Antenna Characteristics

1 Radiation Pattern

The radiation pattern of an antenna is a graphical representation of the radiation properties of the antenna. Graphically, we surround the antenna by a sphere and evaluate the electric / magnetic fields (far field radiation fields) at a distance equal to the radius of the sphere.

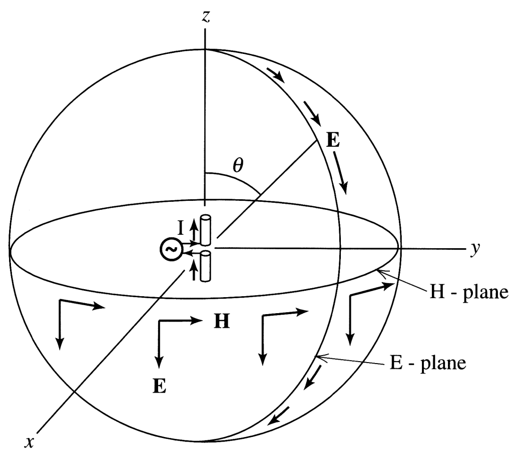


Figure 1: Radiated fields evaluated on an imaginary sphere surrounding a dipole [1]

Usually we will focus on one field component ($\boldsymbol{E}_{ff}$ or $\boldsymbol{H}_{ff}$) radiated by the antenna. Usually we plot the dominant component of the E-field (e.g. E_θ for a dipole). This can be done by plotting the field component over all angles (θ, ϕ), yielding a 3D plot. For a dipole, this leads to the doughnut pattern in 3D because of the dependence of E_θ on $\sin\theta$.

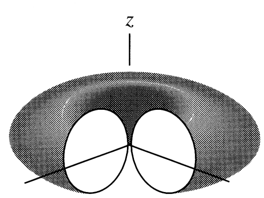


Figure 2: 3D radiation pattern of an ideal dipole [1]

Usually, it is easier and more meaningful to plot a field in a number of principal planes in 2D in a polar plot. A plane containing the electric field vector is called the E-plane of the antenna. For a dipole, $\phi =$ constant is the E-plane; the constant can be any angle since there is no field variation with ϕ. The corresponding plot is a cross section of the doughnut:

Similarly, a plane containing the H vector is called the H-plane. For the dipole, $\theta = 90^\circ$ is appropriate:

Figure 3: Radiation pattern of a dipole in the E-plane (polar form) [1]

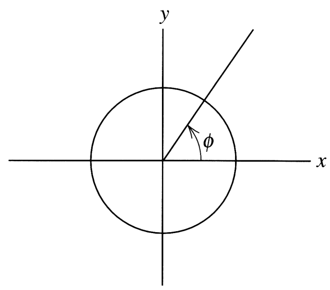


Figure 4: Radiation pattern of a dipole in the H-plane (polar form) [1]

Since there is no variation of the radiated E/H fields with azimuth angle (ϕ), we call this type of antenna pattern *omnidirectional*[1].

Often, we will plot the normalized field pattern, which has a maximum value of 1. In general, it is denoted $F(\theta, \phi)$ and obviously depends on both angles in the spherical coordinate system. For an ideal dipole, it is defined as

$$F(\theta) = \frac{E_\theta}{\max(E_\theta)} = \sin\theta. \tag{1}$$

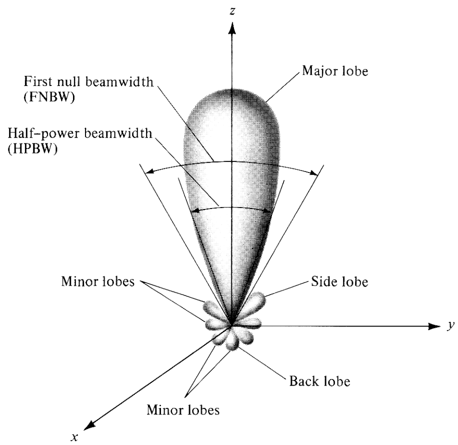
We can also plot the *normalized power pattern* $|F(\theta, \phi)|^2$ of an antenna, which is useful for defining a number of parameters. Refer to Figure 5, which shows an example of a normalized power pattern in 3D form and also in a slice on a rectilinear plot. A few interesting features are observable:

- **Minor lobes**, which are any lobes other than the main lobe in the pattern, which includes sidelobes and back lobes. They are generally undesirable since radiation in the sidelobes reduces power radiated in the desired direction.
- When characterizing the main lobe, it is possible to quantify it according to its **half-power beamwidth** (HPBW), which is analogous to the half-power bandwidth (-3 dB point) we are used to finding for filters, except that it is for spatial angles.
- Similarly, another important parameter about the radiation pattern is the **first null beamwidth** (FNBW), which is the angular spread between the first two nulls in the pattern.

