter intuitive behavior, it is widely accepted now that such behavior does not violate the requirements of relativistic causality, since under all circumstances the precursors propagate with the speed of light in vacuum, c [3]. The fact that precursors precede the evolution of the main signal in a dispersive medium makes them an essential part of describing the signal arrival and signal velocity. Therefore, to have a comprehensive description of signal velocity and accordingly information velocity, complete understanding of the precursor fields is necessary.

In the case of the ultrashort pulses precursors could also affect the steady-state appearance of the pulse [4]. With the growing trend towards the use of shorter pulses in different fields, this is another reason that motivates the study of the precursors.

The asymptotic theory provides a complete description of the pulse propagation in simple dispersive media and a clear physical insight into the observed dynamics of the pulse propagation [5]. On the other hand, numerical techniques, such as the Finite Difference Time Domain (FDTD, [6]), offer an alternative way for the description of pulse propagation in media with temporal or spatial dispersion, including artificial dielectrics and metamaterials.

A more interesting question, that FDTD by itself cannot address, is what frequency components of the signal give rise to which precursor fields. This question can be addressed using the signal-processing

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theory of joint time-frequency analysis [7], which provides the temporal localization of the frequency components of a signal. Considered as such, joint time-frequency analysis is a systematic method, applicable to any arbitrary dispersive medium. The combination of FDTD with time-frequency analysis provides an effective and versatile tool for an insightful characterization of pulse propagation dynamics.

Simulation of the precursor fields in time-domain has been done using FDTD [8]. In this paper, the Auxiliary Differential Equation (ADE, [6]) method is used for the FDTD modeling of temporally dispersive media. As a time-frequency analysis tool, the Wigner-Ville distribution is employed. The latter is briefly introduced in the next section. To show the effectiveness of the combination of FDTD and time-frequency analysis in the study of precursor fields, propagation of a modulated Gaussian pulse inside a highly absorptive single resonance Lorentzian medium is simulated. Simulation results that show the time-frequency evolution of the precursor fields are presented and corroborated through previously published analytical studies.

2 Time-Frequency Analysis (TFA)

Among many applicable time-frequency distributions, the Wigner-Ville distribution (WVD) has been chosen, because it provides the highest resolution in the time-frequency plane. The WVD of a signal $x(t)$, with a Fourier transform $X(f)$, can be defined either as:

$$W_x(t,f) = \int_{-\infty}^{+\infty} x(t+\tau/2)x^*(t-\tau/2)\exp(-j2\pi f\tau)d\tau, \tag{1}$$

or as:

$$W_x(t,f) = \int_{-\infty}^{+\infty} X(f+\xi/2)X^*(f-\xi/2)\exp(-j2\pi\xi t)d\xi. \tag{2}$$

The main drawback of the WVD is the appearance of relatively large spurious interference (cross) terms. These interference terms usually can be readily recognized from their oscillatory form.

3 Evolution of a modulated Gaussian pulse in Lorentzian medium

A modulated Gaussian input pulse of a constant carrier frequency ω_c and pulse width $2T_s$, centered around time $t_0 > 0$, at the $z = 0$ plane is considered. Its mathematical expression is given by

$$S(t) = exp\left(-\left(\frac{t-t_0}{T_s}\right)^2\right) sin(\omega_c t). \tag{3}$$

The pulse is propagating in the $+z$ direction, through a linear dielectric whose frequency dispersion is described by the single resonance Lorentz model with the complex index of refraction:

$$n(\omega) = \left(1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + 2i\delta\omega}\right)^{1/2}, \tag{4}$$

which occupies the half space $z > 0$. Here, ω_0 is the resonance frequency, ω_p is the plasma frequency, and δ is the damping constant of the dispersive, lossy dielectric. The material absorption band is defined over the approximate angular frequency domain $\left[\sqrt{\omega_0^2 - \delta^2}, \sqrt{\omega_1^2 - \delta^2}\right]$, where $\omega_1 = \sqrt{\omega_0^2 + \omega_p^2}$. The Brillouin choice [1] of the material parameters $\omega_0 = 4 \times 10^{16} Hz$, $\omega_p = \sqrt{20} \times 10^{16} Hz$ and $\delta = 0.28 \times 10^{16}$, is used in the numerical calculations. The parameters of the modulated Gaussian pulse of (3) are chosen as follows: $\omega_c = 5.75 \times 10^{16} Hz$, $2T_s = 2 \times 10^{16}$ and $t_0 = 3.035 \times 10^{-16}$. The frequency components of the propagated field that lie within the medium absorption band $\omega_0 \leq \omega \leq \omega_1$ will be greatly attenuated as the Gaussian pulse propagates inside the Lorentzian medium. Practically, they are absent from the propagated field structure.

