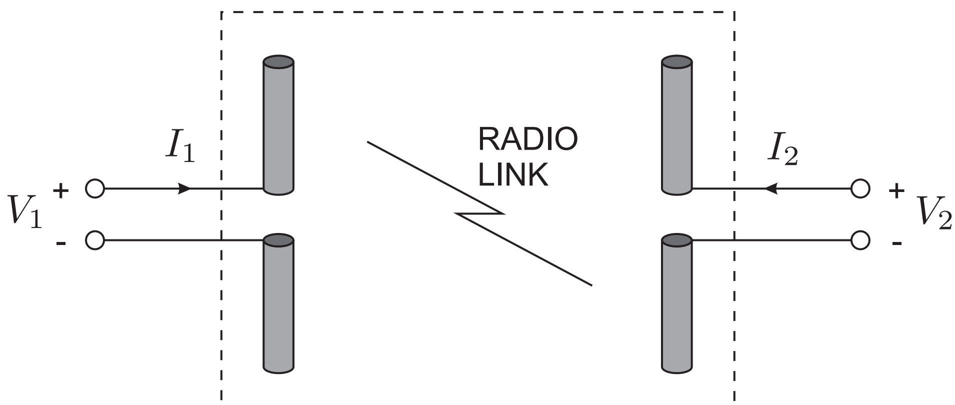
Free Space Radio Links

We have derived the radiation characteristics of a few antenna types acting alone in free space. Now, we wish to study the relationship between two antennas in a radio link, with one antenna used as a transmitting antenna, and the other as a receiving antenna. Specifically, we would like to determine the received power at the receiving antenna knowing the transmitter power and the distance between the two antennas.

1 Equivalent Circuit Model of a Two-Antenna Link

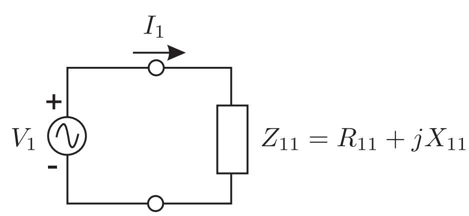
We have previously examined the equivalent circuit of a two-antenna link when we considered receiving antennas. The network under consideration was a 2-port "black box" containing the transmitting antenna, receiving antenna, and space between the antennas.



We previous found the input impedance seen looking into port 1 is

$$Z_1 = \frac{V_1}{I_1} = \frac{Z_{11}I_1 + Z_{12}I_2}{I_1} = Z_{11} + \frac{Z_{12}I_2}{I_1} \approx Z_{11}, \tag{1}$$

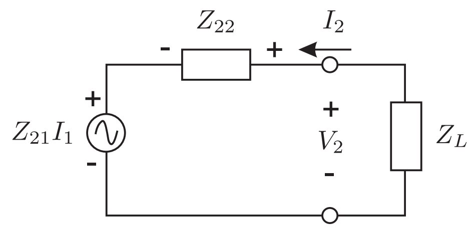
where the approximation holds when a large distance separates the two antennas. This yields the equivalent circuit for the transmitter shown below.



We now wish to use the equivalent circuit to help us predict the power delivered to a load attached to the receiving antenna when the other antenna is used to transmit a signal. Let's begin by using antenna 1 as the transmitting antenna and antenna 2 as the receiving antenna. We can model the transmitter as follows, if the receiver is located a large distance away. We terminate the receiving antenna in a load impedance Z_L which represents the input impedance to the receiver circuit. We wish to find the power delivered to a load impedance Z_L, and find its relationship to the transmitter power. We know that

$$V_2 = Z_{21}I_1 + Z_{22}I_2, \tag{2}$$

suggesting the following equivalent circuit.



