1. The input impedance of a short-circuited lossy transmission line of length 1.5 m (<\lambda/2) and a characteristic impedance of 100 Ω (approximately real) is 40 - j280 Ω.

a) Find α and β of the line.

b) Determine the input impedance if the short-circuit is replaced by a load resistance \( Z_L = 50 + j50 \) Ω.

c) Find the input impedance of the short-circuited line for a line length of 0.15λ.

Solution:

1. a) compute \( \Gamma_{in} \) and write it in terms of \( \Gamma_{short} \) for a lossy transmission-line

\[
\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = 0.915 e^{-j0.675} = \Gamma_{short} e^{-2\alpha \ell} e^{-j2\beta \ell}
\]

knowing that \( \Gamma_{short} = -1 = e^{j\pi} \), we can match magnitudes and phases

\[
e^{-2\alpha \ell} = 0.915 \quad \Rightarrow \quad \alpha = \frac{1}{2(1.5)} \ln \left( \frac{1}{0.915} \right) = 0.0297 \text{ Np/m}
\]

\[-2\beta \ell + \pi = -0.675 \quad \Rightarrow \quad \beta = \frac{0.675 + \pi}{2(1.5)} = 1.27 \text{ rad/m}
\]

b) compute \( \Gamma_L \) from which you can calculate \( \Gamma_{in} \)

\[
\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.2 + j0.4
\]

\[
\Gamma_{in} = \Gamma_L e^{-2\alpha \ell} e^{-j2\beta \ell} = \frac{\Gamma_L}{\Gamma_{short}} \Gamma_{in}^{\alpha} = \left(0.2 - j0.4\right) \left(0.915 e^{-j0.675}\right) = -0.0857 - j0.4
\]

now convert this reflection coefficient to an impedance

\[
Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 62.2 - j59.8 \Omega
\]

c) we apply the same equation as in part a) but with a new \( \ell \) and using \( \lambda = 2\pi/\beta = 4.94 \) m

\[
\Gamma_{in} = \Gamma_{short} e^{-j2\beta \ell} e^{-2\alpha \ell} = -e^{-j2(2\pi)(0.15)} e^{-2(0.0297)(0.15)(4.94)} = 0.296 + j0.910
\]

and convert to

\[
Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 6.36 + j137 \Omega
\]
2. A lossless transmission line with a characteristic impedance of 50 \( \Omega \) is terminated in a load impedance 40 + j30 \( \Omega \). Using Smith Chart techniques, determine:

a) The reflection coefficient \( \Gamma \) (magnitude and phase), and the VSWR

b) The input impedance seen looking into the line if it is 0.2\( \lambda \) long.

c) The length of line needed to make the input impedance look real (one solution is sufficient), and the associated resistance value.

Solution:

\[
Z_L = \frac{Z_L}{Z_0} = 0.8 + j0.6.
\]

a) \( \Gamma = j0.333 = 0.333\angle 90^\circ \), \( S = \frac{A_{\text{max}}}{A_{\text{min}}} = 2 \)

b) Travel 0.2\( \lambda \) along green arrow to B: \( Z_{in} = 1.25 - j0.75 \Rightarrow Z_{in} = 62.5 - j37.5 \) [\( \Omega \)]

c) Travel along red arrow to Point C: \( l = 0.25\lambda - 0.125\lambda = 0.125\lambda \); \( Z_{in} = 2 \Rightarrow Z_{in} = 100 \) [\( \Omega \)]
3. Match a load impedance $Z_L = 100 + j80 \, \Omega$ to a line with characteristic impedance $Z_0 = 75 \, \Omega$ using a shunt single-stub tuner. Find one solution using an open-circuited stub and another using a short-circuited stub.

Solution:

$$z_L = \frac{Z_L}{Z_0} = \frac{100 + j80}{75} = 1.33 + j1.07.$$  

- Enter $z_L$ (P1) on the Smith chart.
- Draw the SWR circle for the P1 (light blue).
- Transform into admittance $y_L$ (P2).
- Travel towards generator until the intersection of the SWR circle and the $g = 1$ circle.
- Intersections:
  - At P3: $y_{B_1} = 1 + j0.98 \Rightarrow y_{s_1} = -j0.98$; $d_1 = (0.161 - 0.432 + 0.5)\lambda = 0.229\lambda$
  - At P4: $y_{B_2} = 1 - j0.98 \Rightarrow y_{s_2} = +j0.98$; $d_2 = (0.339 - 0.432 + 0.5)\lambda = 0.407\lambda$

a) Using an open-circuited stub:
   - Match from P3: $l_{1,OC} = (0.377 - 0)\lambda = 0.377\lambda$; (red arrow)
   - Match from P4: $l_{2,OC} = (0.123 - 0)\lambda = 0.123\lambda$; (red dashed arrow)

b) Using a short-circuited stub:
   - Match from P3: $l_{1,SC} = (0.377 - 0.25)\lambda = 0.127\lambda$; (dark blue arrow)
   - Match from P4: $l_{2,SC} = (0.123 - 0.25 + 0.5)\lambda = 0.373\lambda$; (dark blue dashed arrow)
Q3. Smith chart
4. Design a quarter-wave transformer is to match a 10 Ω resistor to a 50 Ω line at 2 GHz. The transmission line used is coaxial cable whose dielectric has a dielectric constant \( \varepsilon_r = 2.25 \). Sketch the final design, specifying the dimensions and impedance of the transformer.