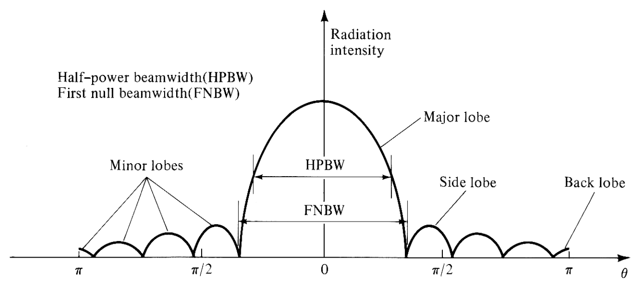
Very often we will plot antenna patterns in dB, which is inherently a power plot. This can be used to extract fine features of the antenna pattern on a logarithmic scale.

$$F(\theta, \phi)|_{\text{dB}} = 20\log|F(\theta, \phi)| = 10\log|F(\theta, \phi)|^2 \tag{2}$$

[1] Not to be confused with isotropic, later



(a) 3D power pattern



(b) 2D power pattern in rectilinear format

Figure 5: Power patterns for an arbitrary antenna [2]

The maximum sidelobe level (SLL) is often of concern to antenna designers, particularly if the antenna was designed to focus power in a specific direction, since sidelobes essentially represent a loss. In dB,

$$SLL_{\mathsf{dB}} = 20 \log \frac{|F(\theta_{\mathsf{SL}})|}{\mathsf{max}(|F|)}. \tag{3}$$

2 Antenna Pattern Types

- **Omnidirectional** – radiation response is constant in *one* of the principal planes of the antenna.
- **Isotropic** – antenna radiates equally in *all* directions in 3D space; theoretically impossible to realize, but a useful reference for quantifying how directive real antennas are.
- **Broadside** – main beam is normal to the plane or axis containing the antenna. An example for an antenna oriented along the z-axis is shown in Figure 6(a).
- **Endfire** – main beam is *in the plane* or parallel to the axis containing the antenna. An example for an antenna oriented along the z-axis is shown in Figure 6(c).

3 Radiated Power and Radiation Intensity

All antennas produce waves that carry power in the far field, as we have seen with the dipole. The time-average radiated power density in the far field is

$$\boldsymbol{P} = \frac{1}{2}\mathsf{Re}(\boldsymbol{E} \times \boldsymbol{H}^*) \tag{4}$$

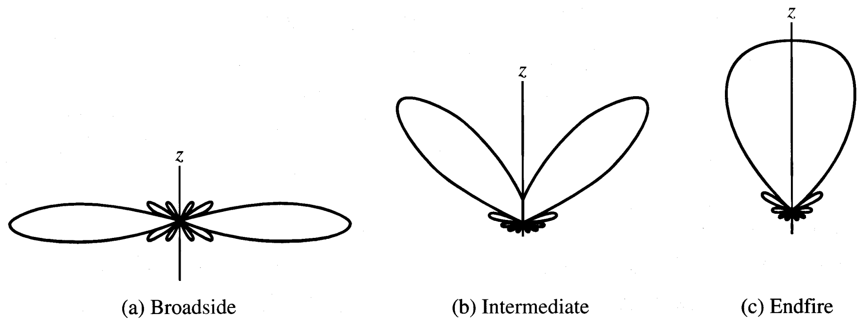


Figure 6: Radiation pattern types [1]

