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A MICROCOMPUTER PROGRAM FOR PREDICTING AM BROADCAST RE-RADIATION FROM STEEL TOWER POWER LINES

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Abstract

Computations of the re-radiation of AM broadcast signals by steel tower power lines are carried out using a simplified method, and compared with computations by moment method. The simplified method is based on transmission line theory and was implemented on a microcomputer in BASIC using 32K of RAM. The computer program (AMPL) includes the effect of the lossy ground. Results show that this simplified method is useful for making an approximate but rapid and inexpensive assessment of the impact of a power line on an AM broadcast antenna's performance.

Introduction

The pattern and location of an AM broadcast antenna are usually optimized in order to maximize the broadcast service while not excessively interfering with other broadcast signals. When a steel tower power line is illuminated by a broadcast antenna, currents are induced in it. These currents will radiate along with the broadcast antenna, sometimes causing the total radiation pattern to be unacceptable. Broadcast engineers and consultants are called upon to evaluate the extent of this problem: the first stage of this evaluation is usually to categorize approximately the severity of the problem, with later stages involving more precise calculations.

It has been shown that moment method analysis of the problem can agree well with physical scale model measurements [1,2], and with full scale measurements [3,4,5]. In the simulation of full scale tests, the effect of the lossy ground had to be included for best results. A drawback of the moment method is that its complexity, computer memory and time requirements preclude its use by most broadcast engineers. This paper describes a simplified method which broadcast engineers can use to determine whether or not a power line poses a threat to the performance of a broadcast antenna.

Analysis

Each tower and each skywire span is represented as a straight uniform wire. On a wire, uniform transmission line theory is applied to obtain the voltage and current distribution. To apply this theory, the characteristic impedance and propagation constant must be found. At wire junctions, Kirchoff's current law and the continuity of voltage are enforced. The phase conductors are neglected because they have been shown to have only a minor effect on the re-radiation [2,6]. Ground effects between towers are represented as a modification of the propagation constant on the skywires, and tower footings are represented as impedances on the tower wires. The AM broadcast antenna is treated as a source, that is, its current distributions are specified as desired, and they are unaffected by the presence of the power line. The electric field strength incident on a wire is approximated by the source field alone: thus the radiation from one wire to another is neglected.

The technique of applying transmission line theory to scattering problems was used extensively in early antenna design, and is covered in [7] and [8]. One simple result is the approximation that a transmitting dipole has sinusoidal current. In the present problem, the accuracy of the method will be determined by comparing the results with the more accurate moment method. Following is a more detailed description of the transmission line method. The units are SI, the coordinate systems have the z axes vertical, and the time dependence is $\exp(j\omega t)$.

A tower is modelled as a uniform cylinder (wire). For a tower wire, its radius a_t is computed as the average equivalent radius of the tower if it had solid faces and no crossarms. The equivalent radius for any tower cross-section is found by equating the capacitances per unit length of two infinite uniform cylinders, one with a circular cross-section and the other with the tower cross-section. For towers of rectangular cross-section, the equivalent radius is given in [9]. For example, if the cross-section is square, the equivalent radius is 0.59 times the width.

The height of the tower wire h_t is taken as the height of the skywire connection point above ground.

The characteristic impedance of the tower wire Z_{ct} is computed as

$$Z_{ct} = 60[\ln(2h_t/a_t)-1] \tag{1}$$

This can be derived by computing the characteristic impedance of a horizontal wire of arbitrary height over a lossless ground plane and averaging this over a height ranging from 0 to h_t.

The propagation constant of a tower wire consists of one term corresponding to a lossless transmission line in free space, and another term to represent power loss due to radiation as shown in (2) below. The propagation constant γ_t for a tower wire in terms of the angular frequency ω and the free-space velocity of light c is

$$\gamma_t = \frac{40}{Z_{ct}\, h_t} \sin^2 \beta_o h_t + j\beta_o \tag{2}$$

where the free-space plane-wave propagation constant β_o is

$$\beta_o = \frac{\omega}{c} \tag{3}$$

The derivation of equation (2) is given in Appendix A where γ and Z_o are used in place of γ_t and Z_{ct}.

