MEDIUM FREQUENCY RE-RADIATION FROM AN UNSTRUNG STEEL POWER LINE TOWER

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Abstract

Computations of AM broadcast re-radiation from a power line tower without phase conductor wires or overhead ground wires are compared with measurements. The tower is a 500 kV type located near Toronto. Computations of re-radiation using a detailed tower model including ground effects due to the footings, compared with measurements done in the near field of the tower, show good agreement in terms of frequency response and variation with distance from the tower. Two simplified computational models are compared with the detailed one in the far field and found to be accurate enough to be of practical use. Methods are given for deriving the models, predicting re-radiation from any tower within certain specifications, and extrapolating near-field measurements to far-field values.

Introduction

The radiation patterns of AM broadcast antennas often have deep nulls toward other stations to avoid interfering with the reception of their signals. In directions of service, the radiation must be strong enough to provide an adequate signal-to-noise ratio. Proof that these specifications are met must be provided to the Department of Communications (DOC) after any new or modified antenna is implemented. Nearby steel-tower power lines can scatter the AM broadcast radiation and severely affect the pattern by filling the nulls and notching the lobes, resulting in interference to other stations and inadequate coverage. This problem is of such concern in Canada that the DOC Working Group on Re-Radiation Problems in AM Broadcasting was created, with power lines as one of its priorities. The group has representatives from the DOC, power line utilities, broadcast consultants, broadcast stations and universities.

Recently, for example, a case study comparing power line re-radiation computations with measurements was presented by Silva, Balmain and Ford [1], in which the power line considered was complete with over-head lightning ground wire or "skywire". Computations have also been compared with physical model measurements by Trueman and Kubina [2]. Furthermore, it has been found that the power carrying cables (phase conductors) can generally be ignored because of weak coupling to the towers, the skywires and the vertically polarized broadcast antenna [3], [4]. Little is available in the literature on towers without skywires in spite of the fact that skywires are not used in regions of low thunderstorm activity. Even with a skywire attached there must still be assurance that the tower to which it is fastened is properly modelled for computation. Furthermore, broadcast engineers sometimes insulate a tower top from the skywires to reduce re-radiation, in which case the tower acts as an isolated structure, with its incident field modified by the rest of the power line. Besides validation of computational models, there remains a need to provide some simplified predictive methods for initial assessments by broadcast engineers.

Equipment

The power line tower tested (Figure 1) was a 52.4 m Ontario Hydro type V1S, number 106 of the Cherrywood-Claireville 500 kV line just north of Toronto. It had adjacent towers 250 m away and had no phase conductors or skywires at the time the tests were carried out. It rests on reinforced concrete foundations (footings) 1.5 m in diameter and 5.3 m in depth in which the peripheral reinforcing rods are wired to the tower bolts for a better lightning ground.

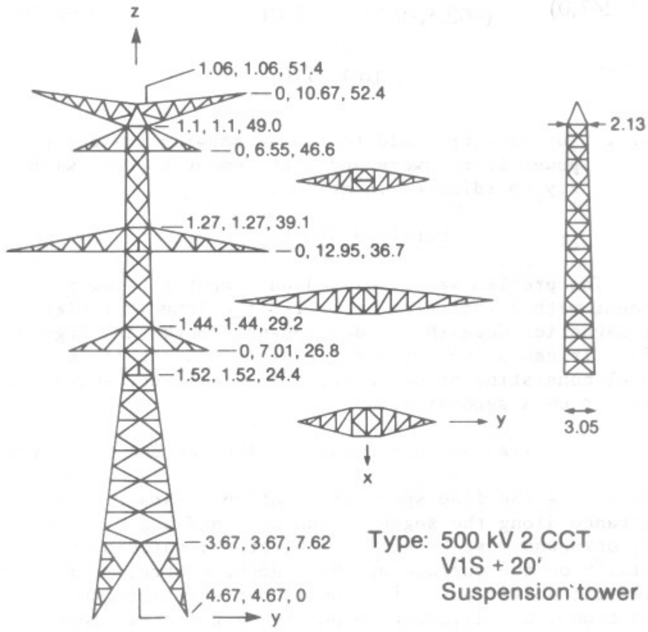

Fig.1 Scale drawing of tower 106 of Claireville-Cherrywood line showing x,y,z coordinates in metres.

The antenna used was a base-insulated telescopic tapered mast with a height of 20 m and diameters of 115 mm at the base and 50 mm at the top. Additionally, it contained a top-loading hat of 6 spokes with lengths of 3 m and diameters of 25 mm. The antenna was fed by a variable-frequency MF generator, wideband amplifier and matching network to cover the entire AM broadcast band.

The field strength meter was a typical type used for AM broadcast measurements. It uses an electrically small loop antenna and thus measures magnetic field strength ($\bar{H}$), although the scale is in units of electric field strength ($\bar{E}$), so the readings were divided by the intrinsic impedance of free space ($\eta_o = 120\pi$ Ohms) to obtain $\bar{H}$.

