Resonant Enhancement of Electro-optic Modulators using Traveling-Wave Ring Resonators

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ABSTRACT

Resonant enhancement of electro-optic modulators has been shown to greatly reduce the required switching voltage ($V_s$) of these devices for narrow-band applications. Standing-wave resonantly enhanced modulators have been widely explored in the literature, but suffer from finite transit time effects and constraints on the active region length. Both of these effects limit their frequency of operation. A traveling-wave resonantly enhanced modulator is presented here that eliminates both of these problems. An implementation of the enhanced modulator using loss compensation is shown to reduce $V_s$ by a factor of 17.8, and reduce link loss in an analog radio-on-fiber link by 25 dB. One implementation of a multiple ring, traveling-wave enhanced modulator is also presented.

Keywords: Electro-optic devices, modulators, traveling-wave resonators, resonance, optical modulation, optical communication, pulse generation, radio-on-fiber, fiber radio

1. INTRODUCTION

The field of microwave photonics has enjoyed a significant amount of growth in recent years and applications abound as electro-optic devices improve. These applications include optically-fed phased array antennas, highly sensitive microwave receivers\textsuperscript{1}, high speed analog to digital converters\textsuperscript{2}, and optical pulse generation\textsuperscript{3}. Optical pulse generation using electro-optic devices has already proven itself as a practical technique, however another application has attracted even more attention. Radio-on-fiber systems have been established as a viable method for the transmission of analog bandpass signals over fiber optic networks. Not only does this method provide an additional means with which to leverage existing fiber optic infrastructure, it also greatly simplifies the interface between digital and analog (wireless) networks\textsuperscript{4}. This simple and inexpensive architecture will be of great benefit as wireless last-mile solutions and other small-area networks, including 802.11, 802.16, and Local Multi-point Communications Systems (LMCS) proliferate.

In each of these radio-on-fiber systems, as well as in the aforementioned applications, inefficient modulation of electrical signal onto the optical carrier is a serious performance limitation. In radio-on-fiber systems, this inefficiency results in a link loss (the conversion loss between the modulator input and the photoreceiver output) of greater than 30dB, with most of this loss concentrated in the modulation process. Link loss is proportional to the square of the switching voltage, or $V_s$, which for present-day commercial modulators is in the 3-6V range. Large switching voltages necessitate the use of high power levels, and consequently large, high-power amplifiers. Furthermore, if the modulators are to be used in harmonic upconversion\textsuperscript{5}, even higher power levels would be required. The addition of these amplifiers to radio-on-fiber systems negates the benefits of the simple and cost-effective architecture mentioned above.

There are presently several research thrusts underway that attempt to reduce the required switching voltage and enhance modulator performance. This paper focuses on one technique that employs traveling-wave resonators. Operational theory measured results are presented that demonstrate the benefits of traveling-wave resonant structures. A cascaded dual ring traveling-wave structure is also presented.

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2. BACKGROUND

2.1 Optical Modulation
External optical modulation has been proven as an effective technique to increase the bandwidth and minimize chirp in both digital and analog fiber optic communications links. The Mach-Zehnder modulator (MZM) is one type of external modulator and operates as an interferometer. An important parameter that characterizes the performance of the MZM is known as the extinction voltage, or $V_\pi$. This is the voltage range required to change the output optical power from maximum to minimum. For an MZM employing a second-order (Pockels) electro-optic material, $V_\pi$ is given by

$$V_\pi = \frac{d}{L} \frac{\lambda_0}{\mathcal{M} \Re n^3},$$

where $d$ is the material height over which the modulating voltage is applied, $L$ is the active region length, $\mathcal{M}$ is Pockels’ coefficient, $\lambda_0$ is the free space optical wavelength, and $n$ is the material’s refractive index when no field is applied.

This expression for $V_\pi$ is only valid for a constant applied voltage that corresponds to a constant, uniform electric field. In most high-speed systems, high-frequency signals are applied to the electro-optic material, and the resulting fields are neither constant nor uniform. A technique used to mitigate this effect is called traveling-wave velocity matching. If the electrode to which the modulating signal is applied is transformed into a transmission line, the speed at which the signal propagates can be made to match the speed of the optical wave in the electro-optic modulator. If the optical and electrical signals travel in the same direction and at the same speed through the active region of the modulator, the optical wave sees a constant electric field and the relation in equation (1) remains valid. Any deviation from velocity mismatch will result in an effective increase in $V_\pi$, and is well characterized.

