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NUMERICAL CALCULATION OF PRECURSOR FIELDS IN ONE DIMENSIONAL PHOTONIC CRYSTAL

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ABSTRACT
The presence of superluminal (in excess of the speed of light in vacuum) or negative group velocities in passive or active Lorentzian medium, one-dimensional photonic crystal (1DPC), undersized waveguide, and other structures has been theoretically predicted and experimentally observed. While due to the temporal or structural dispersions the group velocity in these media is abnormal, it has been argued that such behavior is not contradictory to the requirements of relativistic causality (Einstein causality), since the earliest field oscillations known as the precursors or forerunner must and will obey Einstein causality. In this paper, for the first time, by using finite difference time domain (FDTD) technique in conjunction with the joint time frequency analysis (JTFA) we present the dynamical evolution of these earliest field oscillations; clearly indicating that despite the observed abnormal group velocities the precursor fields are indeed subluminal, and as such, must be associated with the arrival of "genuine information". Moreover, this work presents the combined FDTD and JTFA as a viable tool in studying the dynamical evolution of the transient and steady-state pulse propagation in dispersive media.

KEY WORDS
Superluminal, Precursors, One Dimensional Photonic Crystal, Joint Time-Frequency Analysis.

1 Introduction

The origin of the classical theory of pulse propagation in linear, homogeneous, isotropic, causally dispersive medium is the classical work of Sommerfeld and Brillouin [1]. In a causally dispersive medium the signal arrival appears in the dynamical field evolution as an increase in the field amplitude from that of the precursor fields to that of the steady-state signal. Therefore, correct description of the signal arrival is closely related to the proper analysis of the precursor fields and their role in the field evolution. Sommerfeld and Brillouin classification of different wave velocities into terms such as phase, group, energy, and precursors velocities continues to be the standard description today. More recently, it has been shown that under some circumstances all of the above velocities, except for the precursor's velocity, can become abnormal (superluminal or negative) [2],[3]. In particular, superluminal or negative group velocities at microwave frequencies [4]-[6], at optical frequencies [7], and in the single-photon limit [8] have been experimentally observed. In addition to Sommerfeld and Brillouin, Oughstun and coworkers have extensively studied the precursor fields in Lorentzian media by refining and generalizing the previous results [9]. Surprisingly, there has been little or no work on the precursor fields in dispersive media other than Lorentzian. For example, while the presence of superluminal group velocity in 1DPC has been theoretically predicted and experimentally verified, there has been few attempts in studying the forerunner propagation in these structures. In this paper the time and frequency evolution of a modulated Gaussian pulse propagating inside a 1DPC is studied. The FDTD method [10] is used to calculate the time evolution of the pulse and JTFA [11] is used as a post processing technique to calculate the frequency evolution of the signal including the transient response (the precursor fields).

2 Analysis of One Dimensional Photonic Crystal (1DPC)

As mentioned earlier, there have been few efforts to study the precursors in dispersive structures other than Lorentzian medium. This is mostly due to the complexity of the asymptotic techniques. Based on the results from our previous work [12] the combination of FDTD and JTFA can be used as a robust and accurate tool to study the precursors, or in general the transient response, of the propagating field in dispersive media. In this section this combination has been used to study the transient response of the wave propagation in a 1DPC. The dispersion in the Lorentzian medium originates from the frequency dependent response of the medium dipoles, but it's the inhomogeneous structure of the 1DPC that makes it a dispersive structure. Superluminal group velocity is a consequence of dispersion in 1DPC and it has been experimentally verified in several experiments. In the following, we have analyzed the superluminal propagation of a modulated Gaussian pulse through a 1DPC. The geometry of the 1DPC is based on the physical experiment in microwave domain by

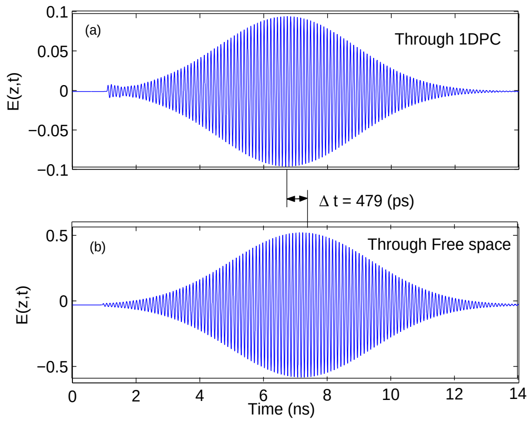

Figure 1. Pulse advancement in 5slab 1DPC.