For maximum power transfer to Z_L, a *conjugate matching condition* must exist. That is,

$$Z_L = Z_{22}^* = R_{22} - jX_{22}. \tag{3}$$

Under this condition, the power absorbed by the load is

$$W_{r,2} = \frac{1}{8}\frac{|V_2^{oc}|^2}{R_{22}} = \frac{1}{8}\frac{|Z_{21}|^2|I_1|^2}{R_{22}}. \tag{4}$$

While it is possible to determine the received power in terms of the mutual impedance Z_{21}, more often the approach is to determine the open-circuited voltage at the receiver terminals, using the vector effective length of the receiving antenna. This allows the power received by the load to be related to the *directivity* of the antenna, which is facilitated by a new quantity we shall introduced called *antenna effective area*.

2 Antenna Effective Area

Before continuing onward with dealing with these ratios, we need to define a new antenna parameters known as *antenna effective area*. To begin, consider an isotropic antenna, which radiates equally in all directions. If the power supplied to the antenna is W_{in}, the radiated power density is

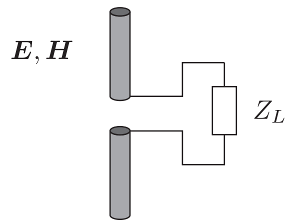
$$P_{iso} = \frac{W_{in}}{A_{sphere}} \tag{5}$$

where A_{sphere} is the surface area of a sphere surrounding the isotropic radiator, of radius r. The surface area of a sphere is $4\pi r^2$ m^2, therefore,

$$P_{iso} = \frac{W_{in}}{4\pi r^2} \qquad \text{(units: W/m}^2\text{)} \tag{6}$$

The gain of an antenna was defined to be the power density radiated in a certain direction normalized to the power density of an isotropic antenna:

$$G = \frac{P}{P_{iso}} = \frac{4\pi r^2 P}{W_{in}} = \frac{4\pi U}{W_{in}} \tag{7}$$



which matches our earlier definition.

A receiving antenna takes radiated fields and converts them into electrical power which is supplied to a load impedance connected to the terminals of the antenna.

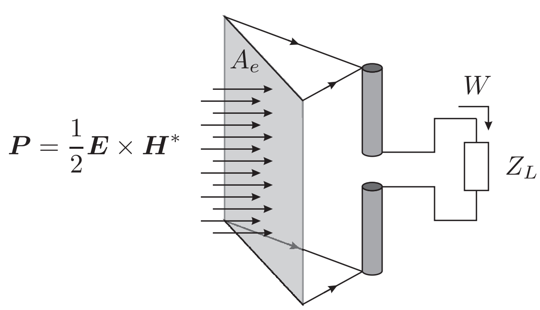
If the incident fields establish a power density around the antenna, we can view the antenna as a type of *collector* that converts power density to power:

$$\text{Power density } [\text{W/m}^2] \rightarrow \text{antenna} \rightarrow \text{Power [W]}$$

Therefore, we can think of the antenna as having an *effective area* that intercepts the incident fields. The greater the collecting area of the antenna, the greater the power generated by the antenna (delivered to the load).

Analogies:

- Sails, windmills – if the wind represents the incident 'power' or 'pressure' density, larger 'area' sails or propellers collect more power
- Solar panels – if the solar intensity represents the incident power density, larger panels obviously collect more power
- Rain catcher – if we have a fixed 'rainfall flux density', then a larger collector (with greater area) collects more rain than a smaller one (a suitable analogy to power delivered would be the flow from a pipe connected to the collector)



The relationship between the power and power density is

$$W = A_e P \tag{8}$$

where A_e is the effective area of the antenna.

We might suppose (correctly) that antennas with more gain have greater effective areas. We will now derive the formal relationship between gain and effective area.