2.2 Analog Fiber Optic Links
Analog fiber optic links are gaining favour as a new method to utilize fiber optic networks and merge optical and wireless systems. A simple analog radio-on-fiber link employing a traveling-wave MZM is presented in Figure 1 below.

![Figure 1: Sample radio-on-fiber link](https://via.placeholder.com/150)

The key measurement of the efficiency of the link is called link gain and is measured from the electrical input, $P_{e,in}$, to the electrical output $P_{e,out}$. For an MZM biased at quadrature (midway between optical maximum and minimum) and assuming that the input magnitude is small compared to $V_\pi$, the link gain can be shown to be

$$G = \frac{P_{e,out}}{P_{e,in}} = \frac{R^2 G_0^2 P_{o,in}^2 \pi^2 R_{M2} R_{DE}}{V_\pi^2}.$$
where $R$ is the responsivity of the photodetector, $G_o$ is the net gain of the optical link, $P_{in}$ is the input optical power, $R_{MZM}$ and $R_{DET}$ are the MZM and photodetector termination resistances respectively. The term that should stand out however is $V_s$. Link gain, and hence modulation efficiency, are inversely proportional to the square of $V_s$, which illustrates the need to reduce the switching voltage of the modulator.

3. MODULATOR ENHANCEMENT

As mentioned in the introduction, there are several research thrusts underway aimed at improving the efficiency of MZMs. In all applications this corresponds to reducing the switching voltage $V_s$, of the modulator. Re-examining equation (1) shows that there are three parameters that can be changed to reduce $V_s$. Firstly, there is Pockels' coefficient, a property of the material, which directly reduces $V_s$. Material research has been underway for some time and Dalton has achieved a great deal of success creating organic polymers with larger electro-optic coefficients. Secondly, increasing the length of the modulator's active region also acts to decrease switching voltage. Long active regions can be impractical to build due to their size as well as increased effects of velocity mismatch and electrode losses, however the apparent length of the active region can be increased using optical resonators. Creating resonant optical structures allow the optical wave to be exposed to the modulating field for a longer period of time before exiting the modulator. This effectively increases the length of the electrode, but exacerbates the problems caused by velocity mismatch. Optical resonant structures include both standing-wave and traveling-wave resonators and are presented by Gheorma. Lastly, there is electrode gap width. Decreasing this value acts to increase the electric field over the electro-optic material for a fixed value of input voltage. A similar effect can be achieved using resonant electrical structures. These structures also act to increase the electric field, but instead of decreasing the gap width, they boost the actual signal voltage itself through their energy storage properties. Like optical resonators, electrical resonators can also be standing-wave or traveling-wave structures. To date, only standing-wave structures have been extensively studied with little effort given to traveling-wave resonators. Standing-wave resonators suffer from finite transit time effects and constraints on active region length, which both act to reduce the maximum frequency of operation. These problems are eliminated using traveling-wave structures, as explained below.

3.1 Traveling-Wave Electrical Resonators

In contrast to standing-wave resonators which have waves traveling in two directions within the resonant cavity, traveling-wave resonators allow propagation in one direction only. For example, energy coupled into an annular cavity using an ideal directional coupler will only propagate in one direction. The proper selection of the input coupler will cause all of the input energy to be trapped inside the ring, a condition known as critical coupling. If the length of the ring is equal to an integer number of wavelengths, new waves entering the ring will add constructively with waves already in the ring and act to increase the net electric field. An MZM can easily be integrated into the ring resonator and exposed to the enhanced field thus improving both modulator and link performance. A visualization of such a circuit is presented in Figure 2; the gain block is used for loss compensation and will be discussed later.
As mentioned at the beginning of the section, this structure eliminates several of the problems associated with standing-wave resonators. Firstly, since the waves in the ring travel only in one direction, the traveling-wave nature of the MZM is preserved. This means that a photon traversing the active region will see a constant field. In a standing-wave resonator, the photon sees a standing-wave envelope profile, and a penalty is incurred if this envelope changes during the optical transit time. Secondly, the requirement that the length of the active region of the modulator be constrained to a half-wavelength is also eliminated, since any number of wavelengths can appear across the active region. Both of these improvements mean that the traveling-wave resonantly enhanced modulator is a completely frequency agile device, i.e. it can operate at any given frequency.