Measurements

The antenna was set up 263 m away from the tower on a bearing of 103° from the power line axis (Figure 2).

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The measurement points were on a circular path passing through, and centred between, the antenna and tower. With this geometry, the bearings from the measurement points to the antenna and tower are 90° apart, so that the primary contribution to H_ϕ was from the tower, and H_ρ was from the antenna. Eight equally spaced measurement points were used ranging from 18 m to 62 metres away from the tower. Measurements of $|H_\phi|$ and $|H_\rho|$ at these points were done for 10 frequencies over the AM broadcast band.

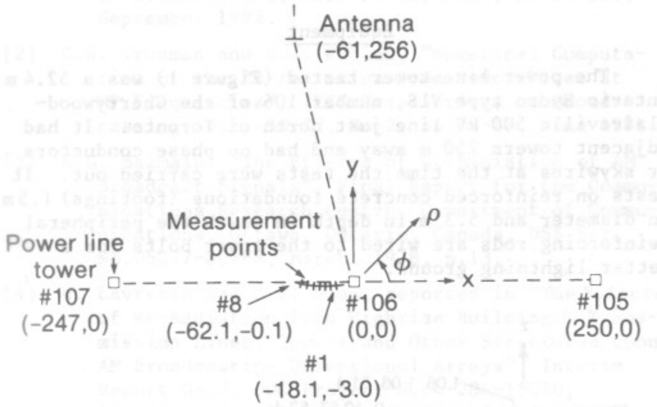

Fig.2 Top view of field test site showing the antenna, power line towers and measurement points, with x,y coordinates in metres.

Detailed Simulation

The problem was simulated on computer using a moment method (MM) program called the Numerical Electromagnetics Code (NEC), developed by Burke and Poggio [5]. It can solve for scattering from a structure model consisting of segmented wires. It represents the current on a segment as

$$I(s) = A + B \sin \beta s + C \cos \beta s \qquad (1)$$

where β is the free space propagation constant, s is distance along the segment, and A, B and C are unknowns. The unknowns must be solved subject to: matching the axial E on the surface of the segment centre; making the electric scalar potential continuous across segment junctions; and allowing no point charges at segment junctions or free ends. If the structure consists of N segments, then an $N \times N$ complex interaction matrix must be filled and solved for a given excitation. To keep computer memory requirements within practical limits it is necessary to use no more segments than required to represent accurately the actual structure. A perfectly conducting ground plane was used and ground effects due to the tower footings were represented by lumped loads.

It was expected that the antenna, which was a top-loaded tapered monopole, could be represented by a taller uniform monopole without top loading. The antenna was first modelled by 5 vertical segments and one segment for each of the 6 spokes on the top hat. The vertical segments had radii equal to the average radius of the real antenna over that portion. Next, a simpler model was tested consisting of 3 vertical segments 41 mm in radius, obtained as an average of the radius of the entire vertical portion of the actual antenna. A height of 29 m was found to give approximately the same incident $\bar{H}$ ($\bar{H}^i$) on the ground at the power line tower location as the detailed model, for a 1V source. The two models were then compared for $\bar{E}^i$ distributions normalized by the previous $\bar{H}^i$ values up to the top of the tower. The 2 models differed by a factor of less than 10^{-3}. The $\bar{E}^i$ approximated a uniform illumination because it varied by less than 6% along the length of the tower.

A detailed power line tower model was developed by placing wires to define the edges of the tower including the cross arms (Figure 3a). The radii of segments were chosen so that the model and the actual tower would have equal distributed capacitances at the segment midpoints. This was accomplished by assuming the actual tower face can be represented by a solid surface. Then, a result of Lo [6] or King [7] allowed conversion of a square cross section to a circular one using

$$a_{eq} = 0.59\ w \qquad (2)$$

where w is the face width, and a_{eq} is the equivalent radius. Next, the four vertical segments of the model were also converted to an equivalent radius from a result of Schelkunoff [8] for the cage antenna,

$$a_{eq} = a_c\left(\frac{na}{a}\right)^{1/n} \qquad (3)$$

where n is the number of wires, 4 in this case, a is the wire radius and a_c is the cage radius, being that from the tower axis to its corner. Equating (2) and (3) yields

$$a = 0.086\ w \qquad (4)$$

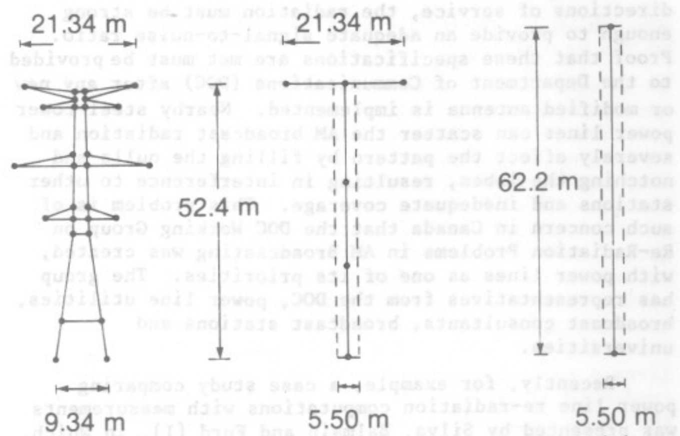

Fig.3 Computational models of the power line tower: (a) detailed model using 92 segments; (b) simple T model using 5 segments; (c) pole model using 1 segment. Solid lines and dots show segment axes and junction points. Dashed lines in (b) and (c) show equivalent vertical member diameters.