Let's consider the received power at an ideal dipole that is perfectly co-polarized with the incident field associated with an incoming plane wave, and also that the antenna is oriented for maximum output (i.e. with direction of propagation of the incident field broadside to the dipole). For a receiving antenna aligned in such a way, it can easily be shown that the open circuit terminal voltage and incident electric field strength are related through

$$V_r^{oc} = E^i \Delta z \tag{9}$$

which we will learn more about when we study the effective length of antennas later on. The incident electric field strength is denoted as E^i, therefore the incident power density is

$$P_r = \frac{1}{2}\frac{|E^i|^2}{\eta} \tag{10}$$

which produced a received power

$$W_r = P_r A_{em}. \tag{11}$$

Here, we have used A_{em} to denote the *maximum* effective area of the antenna; the assumption for now is that the antenna is lossless and hence this area represents the maximum achievable and we expect it to decrease if losses are introduced (we will show this formally shortly).

$$A_{em} = \frac{W_r}{P_r} \tag{12}$$

From our circuit model, if the load is conjugately matched to the antenna, and the antenna is lossless, we know that

$$W_r = \frac{1}{8}\frac{|V_r^{oc}|^2}{R_r} \tag{13}$$

where R_r is the radiation resistance of the receiving antenna. Therefore,

$$\begin{aligned} A_{em} &= \frac{1}{4}\frac{|V_r^{oc}|^2\eta}{R_r|E^i|^2} && (14)\\ &= \frac{1}{4}\frac{|E^i|^2\Delta z^2\eta}{R_r|E^i|^2} && (15)\\ &= \frac{\Delta z^2\eta}{4R_r} && (16) \end{aligned}$$

For an ideal dipole, we know that

$$R_r = \frac{2}{3}\eta\pi\left(\frac{\Delta z}{\lambda}\right)^2 \tag{17}$$

We have

$$\begin{aligned} A_{em} &= \Delta z^2\eta / \frac{8}{3}\eta\pi\left(\frac{\Delta z}{\lambda}\right)^2 && (18)\\ &= \frac{3}{8\pi}\lambda^2 && (19)\\ &= \frac{3}{2}\frac{\lambda^2}{4\pi}, && (20) \end{aligned}$$

and since the directivity of an ideal dipole is $3/2$, we have proved that

$$A_{em} = D\frac{\lambda^2}{4\pi} \tag{21}$$

If the antenna is not lossless, we the effective area of the antenna is reduced by the efficiency factor,

$$A_e = e_r A_{em}, \tag{22}$$

due to the relationship between directivity and gain derived previously. Multiplying this effective area by the incident power density represents the available power that can be delivered to the load. If we multiply equation 21 by e_r, we obtain

$$e_r D = G = \frac{4\pi}{\lambda^2} e_r A_{em} = \frac{4\pi}{\lambda^2} A_e \tag{23}$$

In general, the effective area of an antenna can be a function of angle (θ, ϕ), just as gain/directivity depend on angle. Here the effective area we have discussed refers to the effective area of the antenna in the direction of maximum radiation intensity. In general, though, the effective area should be denoted as a function of angle, $A_{em}(\theta, \phi)$ and $A_e(\theta, \phi)$.

Finally, you may wonder if the effective area is related to the physical area of an antenna. This seems a bit meaningless for thin antennas we have studied so far, but electrically large antennas, like reflector antennas (e.g. satellite dishes) have areas in many square wavelengths. For these antennas, the effective area is related to the physical area through

$$A_e = e_{ap} A_p \tag{24}$$

where A_p is the physical area of the antenna, and e_{ap} is the *aperture efficiency* of the antenna.

3 Friis' Transmission Equation

Let's return to the case where antenna 1 is the transmitting antenna and antenna 2 is the receiving antenna, located a distance R away from the transmitter. If antenna 1 was an isotropic antenna, the power density at the position of the receiver would be

$$P_{r,2} = \frac{W_{t,1}}{4\pi R^2}. \tag{25}$$

If instead antenna 1 had a gain G_1 we know that

$$P_{r,2} = \frac{G_1 W_{t,1}}{4\pi R^2}. \tag{26}$$

The received power at antenna 2 is found by multiplying the incident power density by the effective area of antenna 2,