The effect of the ground on the tower is based on the application of the compensation theorem to a transmitting monopole in two cases, the first case with the monopole on a perfectly conducting ground plane, and the second case with a perfectly conducting ground post (footing) surrounded by earth. The difference between these two impedances is called the footing impedance Z_f. This is used in the present method to represent ground effects on a power line tower by inserting Z_f as a lumped load between the base of the tower wire and a

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perfect ground. The details are given in [3]. The results have been approximated for the present method by

$$Z_f = \eta\left\{20\beta_o h_t + 60[\ln(h_t/a_f)-1]-j20(\beta_o h_t)^2\right\} \quad (4)$$

$$\eta = 120\pi[(60\sigma\lambda_o)^2+\varepsilon_r^2]^{-\frac{1}{4}}\, e^{j\frac{1}{2}\tan^{-1}(60\sigma\lambda_o/\varepsilon_r)} \quad (5)$$

where a_f is the tower footing radius, σ is the earth conductivity, ε_r is the earth relative permittivity, η is the earth intrinsic impedance and λ_o is determined from c and the frequency f as

$$\lambda_o = c/f \quad (6)$$

The footing radius a_f should be chosen to be equivalent to the four footings (usually) found on the actual tower. The actual footings support each tower leg and are often of a circular cross-section of radius a_f' consisting of concrete with vertical reinforcing rods around the perimeter. It was shown in [3] that for four actual footings arranged in a square of width w, a reasonable choice of a_f is

$$a_f = w(\sqrt{2}\, a_f'/w)^{\frac{1}{4}} \quad (7)$$

In a span, the two tower tops are usually joined by two skywires. In the present method, these are represented by a single straight equivalent skywire whose radius a_s is chosen to produce the same characteristic impedance as the two skywires. If the actual skywires have a radius a_s' and a spacing d which is much less than twice their height above ground (i.e. their image distance), then

$$a_s \doteq (a_s' d)^{\frac{1}{2}} \quad (8)$$

The transmission line characteristic impedance Z_{os} and propagation constant γ_{os} for a horizontal wire of height h_s and radius a_s over a perfect ground plane are

$$Z_{os} = 60\ \ln(2h_s/a_s) \quad (9)$$

$$\gamma_{os} = j\beta_o \quad (10)$$

The skywire propagation constant γ_s and characteristic impedance Z_{cs} in the presence of a lossy ground plane are taken from Knight [10] who used the compensation theorem for the problem of a horizontal wire transmission line over a lossy ground plane. His result is used here directly as follows

$$\gamma_s = \gamma_{os}\left(1+\frac{\eta}{\gamma_{os}2\pi h_s Z_{os}}\right)^{\frac{1}{2}} \quad (11)$$

$$Z_{cs} = Z_{os}\left(1+\frac{\eta}{\gamma_{os}2\pi h_s Z_{os}}\right)^{\frac{1}{2}} \quad (12)$$

Radiation from the skywires is neglected in the computation of γ_s.

The axial component of the incident electric field $\bar{E}_{in}$ on a wire acts as a distributed voltage generator. In the present method, the $\bar{E}_{in}$ is approximated by the antenna field $\bar{E}_a$. Furthermore, the $\bar{E}_a$ on the skywires is neglected, and the $\bar{E}_a$ on a tower is taken to be constant given by

$$\bar{E}_a = -\hat{z}\sum_{i=1}^{N_a} F_{io}\,\frac{e^{-j\beta_o R_i}}{R_i} \quad (13)$$

where ^ denotes a unit vector, R_i is the distance from the i^{th} antenna element to a power line tower, N_a is the number of antenna elements, and F_{io} is the θ component of the far E from the i^{th} tower on the surface of a lossless ground plane, given as follows in terms of both its z directed complex current distribution $I_i(z)$ and its height h_{ai}

$$F_{io} = j\beta_o\, 60\int_0^{h_{ai}} I_i(z)\, dz \quad (14)$$

The magnitude of this F_{io} is commonly referred to as the unattenuated field at one kilometre in mV/m (from the i^{th} antenna element). The official antenna description sheet for a broadcast station specifies the magnitude and phase of F_{io} for each tower rather than the terminal current.