Then, the total power radiated (transmitted) by the antenna can be found by integrating the power density over an imaginary sphere placed around the antenna,

$$W_{rad} = \oiint_S \boldsymbol{P} \cdot d\boldsymbol{S} = \frac{1}{2}\text{Re}\left[\oiint_S (\boldsymbol{E} \times \boldsymbol{H}^*) \cdot d\boldsymbol{S}\right] = \frac{1}{2}\text{Re}\left[\int_0^{2\pi}\int_0^{\pi} (E_\theta H_\phi^* - E_\phi H_\theta^*) r^2 \sin\theta d\theta d\phi\right]. \quad (5)$$

Since $H_\phi = E_\theta/\eta$ and $H_\theta = -E_\phi/\eta$ in the far field,

$$W_t = \frac{1}{2\eta}\int_0^{2\pi}\int_0^{\pi} (|E_\theta|^2 + |E_\phi|^2) r^2 \sin\theta d\theta d\phi. \quad (6)$$

We pause to make an important point about the behaviour of the fields and power density in the far field. As we have seen for the dipole, the magnitude of the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields decays according to a $1/r$ relationship. Hence, the radiated power density $\boldsymbol{P}$ decays with $1/r^2$. We will see that the same is true of *any* antena in the far-field.

Often, when plotting or measuring antenna patterns, we are interested in the radiated power rather than the field. Plotting the power density as a function of angle can be done but we notice that it depends on the distance at which this quantity is measured. Since radiation patterns are measured at a fixed distance, this won't affect the pattern. However, it would be nice to have a quantity that was distance-invariant. Such a quantity is called *radiation intensity* and it is found using

$$U(\theta, \phi) = \frac{1}{2}\text{Re}(\boldsymbol{E} \times \boldsymbol{H}^*) \cdot r^2\, \hat{\boldsymbol{r}} = P_r(\theta, \phi) r^2. \quad (7)$$

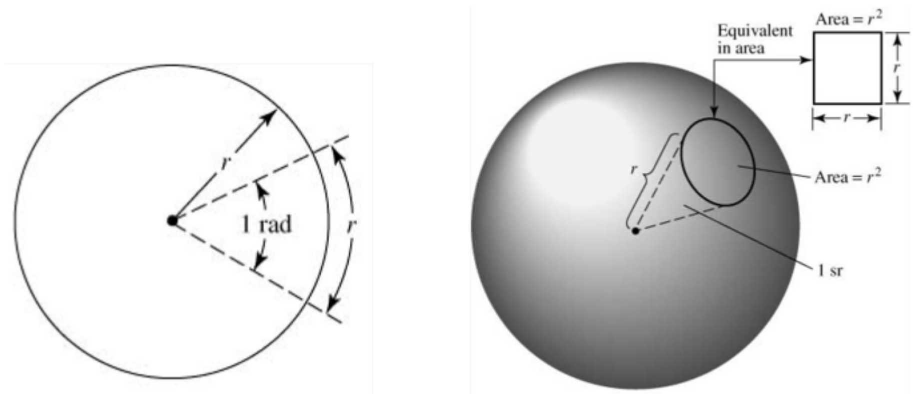
We can see that the r^2 term removes the dependency on distance. Meanwhile, only the radial component of $\boldsymbol{P}$ is used in the calculation, which is the only component that should exist in the far-field anyway. Hence, radiation intensity is defined as a scalar quantity. The units of radiation intensity are watts per solid angle (W/sr). To understand the concept of solid angle, compare now the calculations for total radiated power found by integrating the power density and radiation intensity over a sphere,

$$W_{rad} = \text{Re}\left[\int_0^{2\pi}\int_0^{\pi} P_r r^2 \sin\theta d\theta d\phi\right] \quad (8)$$

$$W_{rad} = \text{Re}\left[\int_0^{2\pi}\int_0^{\pi} U(\theta, \phi) \sin\theta d\theta d\phi\right]. \quad (9)$$

We see that when integrating the power density over a sphere, the differential area element is $r^2 \sin\theta d\theta d\phi$ which has units of area. However, when we integrated the radiation intensity, the differential element is $\sin\theta d\theta d\phi$ which seems to have units of radians squared. When we have the product of two plane angles in radians, we produce a solid angle, whose units are *steradians*.