Solution:

4. Transformer impedance:
Given: \( R_L = 10 \); want: \( R_0 = 50 \). So, \( R_{\lambda/4} = \sqrt{R_L R_0} = \sqrt{500} = 22.36 \).

Transformer’s physical length:
\[ \lambda = \frac{v_p}{f} = \frac{c}{\sqrt{\varepsilon r} \cdot \omega/2\pi} = 0.6283[m] \Rightarrow l = \frac{\lambda}{4} = 0.1571[m]. \]

Ratio between Outer and Inner Conductors
For a (lossless) coaxial cable (see Ulaby 6/E, p. 53):
\[ Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu}{\varepsilon}} \ln \left( \frac{D}{d} \right) = \frac{\eta_0}{2\pi \sqrt{\varepsilon_r}} \ln \left( \frac{D}{d} \right), \text{ where } \eta_0 = \frac{\mu_0}{\sqrt{\varepsilon_0}} \approx 377[\Omega] \] is the characteristic impedance of free space, while \( D \) and \( d \) are the diameters of the inner and outer conductors respectively. In our case, we have
\[ 22.36 = \frac{377}{2\pi \sqrt{2.25}} \ln \left( \frac{D}{d} \right) \Rightarrow \ln \left( \frac{D}{d} \right) = 0.5590 \Rightarrow \frac{D}{d} = 1.7489. \]

Diagrams: ¼-wave transformer design and cross-sectional view of the coaxial cable

![Diagram of ¼-wave transformer design and cross-sectional view of the coaxial cable.](image-url)
5. A shunt single stub tuner is used to match a load impedance to a 50 Ω transmission line at 1 GHz. The load consists of a series circuit composed of a 25 Ω resistor and a 4 nH inductor.

a) Find the required length and position, in wavelengths, of a short-circuited stub made from a section of the same 50 Ω line.

b) Repeat (a) assuming that the short-circuited stub is made of a section of a line that has a characteristic impedance of 75 Ω.

c) Calculate the lengths and positions in parts b and c if a transmission line with a phase velocity of $2 \times 10^8$ m/s is used for the tuner.

Solution:

a) $Z_{stub} = 50 \Omega$

Step 1: Find a normalized load impedance/admittance (A on Smith Chart)

$$Z_L = R_L + j\omega L_L = 25 + j(2\pi \times 10^9)(4 \times 10^{-9}) = 25 + j8\pi ; z_L = \frac{Z_L}{Z_0} = 0.5 + j0.50 .$$

Step 2: Transform into admittance (Point B on the Smith Chart)

Step 3: Move towards generator until $g = \Re \{ y \} = 1$ (black circle)

Option 1: $d_1 = 0$ (stay at point B)
Option 2: $d_2 = 0.5 - 0.338 + 0.162 = 0.324 \Rightarrow d_2 = 0.324[\lambda]$ (move from point B to C following the green arrow)

Step 4: Determine the admittance required from the stub

For matching, we require $1 = y_i = y_B + y_s \Rightarrow b_s = -b_B$.

Option 1: From point B we have $y_B \cong 1 - j$, so $b_s \cong 1$. This can be achieved with a short-circuited stub of length $l_1 = 0.5\lambda - 0.25\lambda + 0.125\lambda = 0.375\lambda$ (start at D, the short circuit point for admittance, and follow the red solid arrow).

Option 2: From point C we have $y_B \cong 1 + j$, so $b_s \cong -1$. This can be achieved with a short-circuited stub of length $l_2 = 0.375\lambda - 0.25\lambda = 0.125\lambda$ (start at D and follow the dark blue solid arrow).
b) \( Z_{stub} = 75[\Omega] \)

Since the characteristic impedance of the stub is different from that of the transmission line, one needs to renormalize before he/she can use the Smith chart to figure out the required length of the stub. Let primed parameters denote renormalization to the stub admittance: \( y'_B = y_B \left( \frac{Z'_0}{Z_0} \right) \).

**Repeat step 4:** Determine the admittance required from the stub

Option 1: From point B we have \( y_B \approx 1 - j \), so \( b_s \approx 1 \Rightarrow b'_s \approx 1 \left( \frac{60}{75} \right) = 0.80. \) This can be achieved with a short circuited stub of length \( l_1 = 0.5\lambda - 0.25\lambda + 0.156\lambda = 0.406\lambda \) (start at D and follow the red dashed arrow).