Now that the transmission line parameters and distributed voltage generators have been specified, the power line currents can be computed. One method of doing so is to replace the distribution of generators on a tower wire by a finite number of generators having the same total voltage over the tower length. Then, each generator can be turned on one at a time, and its resultant current computed over the whole power line at a finite number of points. The final power line current is a superposition of all the current distributions. This method requires the current to be computed at many points. However if the skywire is neglected in the final radiation patterns, current points only need to be specified on the towers, with a great reduction in memory requirements. In the present method, an alternative approach was used. In Figure 1(a) a power line model is shown with the source current turned on, inducing currents in the power line including I_1 to I_N on the skywire at the tower junctions (where terminals are added). Current generators are inserted at locations 1 to N with strengths equal to I_1 to I_N. This does not change the power line current distribution (this is related to the circuit theory concept that any impedance in a circuit with current I flowing through it and potential V across it can be replaced by either a voltage source of strength V or a current source of strength I without affecting the circuit). Now the problem can be analysed by superposition of sources. Figures 1(b) and 1(c) show two cases to be superimposed. It is convenient to refer to the case with the antenna current source turned on and the power line current sources turned off (open circuited) as the open circuit (OC) case, while the case with the antenna source turned off and the power line sources turned on is called the generating (G) case. The details of applying transmission line theory to both these cases follow.

In the OC mode, the current on a tower is not affected by the other towers or skywires according to transmission line theory. The tower current which satisfies the basic transmission line equations for a uniform illumination (the third equation of (17), and (18)) has the form

$$I(z) = A + B\sinh\gamma_t z + C\cosh\gamma_t z \quad (15)$$

$$\left.\begin{aligned} \text{where} \quad & A = E_a/(\gamma_t\, Z_{ct}) \\ & C = -A\,\frac{1 + (Z_f/Z_{ct})\,\sinh \gamma_t\, h_t}{\cosh \gamma_t\, h_t + (Z_f/Z_{ct})\,\sinh \gamma_t\, h_t} \\ & B = (A + C)\, Z_f/Z_{ct} \end{aligned}\right\} \qquad (16)$$

The coefficients A, B and C were derived as follows.

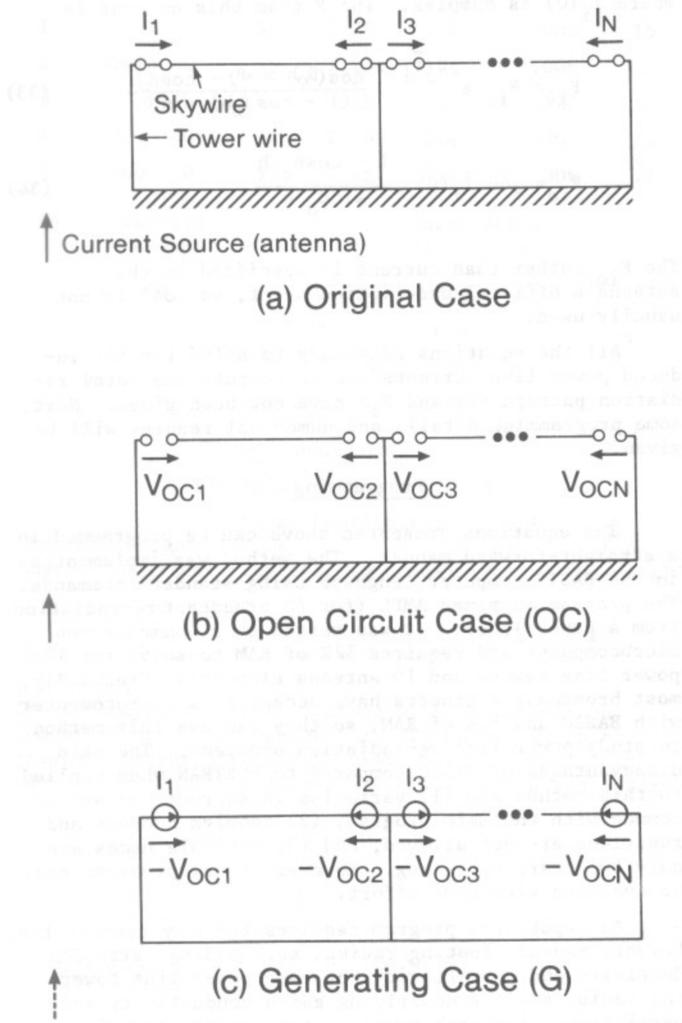

Fig.1 Decomposition of original problem with terminals added (a) into two cases, one with antenna present and terminals open-circuted (b), and the other without antenna but with terminal current sources (c).

