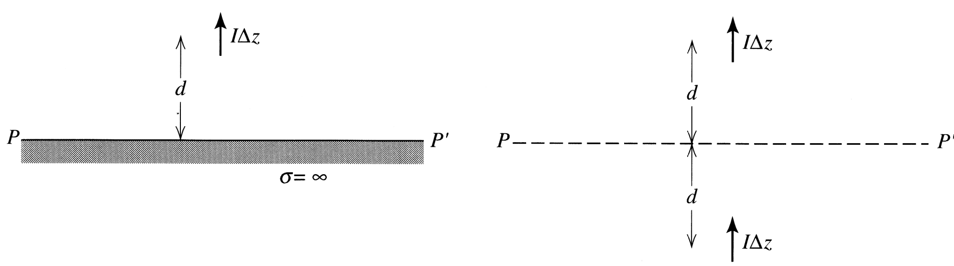
Image Theory / Monopoles

The behaviour of antennas over infinite ground planes is of interest because:

- Antennas can be designed to make use of the ground plane
- The earth's surface can be though of as an infinite ground plane under the right conditions, and we will be interested in what happens when waves interact with the earth (later).

Consider an ideal dipole, vertically oriented over a ground plane that is infinite in extent and perfectly conducting, as shown in Figure 1(a). We will now show that an *equivalent model* of the system corresponds to the situation shown in Figure 1(b), using what we know of boundary conditions at PECs. The virtual dipole below the ground plane is known as the *image* of the primary dipole.



(*a*) Physical model. (*b*) Equivalent model using image theory.

Figure 1: Image theory for vertically oriented dipole [1]

The ground plane need not be in the far field of the dipole. Hence, we need to consider all the fields produced by the dipole. Recall that the dipole has both $\hat{\boldsymbol{r}}$ an $\hat{\boldsymbol{\theta}}$ components for its electric field,

$$\boldsymbol{E} = \frac{I_0 \Delta z}{2\pi}\eta\left(\frac{1}{r} - \frac{j}{kr^2}\right)\frac{e^{-jkr}}{r}\cos\theta\,\hat{\boldsymbol{r}} + \frac{I_0\Delta z j\omega\mu}{4\pi}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]\frac{e^{-jkr}}{r}\sin\theta\,\hat{\boldsymbol{\theta}}, \qquad (1)$$

which can be written more simply as

$$\boldsymbol{E} = C\cos\theta\,\hat{\boldsymbol{r}} + D\sin\theta\,\hat{\boldsymbol{\theta}}. \qquad (2)$$

Let's consider the radial components first.

The radial component for the top dipole is

$$E_{r1} = C\cos\theta_1 \qquad (3)$$

and is

$$E_{r2} = C\cos\theta_2 \qquad (4)$$

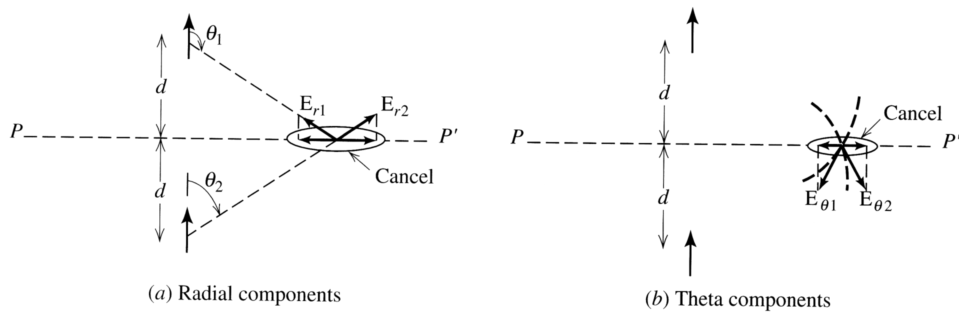


Figure 2: Components of vertical dipole fields from image theory [1]

for the bottom dipole. Note the constant C is the same for both dipoles, since they are equidistant from a point along the imaginary plane that used to be the ground plane. This fact also produces

$$\theta_1 + \theta_2 = 180^\circ \tag{5}$$

from which it follows that

$$E_{r1} = -E_{r2}. \tag{6}$$

Drawing the radial components of the resulting vectors, it is clear that the equivalent setup of dipoles produces tangential electric fields along the image plane that cancel, thus satisfying boundary conditions if a PEC had been present at the location of the plane.

Similarly, it can be shown that for the $\hat{\boldsymbol{\theta}}$ components that

$$E_{\theta 1} = D \sin\theta_1 = D \sin\theta_2, \tag{7}$$

$$E_{\theta 2} = D \sin\theta_2, \tag{8}$$

and along the boundary,

$$E_{\theta 1} = E_{\theta 2} \tag{9}$$

and the direction of the components is also such that the tangential components of the electric fields cancel.

The resulting system is thus equivalent to the original situation. This is to say that the fields above the ground plane are identical to those produced in the equivalent situation. Note that the fields beneath the ground plane in the physical situation are zero, and hence the model is not valid below the image plane. Hence, image theory can be summarized succinctly as [1]:

> The fields above a perfect ground plane from a primary source acting in the presence of the perfect ground plane are found by summing the contributions of the primary source and its image, each acting in free space.