$$W_{r,2} = \frac{G_1 W_{t,1} A_{e2}}{4\pi R^2}. \tag{27}$$

Therefore,

$$\frac{W_{r,2}}{W_{t,1}} = \frac{G_1 A_{e2}}{4\pi R^2}. \tag{28}$$

If we reversed the transmitting and receiving antennas, we easily find that

$$\frac{W_{r,1}}{W_{t,2}} = \frac{G_2 A_{e1}}{4\pi R^2}. \tag{29}$$

By reciprocity, we know that

$$\frac{W_{r,1}}{W_{t,2}} = \frac{W_{r,2}}{W_{t,1}}, \tag{30}$$

therefore,

$$\frac{G_2 A_{e1}}{4\pi R^2} = \frac{G_1 A_{e2}}{4\pi R^2}. \tag{31}$$

Dividing both sides by $A_{e1}A_{e2}$,

$$\frac{G_1}{A_{e1}} = \frac{G_2}{A_{e2}} = \text{constant} \tag{32}$$

We have seen previously in equation (21) that this constant is equal to

$$\frac{G}{A_e} = \frac{4\pi}{\lambda^2}. \tag{33}$$

Using this result in equation (28),

$$\frac{W_{r,2}}{W_{t,1}} = \frac{G_2 G_1 \lambda^2}{(4\pi R)^2}, \tag{34}$$

which we can generalize to a general transmitting-receiving antenna pair as

$$\frac{W_r}{W_t} = \frac{G_r G_t \lambda^2}{(4\pi R)^2}. \tag{35}$$

This equation is called *Friis' transmission formula* since it allows us to calculate the received power knowing the transmit power, transmitter-receiver distance R, frequency (or wavelength λ), and the gains of both antennas. It is the basis of *link budgets* which we will study extensively.

Notice that if both antennas were isotropic ($G_t = G_r = 1$),

$$\frac{W_r}{W_t} = \left(\frac{\lambda}{4\pi R}\right)^2 \tag{36}$$

This term is known as *free space loss* since it represent the factor by which the transmit power has been attenuated. It comes about because the receiving antenna only intercepts a small fraction of the power density that is broadcast by the transmitter. The rest of the power is effectively lost. We can intercept more power (reduce the free space loss) by capturing more power density using an antenna with a larger effective area (i.e., a higher gain), as the Friis' formula shows.

$$W_r = W_t G_t \left(\frac{\lambda}{4\pi R}\right)^2 G_r \tag{37}$$

The product $W_t G_t$ is called *effective isotropically radiated power (EIRP)* of the transmitter. It is called this because at the receiver, the same power density could be produced by replacing the transmitter antenna with an isotropic one and increasing the transmitter power to EIRP watts. EIRP is an important quantity because the antenna effectively "boosts" the transmit power without the need for a transmit power amplifier. In all licensed (and unlicensed) radio systems, regulations always limit the *EIRP* of the transmitter, not the actual transmit power, because it is the EIRP that really counts in the end when you are trying to prevent interference with other radio systems.

A Note on Units

You can use standard units in Friis' transmission formula. But it is much more common to work on a logarithmic (dB) scale. Why? Working in dB has many advantages when the quantities you are working with span many orders of magnitude. For example, a 1 W isotropic transmitter will only produce 5.7×10^{-8} W at an isotropic receiver located only 100 m away at 1 GHz. That is,

$$\frac{W_r}{W_t} = 5.7 \times 10^{-8}, \text{ a very small number.} \tag{38}$$

When dealing with power ratios is is easier to use dB:

$$\left.\frac{W_r}{W_t}\right|_{dB} = 10 \log_{10}\left(\frac{W_r}{W_t}\right) = -72.4 \text{ dB.} \tag{39}$$