The concept of a steradian can be understood by considering Figure 7. In Figure 7(a) we see the familiar definition of a radian as a measure of *plane angle*: 1 rad subtends an arc of length r. Since the circumference of a circle is $2\pi r$, the number of radians in a complete circle is $C/r = 2\pi$ rad. By extension, steradians are a measure of *solid angle*, as shown in Figure 7(b). 1 sr subtends a spherical area of area r^2. Since a complete sphere has a surface area of $4\pi r^2$, there are $A/r^2 = 4\pi$ sr in a sphere.



(a) Plane angle (b) Solid angle

Figure 7: Definitions of plane and solid angle

Hence, the differential element in (9) can be rewritten so that

$$W_{rad} = \text{Re}\left[\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi) d\Omega\right]. \tag{10}$$

where $\Omega = \sin\theta d\theta d\phi$. The concept of solid angle is very important when we discuss the quantification of an antenna pattern in terms of directivity, discussed shortly.

Finally, we note that radiation intensity can also be expressed as

$$U(\theta,\phi) = U_m |F(\theta,\phi)|^2, \tag{11}$$

where $U_m = U(\theta_{max}, \phi_{max})$ is the maximum radiation intensity produced by the antenna at some angle $(\theta_{max}, \phi_{max})$. Hence, it can be related to the normalized (field) pattern factor introduced earlier.

4 Directivity

The *directivity* of an antenna is a measure of how much it concentrates power in a given direction, assuming the antenna is 100% efficient. The directivity of the antenna is always taken *with respect* *to a known antenna*, which is usually an isotropic radiator. On rare occasions, sometimes a half-wave dipole is used as a reference.

Mathematically, the directivity of an antenna is defined as

$$D(\theta, \phi) = \frac{U_m}{U_0} \text{ [dimensionless]}, \tag{12}$$

where U_0 is the *average radiation intensity* that would be produced by the antenna had it been isotropic. Since an isotropic source produces a radiated power density evenly spread out over a sphere,

$$U_0 = \frac{W_{rad}}{4\pi} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} U(\theta, \phi) d\Omega. \tag{13}$$

Hence, the *directivity of an isotropic antenna is 1*. Directivity can also be re-written in terms of power densities and radiated power,

$$D(\theta, \phi) = \frac{U(\theta, \phi)}{U_{iso}} = \frac{P_r(\theta, \phi)}{P_{r,iso}} = \frac{P_r(\theta, \phi)}{W_{rad}/4\pi r^2} = \frac{4\pi U(\theta, \phi)}{W_{rad}}. \tag{14}$$

Example: Ideal dipole

Recall in the far field,

$$\boldsymbol{P} = \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 \omega\mu k \frac{\sin^2\theta}{r^2}\hat{\boldsymbol{r}}. \tag{15}$$

Therefore,

$$U(\theta) = \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 \omega\mu k \sin^2\theta. \tag{16}$$

The maximum radiation intensity occurs when $\theta = 90^\circ$, therefore,

$$U_m = \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 \omega\mu k. \tag{17}$$

Meanwhile, we previously found the power radiated by the dipole to be

$$W_t = \frac{\omega\mu k}{12\pi}(I\Delta z)^2. \tag{18}$$

Therefore,

$$U_{iso} = \frac{W_t}{4\pi} = \frac{\omega\mu k}{3}\left(\frac{I\Delta z}{4\pi}\right)^2 = \frac{2}{3}U_m. \tag{19}$$

That is, the average radiation intensity is only 2/3 of the maximum value; or, the peak radiation is 50% more than the average value. ◇

We can also express directivity as

$$\begin{aligned} D(\theta,\phi) &= \frac{U(\theta,\phi)}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta d\theta d\phi} \\ &= \frac{U_m|F(\theta,\phi)|^2}{\frac{U_m}{4\pi}\int_0^{2\pi}\int_0^{\pi}|F(\theta,\phi)|^2\sin\theta d\theta d\phi} \\ &= \frac{4\pi|F(\theta,\phi)|^2}{\int_0^{2\pi}\int_0^{\pi}|F(\theta,\phi)|^2\sin\theta d\theta d\phi}, \end{aligned} \tag{20}$$

which shows that the directivity in no way depends on U_m; whether we normalize the pattern or not should, obviously, not make any difference on the directivity. Further, we can express the directivity as

$$D = \frac{4\pi}{\Omega_A}|F(\theta,\phi)|^2 \tag{21}$$

where Ω_A is the *beam solid angle* of the antenna defined by

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi}|F(\theta,\phi)|^2\sin\theta d\theta d\phi \tag{22}$$

and can be thought of as the solid angle through which all the power would be radiated if the radiation intensity equaled the maximum value over the beam area.