Using similar reasoning, an arrangement of dipoles parallel to the ground plane produces the equivalent model as shown in Figure 3. Also, if the dipoles were arbitrary oriented, the currents could be decomposed into vertical and horizontal components (with suitable images).

The implications of this for dipoles are as follows:

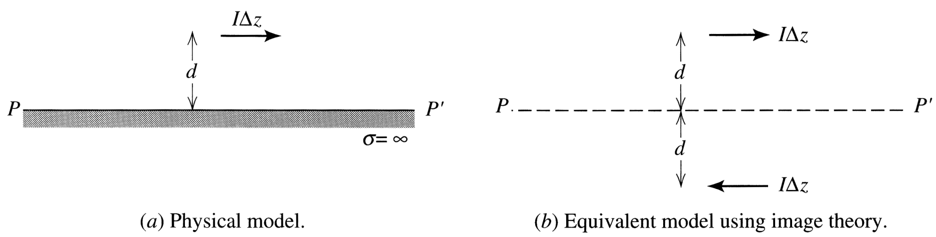


Figure 3: Image theory for horizontally oriented dipole [1]

- As $d \to 0$, the vertical dipole over ground will still work (the two currents effectively meld into one)
- As $d \to 0$, the horizontal dipole over grounded ceases to radiate because the real and image currents cancel (looks like a transmission line!)
- For both situations, you could use array theory to work out the exact response at some point above the ground plane.

The Monopole

We now turn our attention to a well-known antenna, the monopole.

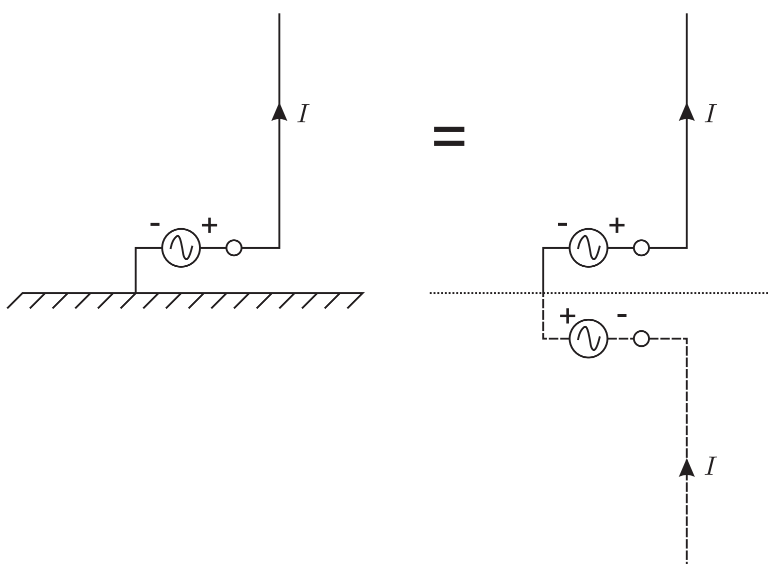


Figure 4: Monopole and equivalent model

By image theory, we have a dipole drive by a voltage source with amplitude V. In reality, we are developing the *same current* on the arm of the dipole (and its image) using only a source with amplitude $V/2$. To see this, we are trying to develop the same electric field across the gap as the dipole case. Since the gap is only half as wide, the voltage need only be half as much to develop the same gap field.

By this reasoning, it is very easy to determine the impedance of a $\lambda/4$ monopole:

$$Z_{monopole} = \frac{V_{monopole}}{I_{monopole}} = \frac{\frac{1}{2}V_{dipole}}{I_{dipole}} \tag{10}$$

and since $I_{monopole} = I_{dipole}$,

$$Z_{monopole} = \frac{1}{2}Z_{dipole} = 36.5 + j21.25\ \Omega \tag{11}$$

or simply $Z_{monopole} = 35\ \Omega$ at resonance.

The same reasoning can be applied to a monopole of any length.

By image theory, a monopole generates the *exact same fields* above the ground plane as a dipole. Therefore, $U(\theta, \phi)$ is the same as a dipole for $\theta \geq 90°$; the fields are zero beneath the ground plane. Hence, the *average power* radiated by a monopole is only half as much as a dipole. This notion is confirmed by the fact that the radiation resistance is only half of that of a dipole.

The directivity of a dipole is defined as

$$D_{dipole} = \frac{U_m}{U_{avg}} = \frac{U_m}{W_d/4\pi}$$

where W_d is the power radiated by the dipole. As a monopole only radiates half the power, $W_m = \frac{1}{2}W_d$, and

$$D_{monopole} = \frac{U_m}{W_d/8\pi}.$$

Therefore,

$$D_{monopole} = 2D_{dipole},$$

i.e., the monopole has twice the directivity of a dipole (3 dB more directivity). The increased directivity is not caused by increased radiation / field intensity, but rather a decrease in the average radiated power density.

Example: A $\lambda/4$ monopole has a directivity of $D = 2 \times 1.64 = 3.28$. In dB, it has 5.15 dBi of directivity (2.15 dBi + 3 dB).

References

[1] W. L. Stutzman and G. A. Theile, *Antenna theory and design*. John Wiley and Sons, Inc., 1998.