Equation (15) is valid for a section of transmission line with both a uniform applied electric field and end loads or sources. For a tower wire in the OC case, the following conditions apply.

$$\left.\begin{aligned} & I(h_t) = 0 \\ & V(0)/I(0) = -Z_f \\ & dV/dz + \gamma_t\, Z_{ct}\, I = E_a \end{aligned}\right\} \qquad (17)$$

In computing the voltage from the current, a useful basic transmission formula is

$$V(z) = -\frac{Z_c}{\gamma}\,\frac{dI(z)}{dz} \qquad (18)$$

When (17) is applied to (15), the results in (16) are obtained. The open circuit voltage V_{oc} is the voltage on the skywire (zero) minus the voltage on the tower top V(h). Thus V_{oc} is

$$V_{oc} = -V(h) = BZ_{ct}\cosh \gamma_t h + CZ_{ct}\sinh \gamma_t h \qquad (19)$$

In the G case, the following conditions apply.

$$\left.\begin{aligned} & I(z = h) = I(h) \\ & V(0)/I(0) = -Z_f \\ & dV/dz + \gamma_t\, Z_{ct}\, I = 0 \end{aligned}\right\} \qquad (20)$$

These result in the following coefficients for the current distribution in (15).

$$\left.\begin{aligned} & A = 0 \\ & C = I(h)/[(Z_f/Z_{ct})\sinh \gamma_t h + \cosh \gamma_t h] \\ & B = CZ_f/Z_{ct} \end{aligned}\right\} \qquad (21)$$

A terminal current in the G case must produce a voltage equal and opposite to the V_{oc} so that when the two cases are summed or superimposed, no voltage exists (the terminals are shorted). The voltage produced across each pair of terminals can be found by computing the mutual impedance matrix in the G case and solving the following matrix equation.

$$\begin{bmatrix} Z_{11} & Z_{12} & 0 & 0 & \cdots \\ Z_{21} & Z_{22} & Z_{23} & 0 & \\ 0 & Z_{32} & Z_{33} & Z_{34} & \\ \vdots & & & & \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \end{bmatrix} = - \begin{bmatrix} V_{oc1} \\ V_{oc2} \\ V_{oc3} \\ \vdots \end{bmatrix} \qquad (22)$$

Only the main diagonal and first adjacent diagonal elements need to be stored because the others are zero.

The self impedance of a pair of terminals involves one skywire span and one tower (other terminals are open circuited). Its formula is

$$\text{self } Z = Z_{cs}\coth \gamma_s \ell_s + Z_{ct}\,\frac{(Z_f/Z_{ct})\cosh\gamma_t h + \sinh\gamma_t h}{(Z_f/Z_{ct})\sinh\gamma_t h + \cosh\gamma_t h} \qquad (23)$$

where ℓ_s is the length of the skywire span.

The mutual impedance between pairs of terminals attached to the same tower is the second term of equation (23).

The mutual impedance between pairs of terminals attached to the same skywire span is

$$\text{mutual } Z = Z_{cs}/\sinh \gamma_s\, \ell_s \qquad (24)$$

When the terminal currents are solved for, numerical values for the A, B and C coefficients can be found for the towers (using (21)) and the skywires. The values for the towers in both the G and OC cases are then added. As a result, the final current distribution on each tower wire and skywire is in the form of (15). Now the radiation pattern can be computed.

The far field will be given for the i^{th} wire in free space. The wire geometry is specified by the location of its reference end $\bar{r}_i$, length ℓ_i, and direction vector $\hat{s}_i$. Its current is given by

$$I(s_i) = A_i + B_i \sinh \gamma_i s_i + C_i \cosh \gamma_i s_i \qquad (25)$$

where $0 \le s_i \le \ell_i$

Let us define a vector $\bar{F}(\hat{r})$ due to currents at position $\bar{r}'(s)$ as

$$\bar{F}(\hat{r}) = -j\beta_o 30 \int_{\substack{\text{all}\\ \text{currents}}} I(s)\hat{s}\, e^{j\beta_o \bar{r}'.\hat{r}}\, ds \qquad (26)$$

The far $\bar{E}$ is related to $\bar{F}$ by

$$\left.\begin{aligned} E_\theta(\bar{r}) &= F_\theta(\hat{r})\, e^{-j\beta_o r}/r \\ E_\phi(\bar{r}) &= F_\phi(\hat{r})\, e^{-j\beta_o r}/r \\ E_r &= 0 \end{aligned}\right\} \qquad (27)$$

Noting that for the i^{th} wire,

$$\bar{r}' = \bar{r}_i + \hat{s}_i s \qquad (28)$$

equation (26) for the i^{th} wire is

$$\bar{F}_i(\hat{r}) = -j\beta_o 30\, e^{j\beta_o \bar{r}_i.\hat{r}}\, \hat{s}_i \int_0^{\ell_i} I(s)\, e^{j\beta_o \hat{s}_i.\hat{r}s}\, ds \qquad (29)$$