The greater advantage of using a logarithmic scale is that multiplications and divisions in Friis' formula reduce to the (easier) operations of addition and subtraction. Hence, a ratio such as

$$\frac{W_r}{W_t} = 5.7 \times 10^{-8} \tag{40}$$

becomes

$$W_r - W_t = -72.4 \text{ dB.} \tag{41}$$

We need a way of expressing absolute power terms like W_r and W_t on a logarithmic scale as well. Such a scale does exist, and like all dB scales, is *relative* to some reference.

$$\text{dBW} = \text{dB with respect to 1 W} = 10 \log_{10}(\text{power in W}) \tag{42}$$

$$\text{dBm} = \text{dB with respect to 1 mW} = 10 \log_{10}(\text{power in mW}) \tag{43}$$

Hence we could represent our previous example as follows:

$$W_t = 1 \text{ W} = 0 \text{ dBW} \tag{44}$$

$$W_r = 0 \text{ dBW} - 72.4 \text{ dB} = -72.4 \text{ dBW.} \tag{45}$$

Note that the *loss ratio* is simply in dB because it is a *relative* measure or ratio of two quantities (W_r/W_t in this case), while dBW and dBm are *absolute* units expressed on a logarithmic scale.

We could have also said

$$W_r = 30 \text{ dBm} - 72.4 \text{ dB} = -42.4 \text{ dBm} = 57.0 \times 10^{-6} \text{ mW}. \tag{46}$$

To aid out dB calculations we can represent the transmission loss as

$$10 \log_{10} \left(\frac{G_r G_t \lambda^2}{(4\pi R)^2} \right) = 10 \log_{10} G_r + 10 \log_{10} G_t + 10 \log_{10} \left(\frac{\lambda}{4\pi R} \right)^2 \tag{47}$$

$$= G_{r,dB} + G_{t,dB} - FSL_{dB} \tag{48}$$

where the minus sign on the free space loss term anticipates that $\lambda/R \ll 1$. Hence,

$$FSL_{dB} = 20 \log_{10} \left(\frac{4\pi R}{\lambda} \right) = 20 \log_{10}(4\pi) + 20 \log_{10}(R/\lambda) \tag{49}$$

$$= 22 \text{ dB} + 20 \log_{10}(R/\lambda). \tag{50}$$

Summarizing,

$$W_r = W_t + G_{r,dB} + G_{t,dB} - 22 - 20 \log(R/\lambda). \tag{51}$$

Now our calculations are very simple – additions and subtractions – which will form the basis of *link budget accounting* later on.

Example: Mars Pathfinder X-band Downlink Budget

Item	Symbol	Value (linear units)	Value (logarithmic units)
Transmit power	W_t	10 W	+10 dBW
Transmit antenna gain	G_t	251.1	+24.0 dBi
Effective isotropic radiated power	EIRP	2.51 kW	+34 dBW
Free space path loss	$(\frac{\lambda}{4\pi R})^2$	220×10^{-30}	-276.6 dB
Received isotropic power	W_r/G_r	554×10^{-27} W	-242.6 dBW
Receive antenna gain	G_r	6.31×10^3	68.0 dBi
Power at receiver	W_r	3.49×10^{-18} W	-174.6 dBW or -144.6 dBm

Details

Item	Value
Operating frequency	8420 MHz
Mars-Earth distance as of 4-Jul-97	191×10^6 km

The operating wavelength is $c/f = 3.56$ cm, which is used in the path loss calculation.

To put things in perspective, a standard cellular phone would require approximately -100 dBm of received power to even properly recover a signal. The NASA receiver is obviously much more sensitive (approximately 45 dB more sensitive, or 31,600 times as sensitive).

The NASA Deep Space Network uses a series of large reflector antennas to receive signals from distance probes. The antenna used here was a 34 m reflector antenna. An ideal aperture of this area would produce $D = \frac{4\pi}{\lambda^2}(\pi \cdot 17^2) = 69.5$ dBi of gain. Most reflectors have an aperture efficiency of 70%, so after you account for this the gain shown in the table matches well with this simple calculation.

Reference: `http://www.ka9q.net/mpf_budget.html`