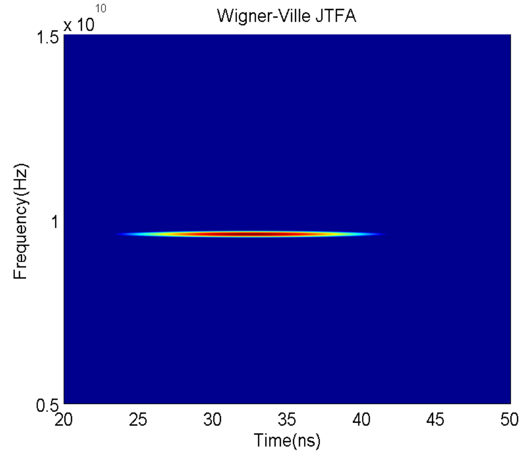

Figure 2. Wigner-Ville JTFA of the output Gaussian pulse in 5slab 1DPC.

Mojahedi *et. al.* [4]. It consists of five dielectric slabs with the width of $1.27cm$ and index of refraction 1.66. The slabs are separated by $4.1cm$ air-gaps. The structure is excited with a modulated Gaussian pulse of temporal width $5.5ns$ (FWHM) ($T_s = 3.11 \times 10^{-9}s, t_0 = 3 \times T_s s$) and carrier frequency $\omega_c = 9.6GHz$. The Gaussian pulse is zero for $t < 0$ and it is defined as

$$S_{(t)} = exp[-(t - t_0/T_s)^2]sin(\omega_c t) \tag{1}$$

for $t > 0$. The center frequency of the modulated Gaussian excitation is inside the bandgap of the 1DPC and hence the contributions of the frequency components outside the bandgap are negligible. Figures 1(a) and 1(b) show a Gaussian pulse propagated through a 1DPC and through vacuum of the same physical length as the 1DPC, respectively. From the figure it is evident that the peak in Fig.1(a) arrives at the output $479ps$ sooner than the vacuum pulse, implying a group velocity of $v_g = 2.71c$ for the pulse traveling in the 1DPC.

The excitations in these simulations have a very smooth turn on (the amplitude of the front is e^{-9}, whereas the peak amplitude is unity). Figure 2 shows the Wigner-Ville JTFA of the pulse propagated through the 1DPC which has the same structure as the pulse propagated in the free space. It is observed that the frequency components are concentrated around the center frequency of the pulse, starting with lower frequency components, evolving to the highest frequency components, and back to the lower ones at the end.

The interesting question is that whether or not these abnormal behaviors are consistent with the requirements of special relativity, which demands no information to be transmitted faster than the speed of light in vacuum. Mojahedi *et.al.* [14] have argued that if information carrying signal is to be presented as an analytical function extending in time from $-\infty$ to $+\infty$, then by definition the signal posses infinite number of derivatives and the future and early behavior of the pulse can be predicted by using Taylor expansion about any point in time. Therefore, a signal without any turn-on point does not convey “genuine” information. They also argue that a signal that conveys information, and is physically realizable, is a causal signal that has a beginning (“front”) in time and space. Consequently, in a noiseless channel (as we have considered in all the simulations) the earliest time that the future value of the information carrying signal can be predicted is $t = 0^+$, since $t = 0$ by definition is a point of non-analyticity for which the Taylor expansion does not exist. Therefore, the genuine information regarding the correct value of a causal signal is contained within the time interval beginning with $t = 0$ (the “front”) and times immediately following it.

Previous simulation shows the superluminal group velocity in the 1DPC for a pulse with a very smooth front, therefore, the effect of the front is not observed in the output. To observe the evolution of the front we have introduced an excitation that enforces the pulse front explicitly. In this excitation, a second order non-analyticity has been introduced in the beginning of a Gaussian pulse similar to the pulse used previously. The excitation is zero at $t = 0$, and the amplitude of pulse increases smoothly with time (the envelope is a second order polynomial). At the point the amplitude of the pulse reaches $e^{-2.25}$ the second order non-analyticity is introduced by matching the second order polynomial to a Gaussian envelope with parameters ($T_s = 3.11 \times 10^{-9}s, t_0 = 1.5 \times T_s s$). Figure 3 shows this excitation and its frequency distribution as compared to the frequency distribution of the excitation with a smooth turn-on used in the first simulation. Comparing the frequency distributions of the two pulses shows that introducing the non-analytic point adds low and high frequency components with small amplitudes to the frequency distribution of the pulse with smooth turn-on. We may note that the frequency distribution of the pulse with enforced front is still narrower than the stop-band of the 1DPC (which is $1.4GHz$ around the center frequency 9.5GHz); therefore, the steady-state dispersion mechanism is similar for both excitations. The only difference between the output pulse