Figure 1 shows the time and frequency evolution of the input modulated Gaussian pulse at propagation distance $z = 0.5\mu m$. The horizontal axis is the dimensionless space-time parameter $\theta = ct/z$, where z is the distance of the sampling point from the source. The Sommerfeld precursor field components first emerge in the propagated field structure. The highest frequencies appear in the beginning and as time goes by lower frequencies appear. There is a downward trend in the beginning for the frequencies above the absorption band. The trend for the frequencies below the absorption band is the opposite. As the propagation distance increases, the Brillouin precursor pulse components emerge. Figure 2 shows the Sommerfeld and Brillouin precursors at $z = 1\mu m$. The Brillouin precursor includes frequencies that are below the absorption band and exhibit an upward trend. There are some cross terms in the absorption band in both figures that can be simply identified from their oscillatory appearance. The frequency distribution of the signal also shows that there are no frequency components in the absorption band. These results are in good agreement with the results calculated using asymptotic techniques in [9]. Using FDTD and time-frequency distributions, the propagation of the precursors in a spatially dispersive Bragg reflector have also been studied. The simulations show some interesting results regarding the time and frequency evolution of the precursors in Bragg reflector that will appear in coming publications.

4 Conclusion

The FDTD technique and time-frequency analysis have been combined to describe the propagation of a modulated Gaussian pulse in passive Lorentzian media, with emphasis in the evolution of Sommerfeld and Brillouin precursor fields. Numerical results validating this technique were presented. Compared to previous analytical studies, the proposed approach presents a robust and versatile framework for the description of pulse propagation dynamics in temporally or spatially dispersive media.

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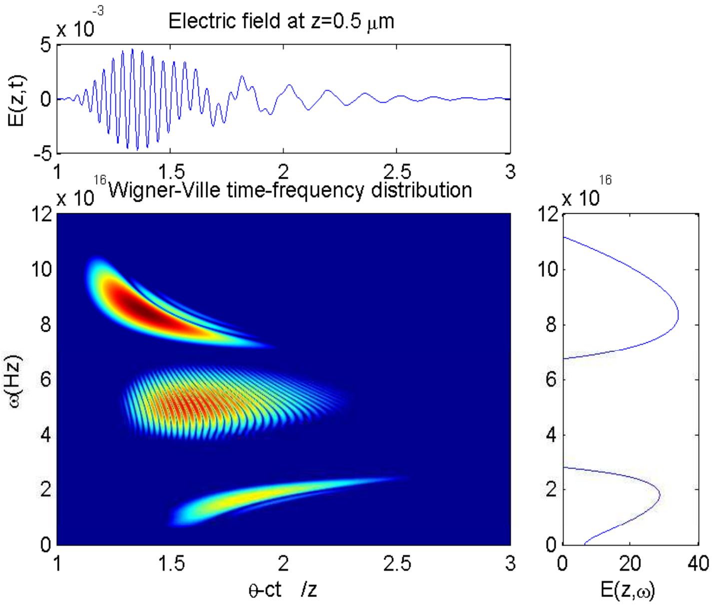

Figure 1: (Top) Modulated Gaussian pulse after propagating $0.5\mu m$ through the dispersive Lorentzian medium. (Right) Frequency distribution of the pulse (Log scale) (Left) time-frequency distribution (Log scale in frequency).

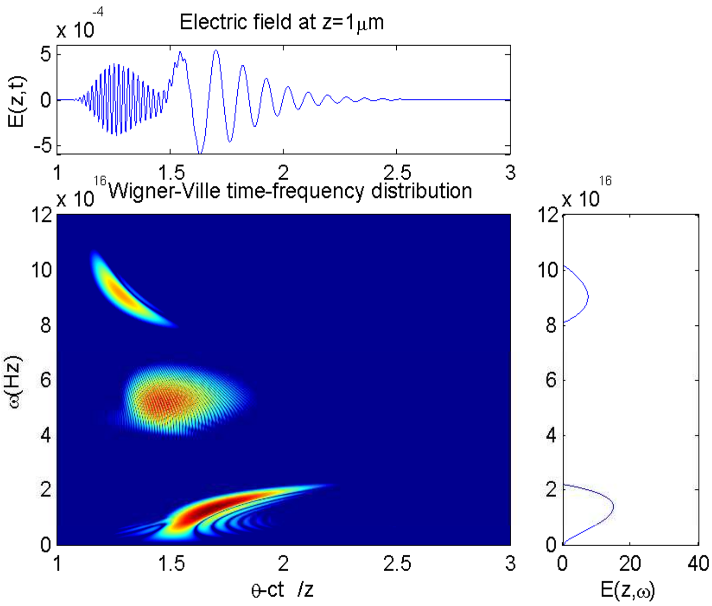

Figure 2: (Top) Modulated Gaussian pulse after propagating $1\mu m$ through the dispersive Lorentzian medium. (Right) Frequency distribution of the pulse (Log scale) (Left) time-frequency distribution (Log scale in frequency).