After performing the integration,

$$\left.\begin{aligned} \bar{F}_i(\hat{r}) &= -j\beta_o 30\, e^{j\beta_o \bar{r}_i.\hat{r}}\, \hat{s}_i \left[A_i \frac{e^{b\ell_i} - 1}{b} + \frac{C+B}{2} \frac{e^{(b+\gamma_i)\ell_i} - 1}{(b+\gamma_i)} \right. \\ &\quad \left. + \frac{C-B}{2} \frac{e^{(b-\gamma_i)\ell_i} - 1}{(b-\gamma_i)} \right] \\ b &= j\beta_o\, \hat{s}_i.\hat{r} \\ F_{i\theta} &= \hat{\theta}.\hat{s}_i\, F_i \qquad F_{i\phi} = \hat{\phi}.\hat{s}_i\, F_i \end{aligned}\right\} \qquad (30)$$

The $\bar{F}$ from the image of the i^{th} wire over a perfectly conducting ground plane is found using (30) but with the following changes:

$$A_i \to -A_i \qquad B_i \to -B_i \qquad C_i \to -C_i$$
$$\bar{r}_i \to \bar{r}_i - 2\bar{r}_i.\hat{z}\hat{z} \qquad \hat{s}_i \to \hat{s}_i - 2\hat{s}_i.\hat{z}\hat{z} \qquad (31)$$

For the i^{th} antenna element, the current is taken as

$$I_i(z) = I_i(0) \sin\beta_o(h - z)/\sin\beta_o h \qquad (32)$$

where $I_i(0)$ is complex. The $\bar{F}$ from this current is

$$F_{i\theta} = F_{io}\, e^{j\beta_o \bar{r}_i.\hat{r}}\, \frac{\cos(\beta_o h \cos\theta) - \cos\beta_o h}{(1 - \cos\beta_o h)\sin\theta} \qquad (33)$$

$$F_{io} = j60\, I_i(0)\, \frac{1 - \cos\beta_o h}{\sin\beta_o h} \qquad (34)$$

The F_{io} rather than current is specified on the antenna's official description sheet, so (34) is not usually used.

All the equations necessary to solve for the induced power line currents and to compute the total radiation pattern (F_θ and F_ϕ) have now been given. Next, some programming details and numerical results will be given.

Programming

The equations presented above can be programmed in a straight-forward manner. The method was implemented in the BASIC computer language using standard commands. The program is named AMPL (for AM broadcast re-radiation from a power line). It was tested on a Commodore-64 microcomputer and requires 32K of RAM to solve for 50 power line towers and 10 antenna elements. Presumably, most broadcast engineers have access to a microcomputer with BASIC and 32K of RAM, so they can use this method to study power line re-radiation problems. The main disadvantages of BASIC compared to FORTRAN when applied to this method are (1) variables in subroutines are in common with the main program, (2) complex numbers and functions are not allowed, and (3) variable names are only two characters long. However, these problems can be overcome with some effort.

As input, the program requires the x, y coordinates, height, radius, footing radius, surrounding earth conductivity and permittivity for each power line tower, the radius and the underlying earth conductivity and permittivity for each skywire, the coordinates of the antenna reference point (local origin), the height, x and y coordinates, and radiation F_o for each antenna element, and the frequency.

The program will output the electric field incident on each power line tower, the radiation F_o of each power line tower, the current distribution on any tower or skywire, the radiation patterns of the antenna F_θ, and the total F_θ and F_ϕ over the hemisphere.

Results

Moment method simulation of a five tower power line and a monopole antenna near Thornhill, Ontario was compared to full scale measurements by Tilston and Balmain [4]. The measurements included near magnetic field magnitudes. These were generally well predicted by the moment method after careful modelling of both the geometry and lossy ground effects. The far field predictions from this model will be used here to compare with the present transmission line method. The specifications for the transmission line model are shown in Table 1. The far field pattern was computed using AMPL.

Table 1 Input data used in the transmission line program for the Thornhill site.

