Time- and Frequency-Domain Measurements for an Active Negative Group Delay Circuit

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Time- and frequency-domain measurements of an active negative-group-delay (NGD) circuit exhibiting gain and NGD at low frequencies are presented. The active design eliminates the problem of high loss, associated with the passive transmission line exhibiting both negative refractive index and NGD [1–3]. A high-frequency, active NGD circuit topology with an operation frequency near 2.5 GHz, is also discussed.[1]

Introduction

We have recently characterized the frequency- and time-domain responses of a passively loaded transmission line exhibiting unusual properties, namely a negative refractive index (NRI) and a negative group velocity (NGV) in the same frequency band [1–3]. The NRI is equivalent to a negative phase delay, and the NGV is equivalent to a negative group delay (NGD); these two functions control the zero- and first-order dispersion terms. The NRI-NGD behaviour of the loaded transmission lines of [1–3] was obtained by periodically loading the transmission line with a series capacitor, shunt inductor and series RLC resonator.

This passive device exhibited a great deal of loss in the NGD band due to the resonant absorption of the RLC, and so is not practical for applications such as NGD-NRI interconnects and buses. In this paper we present a solution to the high-loss problem: active loading elements that produce a negative group delay in a band of low gain. We will briefly describe the design procedure used to obtain the transfer function, and then will present simulated and experimental results in the frequency and time domains for a low-frequency implementation. We will also describe a circuit topology that can achieve similar results for frequencies up to 10 GHz. While the results presented here are for the active loading elements alone, a transmission line loaded with these elements and an appropriate series capacitor and shunt inductor could exhibit both NRI and NGD without any loss.

Transfer Function Design

A NGD transfer function with gain can be designed by engineering pole- and zero-positions in the complex frequency plane. It can be proven that zeros in the left-half of the complex frequency plane (LHP) contribute negative values to the system group delay [4]. Also, it is well-known that the poles of a system provide gain at some frequencies, and that in order for the system to be stable, the poles must be in the LHP. Thus in order to have a stable system exhibiting NGD and gain in the

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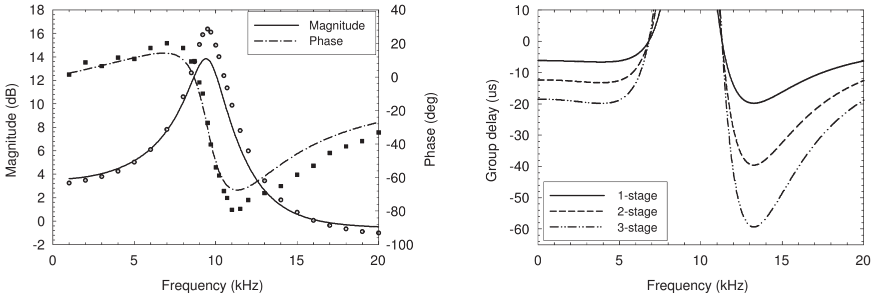
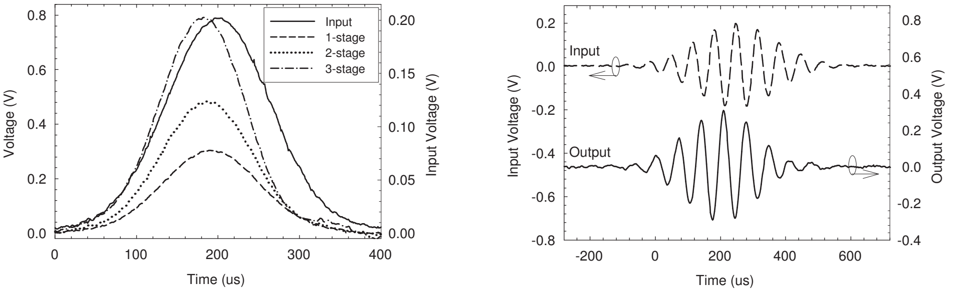


(a) Low-frequency magnitude and phase (simulations and measurements)

(b) Low-frequency group delay (calculated)

Figure 1: (a) Simulated magnitude (solid line) and phase (dot-dash line), and experimental magnitude (circles) and phase (squares) for the low-frequency NGD circuit. (b) Calculated group delay for 1, 2 and 3 stages of Friend circuit.