Of primary interest here is the extent of peak field enhancement measured from the input to the coupler to the MZM. For MZMs driven with signals much less than \( V_m \), this field enhancement factor is equivalent to the improvement in link loss. A detailed treatment of ring resonators is presented by Tischer\cite{4} and gives the field enhancement for a perfectly directive coupler with no gain inside the ring as

\[
F = \frac{E_{\text{max}}}{E_o} = \frac{C e^{-\gamma L/2}}{1 - T e^{-\gamma L}},
\]

(3)

where \( C \) is the coupling, (equal to S31) and \( T \) (S21) is the transmission factor. \( \gamma = \alpha + j\beta \) is the propagation constant for waves traveling inside the ring, and \( L \) is the length of the ring. \( C \) and \( T \) are related by conservation of energy (\( C^2 + T^2 = 1 \)), assuming infinite directivity. This equation is maximized when \( \beta L \) is equal to \( N2\pi \), or equivalently when \( L \) is equal to \( M \lambda \), where \( N \) is the mode number. The equation is transformed to a point half way around the ring from the midpoint of the directional coupler, which is taken to be the location of the center of the modulator’s active region, hence the \( e^{-\gamma L/2} \) term. The frequency dependence of the equation is evident, and it is instructive to derive an expression for the bandwidth of the resonant peaks. This is also derived by Tischer\cite{4} and is given as

\[
\text{BW} = \frac{1 - |T| e^{-\alpha L}}{|T| e^{-\alpha L}} \left( \frac{\lambda_{\text{res}}}{\lambda_0} \right)^2 \frac{1}{\beta L}.
\]

(4)

In both equations (3) and (4) there are two main parameters of interest: the coupling \( C \) and consequently \( T \), and ring propagation loss \( \alpha L \). In order to better illustrate the effects of these parameters on the level of field enhancement and resonant bandwidth, a set of design curves can be generated as the parameters are allowed to vary. The level of field enhancement is presented in Figure 3a as a contour plot for varying
levels of coupling and propagation loss. The contours represent lines of constant field enhancement in dB. Also of interest is the bandwidth that can be achieved for given levels of enhancement. Figure 3b shows a contour plot of the resonant percent bandwidth as coupling and propagation loss are varied for a ring of length λ (first order resonant mode).

![Figure 3a: Ring resonator field enhancement (dB)](image1)

![Figure 3b: Ring resonator percent bandwidth](image2)

There are several points of interest in the above graphs. Firstly examining the plot of field enhancement, for any value of coupling, there exists a value of propagation loss above which no field enhancement can be achieved. This value is higher for smaller values of coupling. For each value of attenuation, there exists a maximum value of field enhancement with an associated coupling parameter that corresponds to the critical coupling point for the resonator. Also note that as loss is decreased, the enhancement increases reaching a maximum when the propagation loss is zero. The level of field enhancement at zero propagation loss is not infinite however; this is due to the finite energy storage capabilities of the ring structure and will be discussed shortly.

Relating bandwidth to field enhancement, it is clear that the greater the field enhancement the smaller the bandwidth, which is to be expected based on the definition of Q. In addition to coupling factor and propagation loss, the length of the ring also affects the bandwidth of the resonator. Examining the expression for bandwidth given in equation (4) reveals that it is inversely proportional to βL. This is important because it means that for higher order modes the bandwidth shown in Figure 3b is reduced by a factor of $\frac{1}{N}$, a detrimental effect for systems requiring a finite usable bandwidth.

Returning to the explanation of finite field enhancement, the plots in Figure 3 only show the propagation losses caused by the transmission lines used to form the ring. For a wave traveling though one complete circuit of the ring, propagation loss is not the only loss encountered. Due to the symmetry of the directional coupler, a loss will occur each time a wave inside the ring passes through the coupler, as energy is essentially coupled out of the ring. This explains why that even for zero propagation loss there is not infinite field enhancement. This also explains why for higher values of coupling the bandwidth increases, since the net loss caused by the coupler is much larger than for propagation loss alone.