Option 2: From point C we have \( y_B \approx 1 + j \), so \( b_s \approx 1 \Rightarrow b'_s \approx 0.667. \) This can be achieved with a short circuited stub of length \( l_2 = 0.34\lambda - 0.25\lambda = 0.094\lambda \) (start at D and follow the dark blue dashed arrow).

c) The wavelength of the wave traveling in the transmission line (and the stubs) is:

\[
\lambda = \frac{v_p}{f} = \frac{2 \times 10^9}{1 \times 10^9} = 0.2[\text{m}].
\]

Thus the physical lengths in parts a and b are tabulated as follows:

<table>
<thead>
<tr>
<th></th>
<th>( d_1 )</th>
<th>( d_2 )</th>
<th>( l_1 )</th>
<th>( l_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: ( Z_{stub} = 50[\Omega] )</td>
<td>0mm</td>
<td>64.8mm</td>
<td>75mm</td>
<td>23mm</td>
</tr>
<tr>
<td>B: ( Z_{stub} = 75[\Omega] )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Determine the parameters of the matching network shown in the figure below, to transform a load impedance $Z_L = 150 + j50$ Ohms into an input impedance $Z_{in} = 20 - j100$ Ohms. The transmission lines of the network have a characteristic impedance $Z_0 = 50$ Ohms.

Is it possible to perform this matching by connecting the open stub at A-A’?

7. Determine the parameters of the matching network shown in the figure below, to transform a load impedance $Z_L = 150 + j50$ Ohms into an input impedance $Z_{in} = 20 - j100$ Ohms. The transmission lines of the network have a characteristic impedance $Z_0 = 50$ Ohms.

Is it possible to perform this matching by connecting the open stub at A-A’?

$$z_L = \frac{1}{50} (30 + j10) = 0.6 + j0.2$$

a) 1. Locate $z_L = 0.6 + j0.2$ on Smith chart (Point P).
   2. With center at O draw a 11/4-circle through $P_L$, intersecting OP at $1.77$. $S = 1.77$.

b) $\Gamma = \frac{1.77 - 1}{1.77 + 1} e^{j146^\circ} = 0.28 e^{j146^\circ}$

c) 1. Draw line OP, intersecting the periphery at $P'$.
   Read 0.046 on “wavelengths toward generator” scale.
   2. Move clockwise by 0.101λ to 0.147 (Point $P_2'$).
   3. Join O and $P_2'$, intersecting the 11/4-circle at $P_2$.
   4. Read $Z_i = 1 + j0.59$ at $P_2$.
   $Z_i = 50 z_i = 50 + j29.5$ (Ω).
Solution (7):

\[ Z_L = 150 + j50 \Omega \rightarrow Z_L = 2 \angle 30^\circ \text{ Ohm} \]

\[ Z_{in} = 20 - j10 \Omega \rightarrow Z_{in} = 0.4 - j1 \text{ Ohm} \]

From the chart, we switch to admittances: \( y_L = 0.3 - j0.1 \), \( y_{in} = 0.1 + j0.08 \).

We can make from \( y_L \) to \( y_B = 0.3 - j1.6 \) by using the susceptance of the open circuited stub. The required change in susceptance is:

\[ \Delta b = j1.6 - (-j0.1) = j1.5 \]

The required stub length is \( L = 0.344\lambda \).

We then follow a constant VSWR circle clockwise from \( y_{in} \) to \( y_B \). The required length is:

\[ l = (0.5 - 0.357)\lambda + 0.071\lambda = 0.234\lambda \]

This is one possible solution. We could have also added susceptance to \( y_L \) to get to \( y_B \) and then moved along constant VSWR circle for the other solution.
8. On the Smith chart, show the normalized impedances \( z_L \) that you can match to a 100 Ohm input impedance, using the matching network of Fig. 1b at 50 MHz. Extract the corresponding range of values for \( R_L \) and \( X_L \). The transmission-line segment shown has a characteristic impedance of 50 Ohms, \( v_p = 3 \times 10^8 \text{ m/sec} \) and the available length you can use is up to 1.5m.

\[
C = 20 \text{ pF} \\
Z_0 = 50 \text{ Ohms} \\
R_L + jX_L \\
100 \text{ Ohms} \\
d \leq 1.5 \text{m}
\]
Let the input impedance seen looking into the line without the series capacitor be \( z_{in}' \). The point \( z_{in}' \) lies at 2 + j3.2, since the capacitor produces \(-j3.2\) units of normalized reactance at 50 MHz. The wavelength is 3e8 m/s / 50 MHz = 6m, so a 1.5m length of line is one quarter-wavelength (half-revolution around the Smith Chart). Moving towards the load up to 0.25 wavelengths, we see the range of match-able load impedances \( z_L \) shown by the dark think line.
The Complete Smith Chart
Black Magic Design