Power Line Data

Tower Number	Coordinates X(m)	Y(m)	Height (m)	Radius (m)	Footing Radius (m)	Adjacent Earth σ(S/m)	ε_r
1	-513	- 70	55	2.25	5.4	.006	15
2	-247	0	55	2.25	5.4	.006	15
3	0	0	52	2.25	5.4	.022	15
4	250	0	51	2.25	5.4	.006	15
5	500	0	52	2.25	5.4	.006	15

Skywire Number	Radius(m)	Earth Below σ(S/m)	ε_r
1	.37	.006	15
2	.37	.006	15
3	.37	.006	15
4	.37	.006	15

Antenna Data

Local origin: x,y(m): -61,256

Tower Number	Height (Degrees)	Local Coordinates X (Degrees)	Y (Degrees)	$\lvert F_o \rvert^{(1)}$ (Volts)	$\angle F_o$ (Degrees)
1	90	0	0	1000	0

Note 1: This is also in units of mV/m at 1 km.

It is plotted in Figure 2 along with moment method computations. The frequency is 825 kHz at which the greatest perturbations were previously observed. It can be seen that the shapes are similar with a standard deviation of 10% and maximum deviation of 25%. The max-to-min ratios agree to within ½ dB or 6%. The accuracy of AMPL is probably good enough for a broadcast engineer to determine if the power line poses a threat to his broadcast operation. When the program was run again neglecting ground effects ($\sigma = 10^{10}$ S/m) the radiation from the power line towers increased by 20% typically.

Another example problem was analysed involving a 500 kV power line and a single-tower AM broadcast antenna near Hornby, Ontario. This antenna transmits CBL on 740 kHz and CJBC on 860 kHz. The power line was modelled with five identical towers 50.9 m high and 2.86 in radius. The towers were on a line (y axis) with equal spacing of 274.3 m. The skywire was 0.385 m in radius (this represents two thinner parallel skywires). The earth was specified as highly conducting ($\sigma = 10^{10}$ S/m). The antenna was located 448.1 m away from the centre tower in a direction perpendicular to the power line (-x axis). This model is similar to one used by Trueman and Kubina [1]. The max-to-min ratio of the radiation pattern on the horizontal plane was computed at frequencies from 200 kHz to 1600 kHz. The results are plotted in Figure 3. Because of the uniformity of the power line model, the resonances (peaks) and antiresonances (valleys) are well defined and regular. Up to 1050 kHz, the agreement between the moment method and AMPL computations is excellent. Above that frequency, AMPL seems to deteriorate and sometimes predicts excessive pattern distortion. Corresponding horizontal radiation patterns are shown in Figure 4. At 500 kHz and 860 kHz the agreement between AMPL and the moment method is usually within 10% and the shape is well predicted. At 1200 kHz and 1600 kHz, the agreement is again usually within 10%, but the pattern ripple is not much more than 10%. The pattern shape is well predicted over 60% of the arc but is exaggerated over 30% and does not show the correct trend over 10%. This degree of accuracy is probably sufficient for a broadcast engineer to assess approximately the impact of a power line on his broadcast operation.

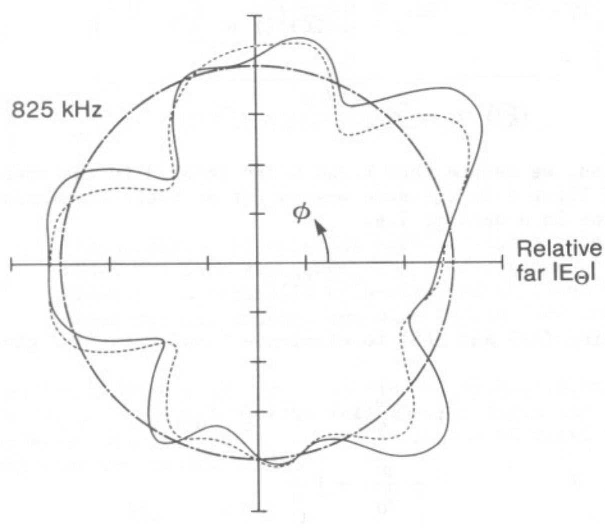

Fig.2 Polar pattern of relative far $|E_\theta|$ at $\theta = 90°$ on a linear scale for antenna alone (-.-) and with power line by moment method (---) and AMPL (——) for Thornhill site.