Often, we are most interested in the maximum directivity of the antenna, and often this is what is quoted by an antenna manufacturer,

$$D_m = \frac{U_m}{U_0}. \tag{23}$$

Technically, directivity is a function of angle and can be plotted as such in a 2D or 3D plot,

$$D(\theta,\phi) = D_m|F(\theta,\phi)|^2. \tag{24}$$

As derived earlier, the directivity of an ideal dipole is $D_m = 1.5$, or in dB units, $D_m = 1.77$ dBi. The "i" is used to explicitly denote that the reference antenna used was an isotropic radiator (more on this shortly).

The directivity of the antenna represents the additional power collected or transmitted in a certain direction relative to an isotropic radiator. So our ideal lossless dipole produces 50% more power in the maximum beam direction ($\theta = 90°$) than an isotropic radiator. It is important to recognize that this extra power does not come from nowhere – an antenna is a passive device and can't create power from nothing (compared to an amplifier, which is active and uses an external source of power). The "extra" power came at the expense of response (directivity) in the other directions. At some angles a dipole is *worse off* than its isotropic counterpart, especially as we near $\theta = 0°$ or $\theta = 180°$ where the is no response whatsoever! So, an antenna is very much like a lens, that focuses intensity at the expense of response in other directions. This can be very handy, as one can imagine, in long range links, as we will see later in the course.

This concept is demonstrated nicely in graphical form below: while an isotropic radiator produced uniform radiation intensity in all directions, a directive antenna produces more in one direction at the expense of responsivity in the other directions.

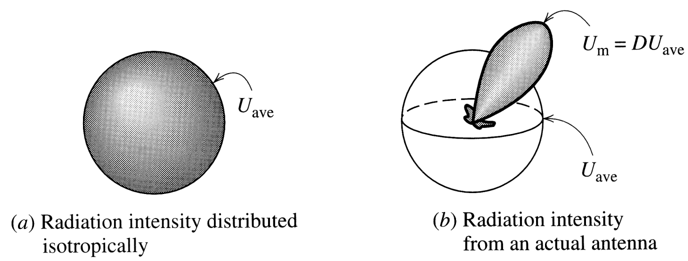


Figure 8: Illustration of directive antenna

5 Gain and Efficiency

Directivity is solely a function of the radiation pattern of the antenna; it assumes all the power supplied to the antenna is radiated (i.e., the antenna is 100% efficient). If this is not the case, then in general the input power supplied to the antenna and the radiated power from the antenna are related through:

$$W_{rad} = e_r W_{in}, \tag{25}$$

where W_{rad} is equivalent to W_t used earlier. The term e_r is the *radiation efficiency* of the antenna, $0 \le e_r \le 1$. Examining the expression for directivity, we have

$$D(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{W_{rad}}. \tag{26}$$

Gain is defined by replacing W_{rad} with the input power to the antenna,

$$G(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{W_{in}} = e_r \frac{4\pi U(\theta, \phi)}{W_{rad}} = e_r D(\theta, \phi). \tag{27}$$

That is, the gain of the antenna is equal to the directivity of the antenna times its efficiency. Since $e_r \le 1$, $G(\theta, \phi) \le D(\theta, \phi)$. Gain represents the focusing power of the antenna with antenna losses (e_r) included. Note that e_r does **not** include polarization or impedance mismatch losses, to be discussed later.

Like directivity, often only the maximum gain of the antenna is quoted. For example, an ideal dipole has a maximum directivity of 1.5. If the dipole was only 70% efficient, then $G_m = 1.05 = 0.21$ dBi.

6 Gain/Directivity in dB

In radio systems it is very common to express quantities in dB units because quantities involved vary over many orders of magnitude. Therefore, dB units are convenient way of representing changes in orders of magnitude, and allow us to replace tedious multiplications and divisions of these quantities with simple additions and subtractions. We will see this more when we study link budgets.