A few horizontal lattice segments were added near any abrupt changes in tower shape. A radius of 0.1 times the length was used because this is the largest radius to length ratio that can be safely used in NEC using the standard thin wire approximations. In total, 92 segments were used.

To represent ground effects due to the footings, the compensation theorem of Monteath [9] was used as done by Silva, Balmain and Ford [1]. In this approach, for a sinusoidal filamentary monopole, the difference in self impedance ΔZ between the case of a perfectly conducting ground plane and the case of a perfectly conducting footing of infinite depth embedded in a lossy ground plane, is calculated and used as a lumped load at the base of the tower model. The monopole was made 65.5 m high to be $\lambda/4$ at the apparent resonant frequency of the tower as determined by the $|\bar{H}|$ measurements.

To represent the four tower footings by one equivalent footing for the monopole, the formula for ΔZ as given by Monteath [10]

$$\Delta Z = \int_s \eta_g \; \bar{H}.\bar{H}\, dS \tag{5}$$

was applied to both situations, where η_g is the ground surface impedance, $\bar{H}$ is for the perfect ground case, and the surface integral is applied over the entire ground plane. The tower footings were assumed to be perfectly conducting. The $\bar{H}$ from the four-footing case was approximated by

$$\bar{H} = \sum_{i=1}^{4} \frac{I_o}{4} \frac{\hat{\phi}_i}{2\pi\rho_i} \tag{6}$$

where ρ_i and ϕ_i are the observation coordinates referred to the i^{th} footing. The $\bar{H}$ for the single footing case was approximated by

$$\bar{H}' = \frac{I_o}{2\pi\rho}\hat{\phi} \tag{7}$$

Placing (7) in (5) and integrating over ϕ and out to ρ yields for the filament,

$$\Delta Z'(\rho) = \frac{I_o^2}{2\pi} \ln \frac{\rho}{a_{eq}} \tag{8}$$

Placing (6) in (5) and integrating numerically over ϕ and out to ρ yields $\Delta Z(\rho)$. The a_{eq} is found by equating $\Delta Z(\rho)$ and $\Delta Z'(\rho)$, which yields

$$a_{eq}(\rho) = \rho e^{-\frac{2\pi}{I_o^2}\Delta Z(\rho)} \tag{9}$$

$$a_{eq} = \lim_{\rho\to\infty} a_{eq}(\rho) \tag{10}$$

The $a_{eq}(\rho)$ was plotted as ρ was increased to test for convergence. At all ρ values tested, from 10 to 80 m, the a_{eq} was 5.4 m, so convergence occurred for ρ less than 10 m. The a_{eq} derived in this manner was found to be equal to that derived using the cage antenna formula (3).

Computed values of ΔZ using Monteath's equation [11] for the monopole with sinusoidal current were multiplied by 4 and placed on the bottom segment of each leg of the NEC tower model with a perfect ground plane. In the computation of ΔZ, the earth's relative permittivity ϵ_r was taken to be 15 and the relative permeability μ_r was 1 as commonly practiced in MF ground wave predictions. The conductivity was initially estimated at 6 mS/m in accordance with a conductivity map published by the DOC.

The ratio $|H_{\phi 1}/H_{\rho 8}|$, being the re-radiated $|\bar{H}|$ at the measurement point nearest the tower, normalized by direct $|\bar{H}|$ at the farthest (8th) point, was plotted as a function of frequency for both measurements and NEC computations. These points were selected because they had the largest ratios of desired to undesired $\bar{H}$, where the desired values are the re-radiation (H_ϕ) from tower 106 and the direct field (H_ρ) from the antenna. The results showed that the computed values were too low, especially near resonance. This was attributed to incorrect conductivity. The conductivity was then adjusted to produce good agreement near resonance. The conductivity required was 22 mS/m, resulting in the ΔZ values plotted in Figure 4. Frequency response was then computed (Figure 5). The agreement is within 10% on 7 of the 10 frequencies. A review of data on soil samples taken under the tower showed that the water table was unusually high, thus explaining the high conductivity, as discussed in Appendix A.