An interesting extension of the analysis presented above is to allow for the possibility of zero net loss inside the ring. This is accomplished by adding a gain to compensate for both propagation losses and coupling losses. The loss axis is adjusted to account for the coupling losses and the resulting graphs are presented in Figure 4.
Figure 4a shows that an infinite amount of field enhancement can be achieved for zero net loss in the ring. Also, the bandwidth now accounts for coupling losses and is essentially independent of coupling factor. The insertion of a loss compensation block inside the ring allows for additional control over the performance of the ring. Both loss and coupling can be selected to provide a required amount of bandwidth for a given value of field enhancement.

3.2 Multiple Resonator Structures
The characteristics of several traveling-wave resonators can be combined to further improve both the magnitude and bandwidth performance of the enhanced modulator. Several topologies of these multi-ring structures are presented in Figure 5.

Each of the above structures comprises three separate resonant cavities. The lengths of these cavities can be adjusted in order to change the resonant frequency. Depending on the application, the rings can be made to have the same resonant frequency, or operate at different frequencies. The three rings operating at the same frequency would provide a more narrowband resonant peak, while slight variations in the frequencies could combine to produce a broadband response. In each case, the field applied to the MZM electrode travels in one direction, hence preserving the traveling-wave nature of the device. This is assured by the use of a directional coupler at the input, and the directional nature of coupled lines.

The design of these structures again depends on the application, but their analysis is relatively straightforward and can be done using signal flow graph theory and the principles of optical modulation presented above. As an example, the cascaded structure in (a) is analyzed using signal flow graph theory for only two rings. Assuming perfect isolation in the directional coupler and the coupled lines, perfectly
matched ports, and a homogeneous ring material, the transfer function from the coupler input to the center of the MZM is given as

\[ F = \frac{E_{\text{max}}}{E_0} = \frac{C_1 e^{-\gamma_1/2} C_2 e^{-\gamma_2/2}}{1 - (T_1 T_2 e^{-\gamma_1} + T_2 e^{-\gamma_2} + C_1^2 e^{-\gamma_1} + C_2^2 e^{-\gamma_2}) + T_1 T_2^2 e^{-\gamma_1} e^{-\gamma_2}}. \] (5)

\( C_{1,2} \) and \( T_{1,2} \) are the coupling (\( S_{21} \)) and transmission (\( S_{31} \)) factors of the directional coupler and coupled line. \( y = \alpha + j \beta \) is the propagation constant inside the rings, and \( l_{1,2} \) are the ring lengths. It is illustrative to plot the above transfer function for a typical set of parameters. Assuming a 10dB coupler and coupled line (\( C_{1,2} = 0.32 \), \( T_{1,2} = 0.95 \)), a net ring loss of 0.25 dB in each ring, and ring lengths \( l_1 = 1.02 \lambda \) and \( l_2 = 0.98 \lambda \), the resulting transfer function is presented in Figure 6. As a reference, the enhancement for a single ring as defined by equation (3) is also presented for the same coupling and loss values and a ring length of \( \lambda \).

![Figure 6: Multi-ring transfer characteristic](image)

Through the proper selection of the ring lengths and by matching the net losses in both rings, a symmetric transfer characteristic can be produced. The 3dB bandwidth of the multi-ring structure is 2.7% while for the single ring structure it is 0.9%, a three times improvement. Passband ripple is equal to 1.5dB, and can be reduced by reducing the difference between the ring lengths. Also of note is the improved value of enhancement over the passband. This is a result of the finite bandwidth of resonance in each ring; one ring will still exhibit resonant enhancement at the other ring’s frequency.

This technique is completely analogous to coupled resonator filter design, and opens the door to customizable electro-optic modulator transfer characteristics. The only caveat is that electrical waves travel in only one direction across the modulator electrode, which is simple to achieve if directional couplers and coupled lines are used.

4. EXPERIMENTAL RESULTS

The theoretical results presented in Section 3 demonstrate the potential benefits of using single and multi-ring resonators in radio-on-fiber links. Field enhancement, bandwidth, and center frequency can be tailored to meet the needs of a particular application. The following sections present measurements of both passive and loss-compensated single ring resonators, as well as a simple multi-ring structure.