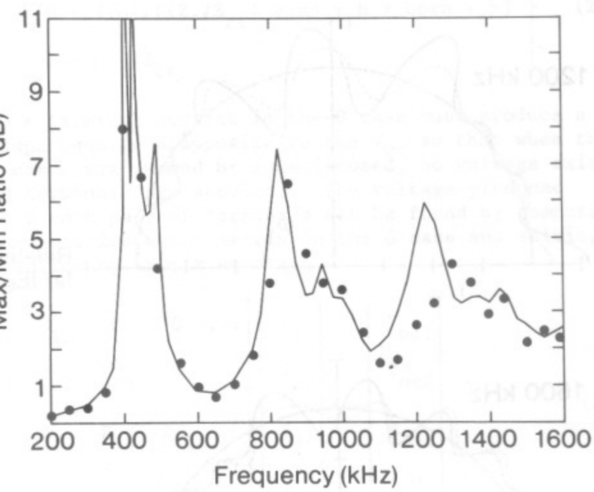

Fig.3 Ratio of maximum to minimum far $|E_\theta|$ at $\theta = 90°$ by moment method (●●●) and AMPL (——) for Hornby site.

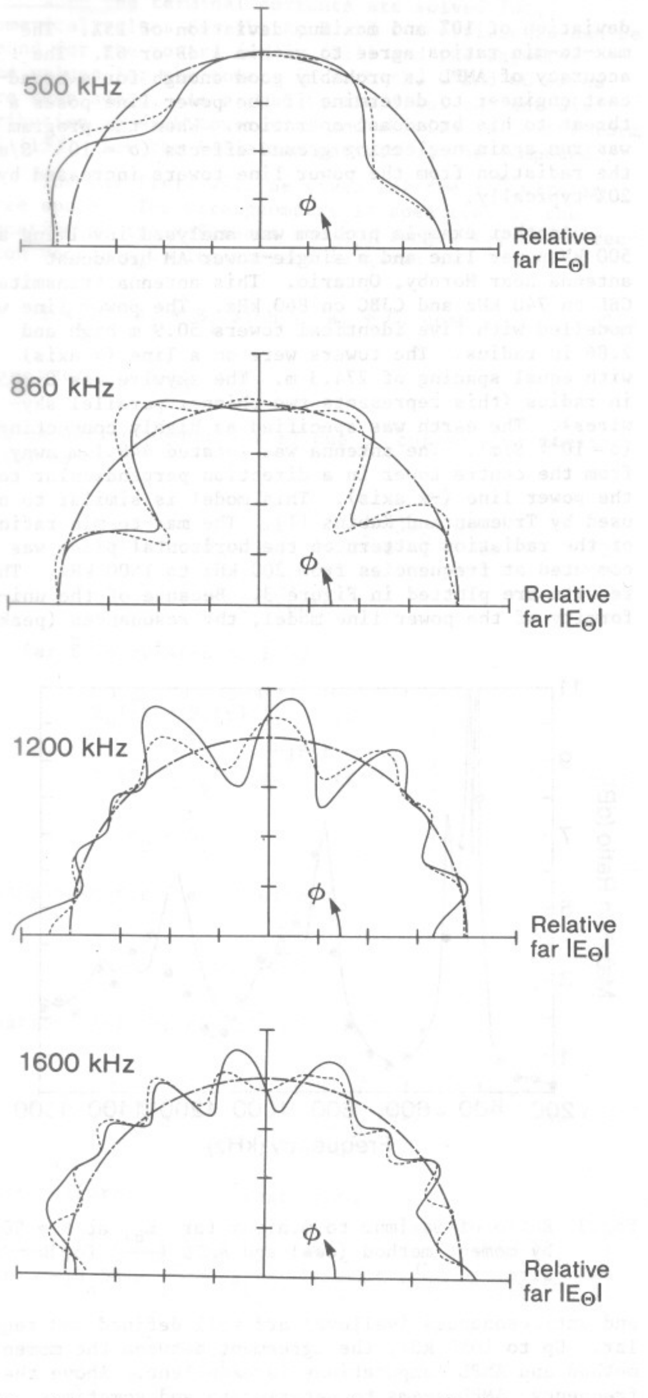

Fig.4 Polar pattern of relative far $|E_\theta|$ at $\theta = 90°$ on a linear scale for antenna alone (-·-) and with power line by moment method (---) and AMPL (——) for Hornby site. Calculations are at the four frequencies indicated.

Conclusions

The transmission line method has been used to compute AM broadcast re-radiation from power lines. It agrees with moment method computations in terms of both the degree of pattern perturbation and the main features of the pattern. However on some small arcs, the two methods do not agree. The transmission line method has been successfully implemented on a microcomputer in BASIC using 32K of random access memory. The computer program (named AMPL) should be of use to broadcast engineers for approximate assessment of the impact of a power line on their broadcast antenna performance. However, the transmission line method is not a replacement for the moment method. The moment method should be used in cases where the most accurate assessment possible is needed, or when it is necessary to study devices to detune the power line.