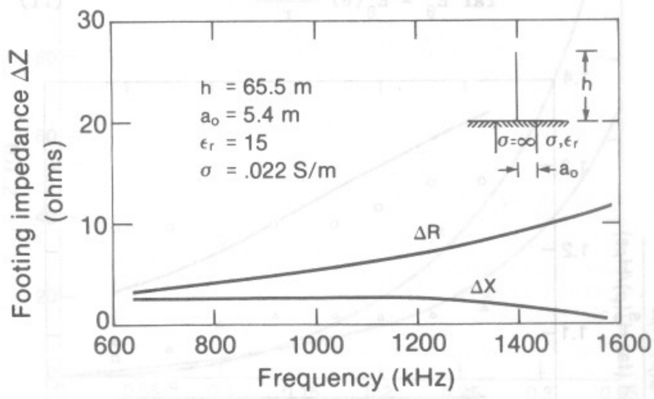

Fig.4 Footing impedance of a monopole with sinusoidal current over a perfectly conducting footing embedded in earth.

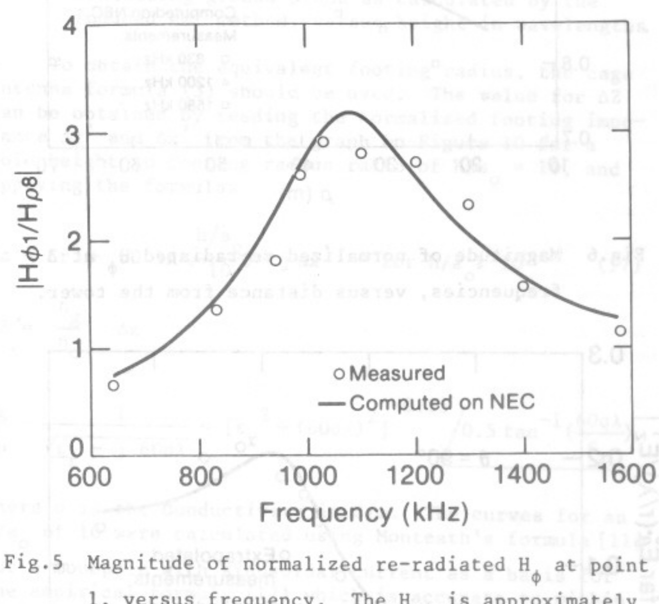

Fig.5 Magnitude of normalized re-radiated H_ϕ at point 1, versus frequency. The $H_{\rho 8}$ is approximately the H incident on the tower.

The near H_ϕ variation with distance was plotted (Figure 6) on 3 representative frequencies for measurements and computations. The values were normalized by the computed re-radiated far field so they would converge to a value of 1 for large distances. All measurement points fall within 10% of the computations. The 3 farthest points show some random variations, probably due to scatter from the other towers. The 5 closest points show that the measured values follow curves similar to the computed ones, and hence the measurements can be extrapolated to far field values using the computed far-to-near field ratios. Further, it was assumed that the measured direct $\bar{H}$ could be extrapolated to $\bar{E}^i$ at the tower using the computed ratio. A plot of far $|E_\theta|$ divided by $|E_z^i|$ as a function of tower height in wavelengths was done (Figure 7) for extrapolated measurements and computations. Because of the extrapolation technique, the agreement is exactly the same as for the near H plot. This plot is in a form most useful for predictions of re-radiation from other towers of similar shape but different heights. Here, far E_θ has been defined as

$$\text{far } E_\theta = E_\theta(\theta)\,\frac{e^{-j\beta r}}{r} \tag{11}$$

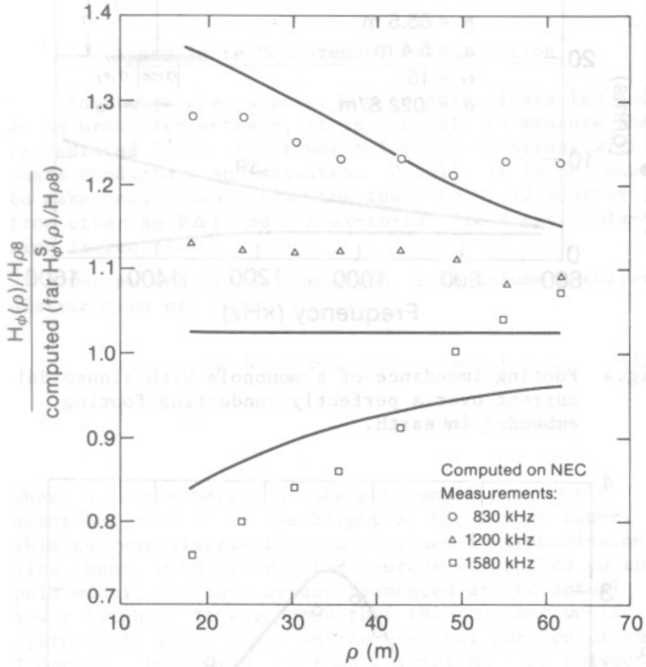

Fig.6 Magnitude of normalized re-radiated H_ϕ at 3 frequencies, versus distance from the tower.

