Field Region Separation

Having just considered the analysis of a half-wave dipole, let us consider for a moment a long wire antenna of length D. The vector potential for a line source directed along the z-axis is found using

$$\boldsymbol{A} = \mu \int \boldsymbol{I}_e(x', y', z') \frac{e^{-jkR}}{4\pi R} dl'. \tag{1}$$

For the infinitesimal and short dipole, we could approximate $R \approx r$. However, as the antenna becomes an appreciable fraction (or larger) of a wavelength, phase errors from the $\exp(\cdot)$ term become significant.

For a dipole on the z-axis,

$$R = \sqrt{x^2 + y^2 + (z - z')^2} = \sqrt{x^2 + y^2 + z^2 - 2z'z + z'^2}. \tag{2}$$

Expressing the field point in spherical coordinates,

$$R = \sqrt{r^2 - 2rz' \cos\theta + z'^2} = r\sqrt{1 + \frac{(-2rz' \cos\theta + z'^2)}{r^2}}. \tag{3}$$

Making use of the binomial theorem,

$$(1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \cdots, \tag{4}$$

we can write

$$\begin{aligned} R &= r\left\{1 - \frac{rz'\cos\theta}{r^2} + \frac{z'^2}{2r^2} - \frac{1}{8}\left(\frac{4r^2z'^2\cos^2\theta - 4rz'\cos\theta z'^2 + z'^4}{r^4}\right) + \cdots\right\} \\ &= r - z'\cos\theta + \frac{z'^2}{2r^2} - \frac{z'^2\cos^2\theta}{2r} + \frac{z'^3\cos\theta}{2r^2} - \cancel{\frac{z'^4}{8r^3}} + \frac{1}{16}\frac{(-8r^3z'^3\cos^3\theta)}{r^5} + \cdots \\ &\approx r - z'\cos\theta + \frac{z'^2\sin^2\theta}{2r} + \frac{z'^3\cos\theta - z'^3\cos^3\theta}{2r^2} \\ &\approx r - z'\cos\theta + \frac{z'^2\sin^2\theta}{2r} + \frac{z'^3\cos\theta\sin^2\theta}{2r^2} \end{aligned} \tag{5}$$

where terms with greater than r^{-3} dependence are assumed to be negligible.

This completes the derivation of the distance R, which matches equation (4-41) in Balanis. Sections 4.4.1 and 4.4.2 show how the error terms in this equation are constrained to determine the Fraunhofer and Fresnel regions.