APPENDIX A

Transmission Line Propagation Constant for Towers

We wish to represent the power radiated by a tower as loss in the transmission line model. To do this, a current distribution must first be specified. Let the rms current I be

$$I(z) = \cos\beta_o z \qquad \text{(A1)}$$

This current is used only to find the transmission line parameters. It is an approximation to the final current obtained by applying transmission line theory to a power line problem.

The power radiated W_r by the current in (A1) was computed and found to be approximately

$$W_r \doteq 40 \sin^2(\beta_o h_t) \qquad \text{(A2)}$$

A transmission line has distributed series resistance R, series inductance L, shunt conductance G, and shunt capacitance C. The propagation constant γ and characteristic impedance Z_o are

$$\gamma = j\omega(LC)^{\frac{1}{2}}([1 + \frac{R}{j\omega L}][1 + \frac{G}{j\omega C}])^{\frac{1}{2}} \qquad \text{(A3)}$$

$$Z_o = (L/C)^{\frac{1}{2}}([1 + \frac{R}{j\omega L}]/[1 + \frac{G}{j\omega C}])^{\frac{1}{2}} \qquad \text{(A4)}$$

We choose to have a distortionless line, so

$$\frac{R}{L} = \frac{G}{C} \qquad \text{(A5)}$$

In this case,

$$\gamma = j\omega(LC)^{\frac{1}{2}}(1 + \frac{R}{j\omega L}) \qquad \text{(A6)}$$

$$Z_o = (L/C)^{\frac{1}{2}} \qquad \text{(A7)}$$

Also, we assume that L and C are related to the speed of light c in the same way as for an ideal transmission line in a vacuum, i.e.

$$c = (LC)^{-\frac{1}{2}} \qquad \text{(A8)}$$

Using (A7) and (A8) to eliminate L and C in (A6) gives

$$\gamma = j\frac{\omega}{c}(1 + R/[j\frac{\omega}{c} Z_o])$$

$$= \frac{R}{Z_o} + j\frac{\omega}{c}$$

$$= \alpha + j\beta_o \qquad \text{(A9)}$$

$$\text{where } \alpha = \frac{R}{Z_o} \text{ and } \beta_o = \frac{\omega}{c}$$

Also, for any transmission line, the voltage V is related to the current by

$$V = -\frac{1}{G + j\omega C}\frac{dI}{dz}$$

$$= -\frac{Z_o}{\gamma}\frac{dI}{dz} \qquad (A10)$$

The power lost W_ℓ in the transmission line for the current in (A1) is

$$W_\ell = \int_0^{h_t} (|I|^2 R + |V|^2 G) dz$$

$$= R\int_0^{h_t} (\cos^2\beta_o z + \frac{\beta_o^2}{|\gamma|^2}\sin^2\beta_o z) dz \qquad (A11)$$

We now assume that

$$\alpha^2 << \beta_o^2 \qquad (A12)$$

So $|\gamma^2| \doteq \beta_o^2$

Equation (A11) can now be integrated to give

$$W_\ell = Rh_t$$

$$= \alpha Z_o h_t \qquad (A13)$$

Equating W_ℓ and W_r gives

$$\alpha = \frac{W_r}{Z_o h_t} \qquad (A14)$$

Combining equations (A2), (A9) and (A14) yields

$$\gamma \doteq \frac{40}{Z_o h_t}\sin^2\beta_o h_t + j\beta_o \qquad (A15)$$

The above approach is similar to a transmission line analysis in Jordan and Balmain [8] for a monopole antenna.

Acknowledgments

The motivation for the present method of analysis came from the work of others who showed that the transmission line method could be successfully applied to power line re-radiation problems. In particular, Ontario Hydro [11] have been using their own transmission-line method for many years, and have obtained encouraging agreement with moment method computations. M.M. Silva (personal communication) at the University of Toronto recently refined the transmission line method to more accurately represent radiation, and to compute the tower current distribution rather than to assume it. With these refinements, she obtained excellent agreement with moment method computations at the lower end of the AM broadcast band. Her work provided some useful background for the present method which has further refinements to improve the high frequency performance, to represent ground effects, and to ease microcomputer implementation. This research program was supported by the Natural Sciences and Engineering Research Council of Canada, under Grants G-0362 and A-4140.

Availability of the Program

A listing of the program AMPL is available from M.A. Tilston with a small charge for handling.

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