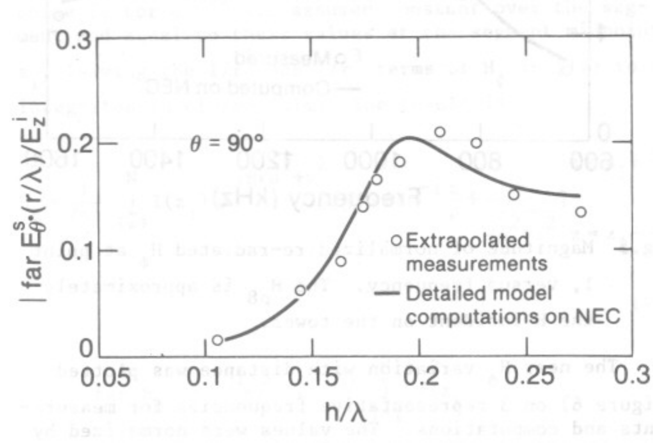

Fig.7 Magnitude of normalized re-radiated far E_θ on the horizontal plane, versus tower height in wavelengths.

Based on the comparisons in this section, the computational technique is considered to be sufficiently accurate for the previously mentioned applications.

Simplified Model

In order to analyse a tower on a small computer, or to analyse a large section of power line on any computer, the tower model must contain no more segments than necessary. One such model is a T shape (Figure 3b) which has 3 vertical segments with radii of 2.25 m and total height of 52.4 m equal to that of the actual tower, and 2 horizontal segments with radii of 0.05 m and lengths of 10.7 m equal to those of the actual top cross arms. The height and cross arm lengths were chosen to meet the sky-wire attachment locations, so that the tower model could be used as part of a complete power line model. The radius of the vertical wire, a, was chosen to have the same average equivalent radius that the actual tower would have if its faces were solid and if it had no cross arms.

An important criterion of equivalence between the T and detailed models is the equality of resonant frequencies. The radius of the horizontal wire on the T model was chosen to meet this criterion. This could have been done by iteration using NEC, but instead a simpler approximate approach was used. Transmission line theory, as done by Schelkunoff [12], was used to calculate base impedances for the T model and a multi-arm model in which an actual tower crossarm or main body portion is represented as a transmission-line segment of constant equivalent radius. In this analysis, segments are treated as centre conductors of coaxial cable with a fictitious outer shield, having characteristic impedance

$$Z_o = N60\left[\ln\frac{2}{\beta a_{eq}} - \gamma\right]$$

$$\gamma = 0.577\ \ldots\ \text{(Euler's constant)} \tag{12}$$

$$\left.\begin{aligned} N &= 1 \\ a_{eq} &= a \end{aligned}\right\}\ \text{except on multiple crossarms (see Appendix B)}$$

where a is the radius of a model segment. Pairs of cross arms are treated as open-circuited lines joined in parallel to the vertical line (tower body). Impedances are transformed along segments according to normal transmission line equations for reactive loads:

$$B_{in} = Y_o\,\frac{B_L\cos\beta\ell + Y_o\sin\beta\ell}{Y_o\cos\beta\ell - B_L\sin\beta\ell} \tag{13}$$

where B_{in} and B_L are input and load susceptances. The tower is resonant if B_{in} is infinite for a voltage source at the base. Resonance was found by iterating the frequency. Although the resonant frequency obtained by this method is approximately 10% higher than NEC computations, it was thought that if the T and multi-arm models resonated at the same frequency using this method, then the same would be true using NEC with an incident field.

The re-radiated far field was computed using NEC for the T model, including the footing impedance previously calculated, and plotted along with the field from the detailed model (Figure 8). It is seen that the agreement is good in spite of their great difference in complexity. The curve of the T model is displaced upward in frequency by approximately 3%. This can be corrected by a minor adjustment of the cross arm radius using iteration on NEC. It is expected that this tower model would be a good one to use in a model of a complete power line including skywires.

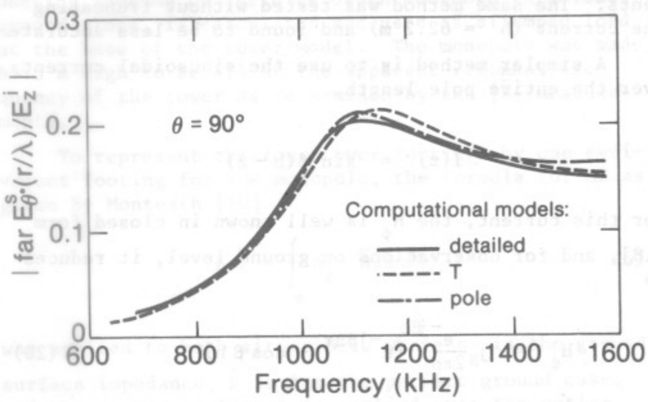

Fig.8 Magnitude of normalized re-radiated far E_θ on the horizontal plane for 3 computational models, versus frequency.

Approximate Predictive Methods

It is desirable to obtain approximate methods to predict power line tower re-radiation for broadcast engineers to use when considering treatment of towers by insulating the skywires, and for application to power lines without skywires.

The best approach tested resulted from a pole model of the tower (Figure 3c). The pole radius was chosen to be 2.25 m, equal to the average equivalent radius of the actual tower if faces are assumed solid and cross arms are neglected, as was done for the T model. The height was chosen to be 62.2 m, to produce the same resonant frequency as the multi-arm model using transmission line theory as done for the T model.