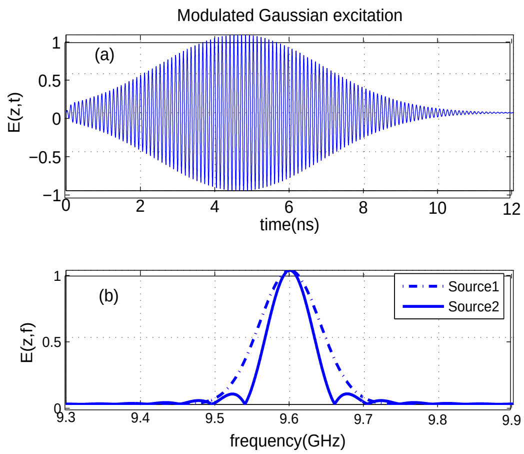

Figure 3. (a) Modulated Gaussian excitation with enforced front (b) frequency distribution of the Gaussian excitations with smooth front (source1) and with enforced front (source2)

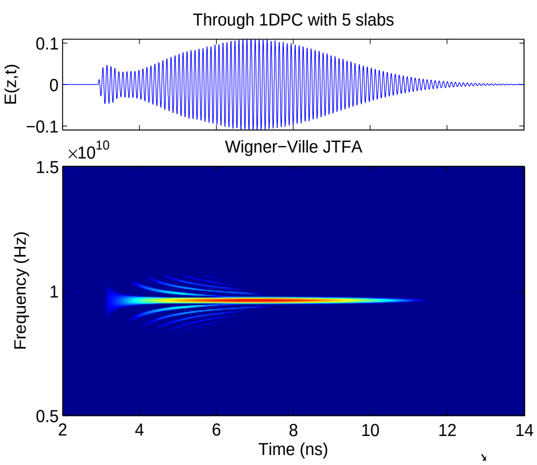

Figure 4. WVD JTFA of the output Gaussian pulse in 5slab 1DPC.

in the 5 slab 1DPC due to the excitation with an enforced front and the output pulse in previous simulation with a smooth front excitation is the oscillations ("precursors") in the early part of the pulse. Figure 4 shows the pulse propagated through a 1DPC with 5 slabs and its Wigner-Ville JTFA. The JTFA shows that these precursors contain high and low frequencies that appear at the same time in the beginning of the pulse and are not present in the remaining part of the pulse. Simulations that are not presented here show that similar behavior is observed when a Lorentzian medium is excited away from the resonance with similar excitation, where low and high frequencies due to the discontinuity in the beginning of the pulse appear concurrently at the output.

The interesting observation is that although group velocity is superluminal there is a subluminal delay in the appearance of the front. Figure 5 shows that the peak of the pulse that has traveled through the 1DPC appears in the output sooner than the peak of the pulse that has traveled the same length in the free space, but as it can be clearly seen in the inset the early oscillations of the free space pulse appears sooner than the early oscillations of the pulse that has traveled through the 1DPC. In other words, the front velocity for the 1DPC pulse is subluminal, and to the extent that information should be associated with the points of non-analyticity, these earliest oscillations travel subluminally.

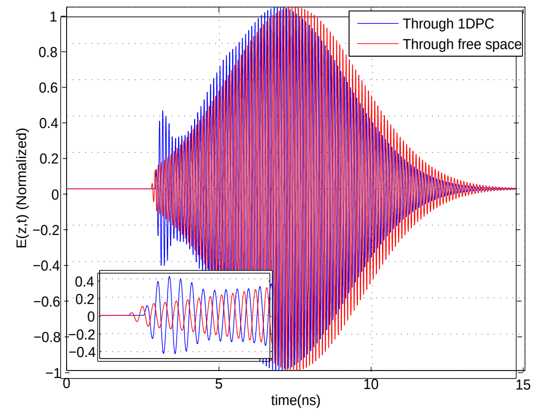

Figure 5. Normalized Gaussian pulse with enforced front at the output of the 1DPC compared to the same pulse that has traveled the same length in free space.