4.1 Passive Ring Resonator
A passive ring resonator based on the schematic in Figure 2 was constructed and characterized. No loss compensation was used and the coupler employed was a 10 dB directional coupler with its isolated port
accessible. An externally terminated MZM traveling-wave modulator and interconnect components required to complete the ring structure were also used. Figure 7 below shows the link loss improvement observed using a ring resonator compared with a standard traveling-wave modulator. Both structures employed the same MZM. The theoretical value of field enhancement using equation (3) along with measured values for ring loss and coupling is also presented. The strong correlation between the two supports the assertion that field enhancement inside the ring translates directly to improvement in link gain.

![Figure 7: Theoretical and measured link loss improvement](image)

Here the frequency selectivity of the structure is clearly evident in the multiple peaks, each corresponding to a separate resonant mode. There is also a correlation between link loss improvement and loss observed in the ring which is verified by the theoretical curve. With respect to this loss, it is evident that there is a threshold value below which there is no benefit to using a ring resonator over a traveling-wave device. Also apparent is the maximum enhancement achievable (5 dB for N = 1), which occurs when ring loss is a minimum. The limitations imposed by the ring losses underscore the need for loss compensation.

4.2 Loss-Compensated Ring Resonator
To further improve modulator performance, an active version of the circuit was constructed that incorporated a gain block for loss compensation and a filter to isolate a single resonant peak. The gain block was tailored to allow the net ring loss to be swept from 0 to 3 dB. Link loss improvement and bandwidth were measured for N = 13 (1.7 GHz) and the results are presented in Figure 8.

![Figure 8: Measured modulator enhancement over attenuation (P_e=45dBm)](image)
Up to 25 dB of link gain improvement (a \( V_a \) reduction of 94\%) was observed, a level hereto unmatched by resonantly enhanced structures in the literature. The levels of enhancement achieved far exceed the net gain added to the ring circuit (a few dB). In essence, the gain of the amplifier is increased beyond its nominal value by the resonance effect of the ring. This has significant implications on amplifier requirements since only inexpensive low-gain amplifiers are required to provide loss compensation in the ring circuit. Achieving the same level of gain by driving the MZM directly would require an amplifier with at least 20 dB more gain.

There is a strong correlation between theoretical and measured link loss improvement in Figure 8. The two main deviations being caused by amplifier saturation (at low loss values) and limited dynamic range in the test equipment (at high loss values). Low input power levels (-45-dBm) were used to maximize the achievable enhancement, at the expense of dynamic range at larger loss values.

Figure 8 also shows that link gain improvement can be traded off against bandwidth. For applications requiring low bandwidths, \( V_a \) can be reduced enormously, and for higher bandwidth applications the enhancement can be reduced to assure that the bandwidth requirements are met. The bandwidth observed in Figure 8 can be substantially increased by reducing the length of the ring. The 13th order mode was a result of the connectorized components that were used to build the active circuit. From equation (4), operating at a first order mode would provide a 13 times increase in bandwidth.

4.3 Cascaded Multi-Ring Resonator

A cascaded double ring structure based on Figure 5a was also constructed. Loss compensation similar to that used in the single ring active circuit was used in each ring, which allowed for amplitude tuning. The frequency tuning was accomplished by placing a line stretcher in one of the rings. The compensation and resonant frequencies were adjusted to create contiguous resonant passbands. The results are compared to those using a single ring with the same net ring loss and presented in Figure 9.

![Figure 9: Measured multi-ring resonator performance](image)

The above plot validates the theoretical plot presented in Figure 6. The two resonant peaks are present, and the bandwidth is increased from 0.38\% for a single ring to 1.24\% for a dual ring, an improvement of greater than three times. Work is underway to further characterize these multi-rings structures.

5. CONCLUSIONS

The traveling-wave ring resonator presented in this paper dramatically improves, and in some cases can eliminate link loss for narrow-band radio-on-fiber and other analog systems. In active realizations of the circuit, the enhancement achieved far exceeds the actual gain added by the active component. The
structure is superior to traditional standing-wave modulators because it incurs no additional velocity mismatch penalty and is not limited in frequency of operation. Theoretically, any traveling-wave field-effect device can benefit from this type of resonant enhancement. Multi-ring resonators can increase the bandwidth of these devices and can be used to customize the link transfer characteristics in radio-on-fiber systems. Work is underway on monolithic integration of these structures in order to further increase the operational bandwidth. Other loss compensation techniques are also being explored. Apart from the performance benefits, resonant enhancement using ring resonators can greatly reduce the cost of future radio-on-fiber base stations.

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