For simplicity, the current was assumed to be sinusoidal

$$I(z) = I_o \frac{\sin\beta(h-z)}{\sin\beta h} \qquad (14)$$

where I_o is the base current and h is the pole height. The short circuit current I_{sc} can be calculated as in Jordan and Balmain [13]. The re-radiated far E_θ can then be found using I_o equal to I_{sc} in (14), and in the far E_θ formula [14]. The resulting formula is

$$\frac{E_\theta^s}{E_z^i} = -\frac{j}{2\pi}\frac{\eta_o}{Z_{in}}\frac{e^{-j\beta r}}{\beta r}\left(\frac{1-\cos\beta h}{\sin\beta h}\right)^2 \quad \text{for } \theta = \frac{\pi}{2}$$

or for graphical purposes,

$$\left|\frac{E_\theta^s \cdot (r/\lambda)}{E_z^i}\right| = \frac{30}{\pi|Z_{in}|}\left(\frac{1-\cos\beta h}{\sin\beta h}\right)^2 \qquad (15)$$

where E_θ^s is the re-radiated far E_θ on the horizontal plane, r/λ is the distance in wavelengths, and Z_{in} is the self impedance Z_{self} of a sinusoidal monopole as calculated using the induced emf method [15], in series with the footing impedance ΔZ as previously calculated.

The re-radiated far field magnitude was computed for this method with the pole model and plotted along with NEC computations for the detailed and T models (Figure 8). The agreement is surprisingly good, and acceptable for its stated application.

To simplify calculation of the self impedance, a method given by Tai [16] is to read values of R and X' from the graph of Figure 9 for the pole height in wavelengths, and compute the self impedance Z_{self} as

$$Z_{self} = R + jX' - j Z_o \cot\beta h \qquad (16)$$

$$Z_o = 60(\ln\frac{h}{a} - 1)$$

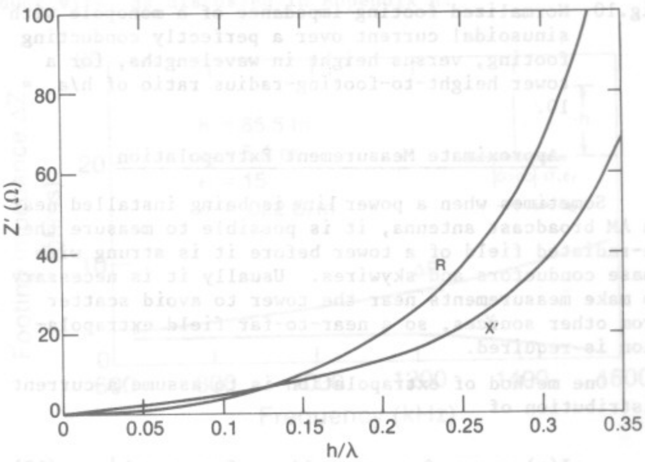

Fig.9 Self resistance and partial self reactance of a monopole with sinusoidal current on a perfectly conducting ground plane as calculated by the induced emf method, versus height in wavelengths.

To obtain the equivalent footing radius, the cage antenna formula (3) should be used. The value for ΔZ can be obtained by reading the normalized footing impedance $\Delta r'$ and $\Delta x'$ from the graph on Figure 10 for a pole height to footing radius ratio of $h/a_o = 10$, and applying the formulas

$$\Delta z = \Delta r' + 60\ln\left(\frac{h/a}{10}\right) + j\Delta x', \quad \text{for } h/a_o \geq 10 \qquad (17)$$

$$\Delta Z = \frac{\eta_g}{\eta_o}\Delta z$$

$$\frac{\eta_g}{\eta_o} = \frac{1}{\sqrt{\varepsilon_r - j\,60\sigma\lambda}} = [\varepsilon_r^2 + (60\sigma\lambda)^2]^{-0.25} \angle 0.5\tan^{-1}\left(\frac{60\sigma\lambda}{\varepsilon_r}\right)$$

where σ is the conductivity in S/m. The curves for an h/a_o of 10 were calculated using Monteath's formula [11] for a monopole with sinusoidal current as a basis for the empirical formula (17) which is accurate to within 3% in Δr and 2Ω in Δx for $h/a_o \geq 10$.

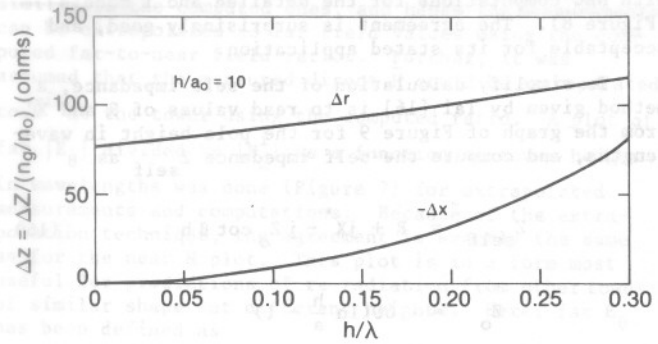

Fig.10 Normalized footing impedance of a monopole with sinusoidal current over a perfectly conducting footing, versus height in wavelengths, for a tower height-to-footing-radius ratio of h/a_o = 10.