A dB quantity is *always* taken in relation to some reference. If I said I had 40 dB of loss, it would be relative to some reference (e.g. the output power of a system is 40 dB lower than the input power, or 4 orders of magnitude (10^4) lower). Note that in this case the ratio was power levels taken at the input and output of the system.

For antennas, we notice that in the definition of directivity, the denominator (reference) is the average radiation intensity produced by an isotropic radiator. Hence, directivity is inherently a comparison between two antennas. Hence, when we evaluate the directivity of an antenna, it is being done with respect to an isotropic unit. If we evaluate gain or directivity in dB,

$$D_{\text{dB}} = 10\log(D/D_{\text{ref}}) = 10\log D - 10\log D_{\text{ref}} \tag{28}$$

$$G_{\text{dB}} = 10\log(G/G_{\text{ref}}) = 10\log G - 10\log G_{\text{ref}}. \tag{29}$$

Notice that when an ideal isotropic radiator is used, $D_{\text{ref}} = G_{\text{ref}} = 1$ and the second terms in each expression are zero. In this case, we explicitly denote that the reference was an isotropic radiator by replacing dB with dBi. This is because sometimes, a reference other than an isotropic radiator is used. For example, a half-wave dipole is often used as a reference, in which case we would use dBd units, and $10\log D_{\text{ref}}$ and $10\log G_{\text{ref}}$ would be nonzero (1.64 dBi to be exact, but we haven't covered that yet).

7 Antenna Impedance

The last important quantity to consider is an antenna's input impedance, since an RF system (with its own impedance) needs to eventually interface to the antenna. It wouldn't be any good if the antenna couldn't be *impedance-matched* to the rest of the system since an impedance mismatch would produce reflection and hence result in inefficient power transfer to/from the antenna.

In general, the antenna's input impedance can be written as

$$Z_A = R_A + jX_A. \tag{30}$$

We see that:

- R_A present a way for real power to be dissipated by the antenna, either as ohmic loss (e_r) or radiation. Real power is dissipated in both cases;
- X_A allows power to be stored by the antenna, which we know happens from near-field analysis.

From this definition,

$$W_{in} = \frac{1}{2}R_A|I_A|^2 = W_{rad} + W_{ohmic}, \tag{31}$$

and the efficiency of the antenna e_r we presented previously can be defined mathematically as

$$e_r = W_{rad}/(W_{rad} + W_{ohmic}). \tag{32}$$

Let's separate R_A such that

$$R_A = R_{rad} + R_{ohmic}. \tag{33}$$

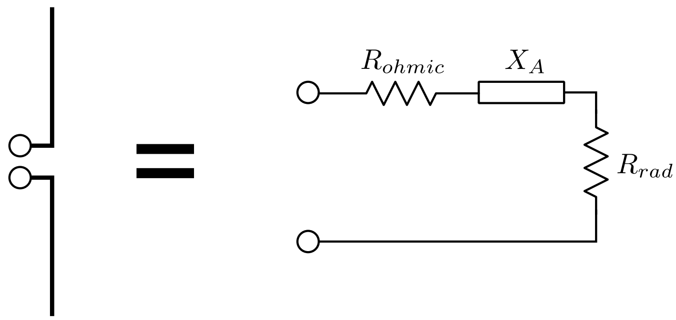


Figure 9: Equivalent circuit for a transmitting antenna

This leads to the equivalent circuit shown in Figure 9.

The input power supplied to the circuit is then

$$W_{in} = \frac{1}{2}R_{rad}|I_A|^2 + \frac{1}{2}R_{ohmic}|I_A|^2. \tag{34}$$

R_{rad} is called the *radiation resistance of the antenna*. Any power "dissipated" in this resistor is actually radiated by the antenna. For a Hertzian dipole,

$$R_{rad} = \frac{2W_{rad}}{I^2} = \frac{2}{I^2}\frac{(I\Delta z)^2}{12\pi}\omega\mu k = \frac{\Delta z^2}{6\pi}k^2\eta \tag{35}$$

