Analysis of Radiating Microstrip Structures Using the Contour Integral Method

Utkarsh R. Patel*, Piero Triverio, and Sean V. Hum
Edward S. Rogers Department of Electrical and Computer Engineering
University of Toronto
Toronto, Ontario, Canada
utkarsh.patel@mail.utoronto.ca

Abstract—In this paper, we discuss the possibility of numerically solving electromagnetic radiation problems from microstrip structures with only contour discretization of the radiating elements. We demonstrate that such a numerical method can be realized by exploiting the underlying physics of microstrip surfaces, and combining the so-called contour integral method with the equivalence principle. Such a numerical technique requires fewer unknowns and can potentially lead to significant computational savings in the simulation of large microstrip structures such as arrays, reflectarrays, metasurfaces, etc. Preliminary results show that the technique can accurately predict the input impedance and radiation pattern of a patch antenna in an array environment.

I. INTRODUCTION

The electromagnetic equivalence principle allows one to represent a complex field distribution inside a closed volume with equivalent electric and magnetic current sources on the enclosing surface. The equivalence principle motivated the development of the integral equation method (IEM) [1] which requires discretization of only the surface of a metallic scatterer. In comparison to finite element method (FEM) [2], which needs volumetric 3D discretization with number of unknowns \( N \) of order \( O(n^3) \), the IEM needs only a 2D mesh which contributes to superior speedups and \( N \sim O(n^2) \).† However, for problems involving electrically large microstrip structures, such as reflectarrays, transmitarrays, and metasurfaces, the computational time and memory requirements can become unreasonable even with IEM. This motivates the need for reduced-order methods that facilitate further dimensionality reduction. Such a method would significantly lower the number of unknowns and enable the simulation of large planar microstrip arrays, including metasurfaces.

In the literature, contour-based methods have been successfully applied to analyze guided-wave microwave structures [3] where radiation loss is of little interest. A contour-based method has been proposed for simulating patch antennas [4], but it requires prior knowledge of field distribution on the edges of the patch, and can therefore be applied only to resonant antennas where the cavity model is accurate [5].

In this paper, we discuss how the physical understanding of microstrip antennas can be combined with the equivalence principle [5] and the so-called contour integral equation method (CIM) [3] to analyze radiating microstrip structures. The presented technique is general and does not assume any prior knowledge of field distribution on the radiating elements.

II. ASSUMPTIONS

Consider the geometry of a sample patch antenna shown in the left panel of Fig. 1. The following two assumptions hold when electrical spacing between the two metallic plates is small [5]:

- The \( z \)-directed electric field is dominant compared to the \( z \)-directed magnetic field, which may be ignored. In other words, the fields satisfy the transverse magnetic field assumption.
- If the two parallel plates are enclosed by an equivalent surface, then the main radiation mechanism is due to the equivalent currents on the periphery. The equivalent currents on the top and bottom surfaces may be ignored.

Additionally, for thin dielectrics we may assume that the fields are invariant along \( z \)-direction, i.e. \( \partial E/\partial z = 0 \).

† \( n \) is the number of basis functions per dimension
III. MAIN IDEA

To solve for fields everywhere in space, we decompose the problem into two subproblems.

A. Contour Integral Method

In the first subproblem, we consider the volume enclosed by the two metallic plates and the sidewalls $\gamma$, and relate the electric and magnetic field using the CIM [3]

$$ E_z(\vec{r}) = \frac{j}{2} \int_{\gamma} \left[ \frac{\partial H_0^{(2)}(k|\vec{r} - \vec{r}'|)}{\partial n'} \right] E_z(\vec{r}') + j \omega \mu H_0^{(2)}(k|\vec{r} - \vec{r}'|) H_t(\vec{r}') \, dr' $$  \hspace{1cm} (1)

where $k = \omega \sqrt{\varepsilon \mu}$ is the wavenumber inside the cavity, and $H_0^{(2)}(.)$ is the Hankel function of the second kind. Other relevant geometrical parameters in (1) are shown in the right panel of Fig. 1. We let $\vec{r}'$ in (1) be on the contour $\gamma$, to obtain a relationship between $E_z$ and $H_t$ on $\gamma$.

B. Integral Equation

In the second subproblem, we apply the equivalence principle and replace the two parallel metal plates by the equivalent currents on the periphery $\gamma$. Furthermore, since fields between the plates are invariant with the $z$-direction, 1D basis functions are sufficient to discretize equivalent currents on the periphery $\gamma$.

Next, we relate $E_z$ and $H_t$ on $\gamma$ through the magnetic field integral equation

$$ H_t(\vec{r}) = \oint_{\gamma} G_j(\vec{r}, \vec{r}') J_z(\vec{r}') \, dr' + \oint_{\gamma} G_m(\vec{r}, \vec{r}') M_t(\vec{r}') \, dr' $$  \hspace{1cm} (2)

where $G_m(\vec{r}, \vec{r}')$ and $G_j(\vec{r}, \vec{r}')$ are the Green’s functions associated with magnetic and electric sources, respectively. Since the equivalent currents in (2) are related to $E_z$ and $H_t$, (2) provides a second relation between $E_z$ and $H_t$.

C. Continuity of Fields

According to the uniqueness theorem, we enforce continuity of fields along $\gamma$ to solve for fields $E_z$ and $H_t$ along $\gamma$ in terms of excited fields. Once $E_z$ and $H_t$ are known along $\gamma$, (1) and (2) can be used to find fields elsewhere.

IV. NUMERICAL RESULTS

For validation, we apply the proposed method to solve for the impedance and radiation from a $2 \times 2$ rectangular patch antenna array. The dimensions of the antenna elements, array spacing, feed position and substrate height are shown in Fig. 2.

A. Input Impedance

Selected entries of the impedance matrix of the patch array obtained with the proposed method are shown in Fig. 3. The impedance obtained with the proposed method matches well against Ansoft HFSS. In particular, both far field radiation losses and near field reactive coupling are accurately captured.

B. Radiation Pattern

Fig. 4 shows the directivity of the antenna array when a uniform excitation is applied across all elements. There is an excellent agreement between directivity calculated with the proposed method and Ansoft HFSS.

V. CONCLUSIONS

In this paper, we presented a novel idea for solving radiation from microstrip structures using the hybridization of the contour integral method and the equivalence principle. The accuracy of the numerical results suggests that contour discretization is sufficient to solve for radiation problems from microstrip antennas. Currently, the method is being extended to include to include dielectrics. Hence, once fully developed, the method is expected to offer a faster means to simulate reflectarray, metasurfaces and other planar microstrip structures with $N \sim O(n)$, compared to $N \sim O(n^2)$ required with the IEM.

REFERENCES