Approximate Measurement Extrapolation

Sometimes when a power line is being installed near an AM broadcast antenna, it is possible to measure the re-radiated field of a tower before it is strung with phase conductors and skywires. Usually it is necessary to make measurements near the tower to avoid scatter from other sources, so a near-to-far field extrapolation is required.

One method of extrapolation is to assume a current distribution of

$$I(z) = \cos\beta z - \cos\beta h \quad \text{for } z \le h' \qquad (18)$$
$$= 0 \quad \text{for } z > h'$$

where h is the height of the pole model previously described, and h' is the height of the actual tower. This current distribution is obtained if transmission line theory with distributed sources is applied to the pole model, and the current truncated at the actual tower height. It was hoped that this current would approximate the current on the vertical portion of the T model. The near H_ϕ of this current must be solved numerically due to the term of constant current ($\cos\beta h$), using [17]. This is easily done if the tower is divided into N segments, and the current I(z) and phase factor $e^{-j\beta r}$ are assumed constant over the segment and equal to their values at the segment midpoints z_i, leaving the $1/r^2$ and $1/r^3$ terms of H_ϕ in [17] to be integrated in closed form. The result is

$$H_\phi = \frac{1}{2\pi\rho}\sum_{i=1}^{N} I(z_i)\, e^{-j\beta\sqrt{\rho^2+z^2}}\left[j\beta\rho\tan^{-1}\frac{z}{\rho} + \frac{z}{\sqrt{\rho^2+z^2}}\right]_{z=z_i-\frac{\Delta z}{2}}^{z=z_i+\frac{\Delta z}{2}} \qquad (19)$$

$$\text{far } H_\phi = \frac{j\beta\Delta z}{2\pi\rho}\, e^{-j\beta\rho}\sum_{i=1}^{N} I(z_i)$$

This formula can be used for any current distribution. For the current of (18) with N = 5, h' = 52.4 m and h = 62.2 m, the ratio of $|H_\phi/\text{far } H_\phi|$ was within 1% of the values obtained by NEC for the detailed model over the range of frequencies and distances of the measurements. The same method was tested without truncating the current (h' = 62.2 m) and found to be less accurate.

A simpler method is to use the sinusoidal current over the entire pole length,

$$I(z) = \sin\beta(h-z)$$

For this current, the H_ϕ is well known in closed form [18], and for observations on ground level, it reduces to

$$H_\phi = j\,\frac{e^{-j\beta\rho}}{2\pi\rho}\left(e^{-j\beta\Delta r} - \cos\beta h\right) \qquad (20)$$

where $\Delta r = \sqrt{\rho^2+h^2} - \rho$ (= 0 for far field)

The near-to-far field magnitude ratio is then

$$|H_\phi/\text{far } H_\phi| = \left[\frac{(\cos\beta\Delta r - \cos\beta h)^2 + \sin^2\beta\Delta r}{1-\cos\beta h}\right]^{0.5} \qquad (21)$$

Computations of $H_\phi/\text{far } H_\phi$ have been done using this formula and found to be within 10% of the NEC results for the detailed model. This accuracy should be acceptable.

Conclusions

The NEC moment method combined with the described tower and footing modelling techniques have accurately simulated re-radiation from an unstrung power line tower.

Considerable simplification of the model is possible without seriously affecting its accuracy. Even a simple uniform pole, with its height adjusted to compensate for the top loading effect of the actual cross arms, can produce good results.

Simple methods for predicting re-radiation including footing ground effects, and for extrapolating near-field measurements to far field values, have been presented in forms which can be implemented on a pocket calculator with good accuracy.

Because the local ground conductivity can vary significantly, some uncertainty in what conductivity to use exists. This uncertainty can be reduced by making use of soil and water table data for the specific location of the tower.

Appendix A

Ground Conductivity

Because the apparent conductivity was much higher than the overall average for this area, measured soil data was reviewed. Prior to installing the footings, Ontario Hydro tested the soil to depths of 10 m or more. This was done in January 1980, five months before the field strength measurements were made. At tower 106, the water table was 2.4 m below the ground surface, whereas the four nearest towers had average water table levels of more than 10 m below surface. Assuming that the soil above the water table has a conductivity of 6 mS/m and relative permittivity of 15, the skin depth at tower resonance (1.1 MHz) is 6.5 m, so that tower 106 had substantial ground current below the water-table.

It is known that moisture content in soil greatly affects conductivity. An empirical formula, referred to as Archie's law [19], states that for a given soil which is moist, the conductivity is approximately proportional to the square of the moisture content.