$$= \frac{\Delta z^2}{6\pi}\left(\frac{2\pi}{\lambda}\right)^2\eta = \frac{2\pi}{3}\left(\frac{\Delta z}{\lambda}\right)^2\eta. \tag{36}$$

where $\eta = \omega\mu/k$ has been used. In free space, $\eta = 120\pi\ \Omega$ and

$$R_{rad} = 80\pi^2\left(\frac{\Delta z}{\lambda}\right)^2. \tag{37}$$

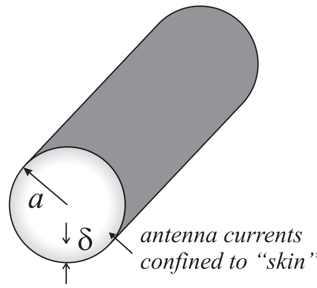
For a Hertzian dipole, R_{ohmic} can be found using Ohm's Law:

$$R_{ohmic} = \frac{L}{\sigma S}, \tag{38}$$

where L is the length of the dipole and S is the area of the conductive part of the wire. Note that because of the skin effect, which we learned about while studying plane waves in good conductors, currents only travel along the "skin" of the conductor. Except at low frequencies, the skin depth δ is usually much less than the conductor radius a. Recall the skin depth for good conductors is

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu}} \tag{39}$$

where σ is the conductivity of the conductor composing the antenna.



The area of the skin is $2\pi a\delta = 2\pi a\sqrt{2/\omega\sigma\mu}$. Therefore,

$$R_{ohmic} = \frac{L}{\sigma 2\pi a\sqrt{2/\omega\sigma\mu}} = \frac{L}{2\pi a}\sqrt{\frac{\omega\mu}{2\sigma}} \equiv \frac{L}{2\pi a}R_s, \tag{40}$$

where $R_s = \sqrt{\omega\mu/2\sigma}$ is called the *surface resistance* of the wire $[\Omega]$.

Knowing R_{rad} and R_{ohmic}, you can estimate the efficiency of the antenna as

$$e_r = \frac{W_{rad}}{W_{rad} + W_{ohmic}} = \frac{\frac{1}{2}|I_A|^2 R_{rad}}{\frac{1}{2}|I_A|^2 R_{rad} + \frac{1}{2}|I_A|^2 R_{ohmic}} = \frac{R_{rad}}{R_{rad} + R_{ohmic}}. \tag{41}$$

Usually, unless the antenna is short, e_r is quite high because $R_{rad} \gg R_{ohmic}$. But very very short antennas (e.g. the Hertzian dipole), the equation for R_{rad} shows that it can be quite small, and can approach the order of R_{ohmic}. This will produce low efficiencies, which is a major challenge when making antennas that are much smaller than a wavelength ("electrically small" antennas).

We will not worry about determining X_A in this course since the calculation are much more involved (often requiring numerical techniques). Plus, it only really factors into designing the matching circuit which is not addressed in this course.

Example: Determine the radiation efficiency of a steel AM radio antenna with $L = 1.575$ m, $a = 0.159$ cm, operating at $f = 1$ MHz.

The wavelength of the AM radio signal considered is $\lambda = 300$ m, meaning that the antenna has a length of $L = 0.00525\lambda$. The antenna definitely qualifies as electrically short and hence we will apply Hertzian dipole formulas here.

Steel has a conductivity of $\sigma = 2 \times 10^6$ S/m, therefore,

$$R_s = \sqrt{\frac{(2\pi \times 10^6)(4\pi \times 10^{-7})}{2 \cdot 2 \times 10^6}} = 1.4 \times 10^{-3}\ \Omega \tag{42}$$

$$R_{ohmic} = 0.22\ \Omega \tag{43}$$

$$R_{rad} = 80\pi^2 \left(\frac{1.575}{300}\right)^2 = 0.0218\ \Omega \tag{44}$$

$$e_r = \frac{R_{rad}}{R_{rad} + R_{ohmic}} = 8.95\% \tag{45}$$

Clearly, the antenna is not very efficient!

References

[1] W. L. Stutzman and G. A. Theile, *Antenna theory and design*. John Wiley and Sons, Inc., 1998.

[2] C. A. Balanis, *Antenna theory: analysis and design*, 2nd ed. John Wiley & Sons, Inc., 1987.