To further study the propagation of the front inside an inhomogeneous structure, in another series of numerical experiments, a 1DPC with the same structure as the previous simulations but with different number of slabs has been studied. While, increasing the number of slabs slightly changes the dispersion characteristics of the structure, it will cause a stronger attenuation of the signal. Therefore, by increasing the number of slabs we can study the attenuation rates of different portions of the pulse. Figure 6 shows the output pulse for 1DPCs with different number of slabs. While adding more slabs increases the attenuation, it can also be seen from the figure that the attenuation rate is not the same for different portions of the pulse. As the number of slabs increases the precursors are separated from the remaining part of the pulse. In the 5-slab 1DPC the peak of the precursors is lower than the peak of the main pulse, but in the case of 9-slab 1DPC the amplitude of the precursors is higher than the main part of the pulse. This in effect shows that the decay rate for the precursors is less than the decay rate for the main pulse.

Figure 7 compares the energy of the precursor field and the main part of the signal calculated for 1DPC with different number of slabs. The energy is defined as

$$E = \int_{t_1}^{t_2} |f_{(t)}|^2 dt \qquad (2)$$

where t_1 and t_2 are the beginning and the end of each portion of the pulse. The beginning of the precursors is the time that pulse appears in the output and the end is defined as the time that the absolute amplitude of the pulse is min-

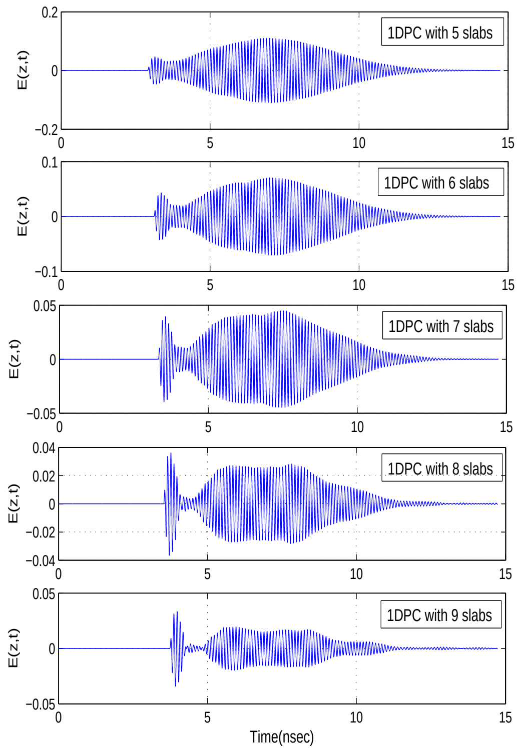

Figure 6. Propagation of the Gaussian pulse inside the 1DPC with different number of slabs.

imum between the precursors and the main signal (it is not necessarily zero). The main signal starts at the end of the precursors and it end as the signal disappears in the output sampling point. It can be seen from Fig. 7 that the energy of the main signal is almost 50 times the energy of the precursors for the 1DPC with 5 slabs. As the number of slabs increases the energy of the main signal decreases drastically but the rate of attenuation for the precursors is lower. For example, in the case of 1DPC with 9 slabs the energy of the main signal and precursors are comparable (the energy of the main signal is 3 times the energy of the precursors).

3 Conclusion

In this paper we have described the propagation of a modulated Gaussian pulse in a 1DPC. Time and frequency evolution of the precursors and the fact that their propagation velocity is subluminal has been verified. Despite the claim by the authors in reference [6], asserting that that they have demonstrated superluminal information velocity using (smooth) signals by means of under cutoff frequencies and without generating further precursors; this does not seem to be true in practice. In order to send a signal some discontinuities have to be created that in turn generates new precursors and if the signal can not overtake its

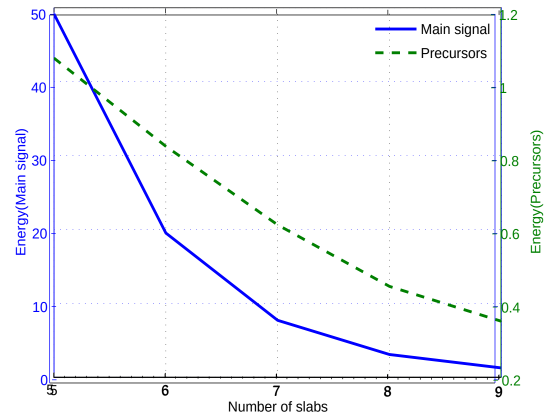

Figure 7. Energy of the precursor and the main part of the pulse for different number of slabs.

precursors, then information can not be transmitted faster than light, in spite of the fact that group velocity can be superluminal. An interesting point in these simulations is that the front of the pulse is subluminally delayed as it travels through the structure. Therefore, although the group velocity is superluminal the precursors, that are the genuine carriers of information, are not. This work also presents the combination of FDTD and JTFA as a tool that can be used to study both transient and steady-state of the time and frequency evolution of a pulse propagating inside a dispersive medium.

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