Ontario Hydro measured the soil moisture of samples extracted during the test augering. However, water was allowed to drain out during extration. It is assumed that for soil above the water table, no drainage occurred, so the measured moisture content of 15% corresponds to the actual value underground. For soil below the water table, some drainage would have occurred, so the measured moisture content does not apply to that region. Under the water table, the soil is saturated with water. Hence the moisture content is equal to the porosity. This was estimated at 45% which is typical for silt. According to Archie's law, the conductivity of soil beneath the water table should be nine times greater than that above (54 mS/m).

The surface impedance of a stratified earth can be found using Wait's method [20]. This surface impedance is equal to the intrinsic impedance of an equivalent homogeneous earth. For a two-layered earth, the equivalent intrinsic impedance η_{eq} is given by

$$\eta_{eq} = \eta_1 \frac{\eta_2 + \eta_1 \tanh \gamma_1 h_1}{\eta_1 + \eta_2 \tanh \gamma_1 h_1} \qquad \text{(A-1)}$$

where η is the intrinsic impedance, γ is the propagation constant, h is the thickness of the earth layer, and the subscripts 1 and 2 refer to soil above and below the water table.

A sample calculation was done at 1.1 MHz. The resulting η_{eq} gave a footing impedance of 7.3 + j 5.7 Ohms which compares well with the 6.2 + j 2.5 Ohms used in the re-radiation computations. The η_{eq} corresponds to an equivalent conductivity of 10 mS/s and relative permittivity of -92. It is seen that the equivalent relative permittivity of a stratified earth does not correspond to an actual soil type. The DOC conductivity map represents inhomogeneous soil as an equivalent homogeneous soil with a relative permittivity of 15 and an effective conductivity chosen to match the measured ground wave attenuation of the inhomogeneous soil. This procedure was followed here, but the effective conductivity was chosen to match the footing resistance at tower resonance obtained for the stratified earth. The resulting effective conductivity was 16 mS/m, which gives a footing impedance of 7.3 + j 2.9 Ohms. The conductivity of 16 mS/m is in reasonable agreement with the value of 22 mS/m used in the re-radiation computations. Obviously, the effective conductivity is very sensitive to water table level. For example, at tower 106, two months after the field strength measurements, the ground was continuously under water, so the conductivity must have been approximately 54 mS/m.

Appendix B

Transmission Line Theory Applied to Tower Crossarms

A simple transmission line representation for the detailed tower model does not include capacitive coupling between the crossarms. This can be compensated for approximately by changing the characteristic impedance, Z_o, used for the individual crossarms in the simple transmission line representation.

Because the characteristic impedance of a uniform transmission line of bare, perfectly conducting wires in free space is related to the distributed capacitance per unit length, C, by the expression

$$Z_o = \frac{\sqrt{\mu_o \varepsilon_o}}{C} \qquad \text{(B-1)}$$

we may consider C in place of Z_o whenever it is convenient to do so. The N crossarms (N = 4 in this case) may be thought of as parts of N long parallel wires over ground, carrying waves which are in phase because the wires are joined at the tower and thus forced to be equipotential. There exists a single wire over ground which is equivalent to the assemblage of N wires. The relevant properties of this single wire are that its $Z_{o_{eq}}$ (or C_{eq}) and its height equal the Z_o (or C) and average height of the N-wire assemblage. The C_{eq} is directly related to the single-wire radius, a_{eq}. If the N wires all have equal charge, and if the interwire spacing is much less than the heights, we can equate the capacitance of the single wire to the N wire assemblage to obtain a_{eq} in terms of the interwire spacing, a_{ij}.

$$a_{eq} = \left[\prod_{i=1}^{N} \prod_{j=1}^{N} a_{ij} \right]^{1/N^2} \qquad \text{(B-2)}$$

The self spacing a_{ii} is the radius of wire i. The radius a_{eq} is the one that would be used at this stage if a single wire equivalent were desired. Rather, an N-uncoupled-wire equivalent is desired, that is, one which would give the same input impedance if the N uncoupled wires were fed in parallel. Each of the N wires would need to have a characteristic impedance of

$$Z_o = N Z_{o_{eq}} \qquad \text{(B-3)}$$

Thus, using the Schelkunoff formula for the characteristic impedance of a thin wire forming part of an antenna, we get

$$Z_o = N60 \left[\ln \frac{2}{\beta a_{eq}} - \gamma\right] \qquad \text{(B-3)}$$

For four coplanar crossarms with spacing d, and radius a, equation (B-2) becomes

$$a_{eq} = d \left(\frac{3.5a}{d}\right)^{1/4} \qquad \text{(B-4)}$$

In the detailed tower model, a is 0.6 m and the average d is 7.7 m, which yields an a_{eq} of 5.6 m for the crossarm assemblage. With N = 4, equation (B-4) yields a crossarm characteristic impedance of Z_o = 520 Ohms at 1.1 MHz, which is about twice that obtained for an isolated crossarm.

In applying transmission line theory, the lengths of the crossarms should be specified as being measured from the tower surface rather than from the tower axis, so that the minimum crossarm length can be zero rather than the tower radius.

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