(a) Baseband

(b) 15 kHz

Figure 2: Experimental time-domain results (raw data) of (a) baseband Gaussian pulses (FWHM 150 μs) through one, two and three stages of Friend circuit; (b) Gaussian pulse modulated at 15 kHz (FWHM 290 μs) through three stages of Friend circuit.

same frequency band, we must have both LHP zeros and LHP poles, arranged in some particular relation with respect to each other. To satisfy these requirements, we have designed a NGD biquadratic transfer function

$$T(s) = H_o \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)} \tag{1}$$

with complex zeros and poles, $z_{1,2} = \gamma \pm j\delta$ and $p_{1,2} = \alpha \pm j\beta$, $\gamma < 0$, $\alpha < 0$, and $\delta \simeq \beta$.

Results

The transfer function (1) with the zeros $z_{1,2} = (-28 \pm j70) \times 10^3$, and poles $p_{1,2} = (-6 \pm j60) \times 10^3$, was implemented with the Friend single-amplifier biquadratic (SAB) active filter [5], using commercially-available components and a LM741 operational amplifier. The simulated and measured transfer function magnitude (solid line) and phase (dot-dash line) for the low-frequency circuit are shown in Fig. 1(a); simulations

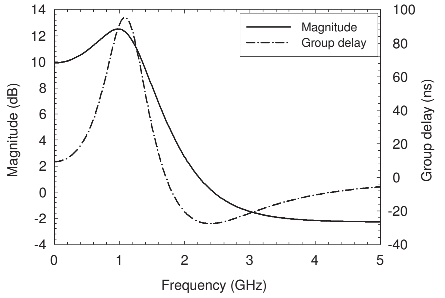


Figure 3: Simulated magnitude and group delay for the high-frequency CCII- topology.

were performed in Agilent Advanced Design System (ADS) using an ideal op amp. Fig. 1(b) shows the group delay response, calculated using (1) and the given values for $z_{1,2}$, $p_{1,2}$. Regions of negative group delay can be seen below and above the central gain line, in the frequency ranges 0–6 kHz and greater than 10 kHz; the gain region extends from DC to 16 kHz.

Time-domain experiments were used to confirm the coexistence of NGD and gain predicted by the frequency-domain results. Fig. 2 shows raw data from the time-domain experiments, where baseband [full-width at half-maximum (FWHM) 150 μs] and modulated (FWHM 290 μs) Gaussian pulses were passed through a cascade of three Friend circuits. The baseband group delays were calculated to be -8.5 ± 0.1 μs, -14.0 ± 0.1 μs and -19.6 ± 0.1 μs, for the 1-, 2- and 3-stage Friend circuits, respectively, which compare favorably to the calculated values (Fig. 1(b)) of -6.2 μs, -12.3 μs and -18.5 μs. The FWHM of the 3-stage pulse is compressed to $82.5 \pm 0.2\%$ of the FWHM of the input pulse. Fig. 2(b) shows the input to and output from the 3-stage device, excited with a modulated Gaussian pulse. Here the group delay was found to be -47.4 ± 1.3 μs, 16% of the input pulse FWHM. This result is in agreement with the frequency-domain group delay curve of Figure 1(b), which predicts a group delay of -48.5 μs. Note that all errors quoted above are not due to experimental error but to the uncertainty in the positions of the pulse peaks, which were estimated by a 4-parameter Gaussian curve fit.

High-Frequency Design

Transfer function synthesis at low frequencies is typically accomplished with op-amp-based circuits due to the availability of high-performance op-amps and numerous filter configurations. However, adapting these topologies to operate at microwave frequencies is difficult due to the bandwidth limitations of op-amps. Thus, to transport the NGD transfer function design presented above to gigahertz frequencies, we require an alternative topology to the op-amp-based Friend SAB circuit. Current research in the Electromagnetics group at the University of Toronto is investigating active microwave filters that exhibit NGD at 2.5 GHz, using monolithic negative second generation current conveyor (CCII-) based filters in GaAs. The decision to use GaAs CCII-s is based on the work of [6–9]. The monolithic nature and low transistor count of GaAs CCII-s both contribute to signal integrity in our design.

A NGD transfer function achieved with this topology is shown in Fig. 3.

Conclusion

Using an active NGD filter design, we have solved the problem of high loss in the passive NGD loaded transmission line of [1–3]. For a design frequency of 15 kHz, simulation and experimental results in the frequency and time domains show that this circuit exhibits the expected NGD and gain; pulse advances of up to 16% of the pulse FWHM were obtained, while maintaining the pulse amplitude. An alternate circuit topology was used to simulate the same transfer function design at gigahertz frequencies.

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