ECE424F MICROWAVE CIRCUITS

Homework #1

1. A plane-wave at 1GHz is propagating through a non-magnetic dielectric with a relative permittivity $\varepsilon_r = 4.0$. The loss tangent of the material is $\tan\delta = 0.001$. Calculate the power loss in dB when the wave travels a distance of 1.5m. Repeat for $\tan\delta = 0.01$ and $\tan\delta = 0.1$.

2. A 100m long copper wire has a radius of a=1mm. Calculate the corresponding D.C. resistance. Compute the skin depth at 3GHz and the associated A.C. resistance. How do the two resistance values compare to each other ?

3. A transmission line is filled with a non-magnetic dielectric of $\varepsilon_r = 2.5$. The line has a capacitance per unit length of $C = 200\text{pF}/\text{m}$ and a resistance per unit length of $R = 2\ \Omega/\text{m}$. Calculate the corresponding phase velocity, characteristic impedance and attenuation constant α (assume G=0).

4. Problem 2.3 in textbook.

Hint: Assume $\overline{E} = \hat{y}E_0 e^{-\gamma z}$

Note:

A very nice CAD tool for analyzing microwave circuits is Puff. Puff was designed at CalTech and runs on PC's under MS-DOS. You can really have a lot of fun with this software while still learning! Therefore, it is highly recommended that you purchase it. The price is about 15 USD and it can be ordered through the Internet at http://www.systems.caltech.edu/EE/Faculty/rutledge/Pufform.html. Have fun!!

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Solutions to HWK # 1

Problem ①:

① The fields vary as $e^{-\gamma z}$ and therefore the power, $P \propto e^{-2\alpha z}$.

$$\gamma \cong j\omega\sqrt{\mu_0 \varepsilon'}\left(1-\frac{1}{2}j\tan\delta\right) = \alpha + j\beta. \text{ Hence}$$

$$\alpha = \frac{1}{2}\omega\sqrt{\mu\varepsilon'}\tan\delta = \frac{\pi}{\lambda}\tan\delta. \ \mathrm{NP/m}$$

The free-space wavelength at 1GHz is $\lambda_0 = \frac{c}{f} = \frac{3\times10^8}{1\times10^9} = 0.3\,m$

Hence in the dielectric $\lambda = \lambda_0/\sqrt{\varepsilon_r} = \frac{0.3}{\sqrt{4}} = 0.15\,m$.

$\tan\delta = 0.001$:

$$\alpha = \frac{\pi}{0.15}\, 0.001 \simeq 2.1\times10^{-2}\ \mathrm{NP/m}$$

But $1\,NP = 10\log_{10} e^2 \cong 8.7\,dB$. Hence the attenuation in dB for a $l = 1.5m$ traveled distance is:

$$-(8.7)(\alpha)(l) = -0.27\,dB$$

$\tan\delta = 0.01$

$$\alpha \cong 2.1\times10^{-1}\ \mathrm{NP/m}$$

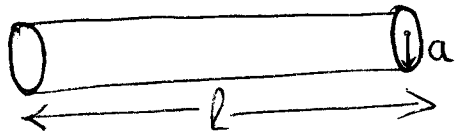
attenuation $= -(8.7)(\alpha)(l) = (8.7)(2.1\times10^{-1})(1.5) = -2.7\,dB$

$\tan\delta = 0.1$

$$\alpha = 2.1$$

$\Rightarrow$ attenuation $= -(8.7)(2.1)(1.5) = -27\,dB$! (All the power is basically lost)

Problem ②:

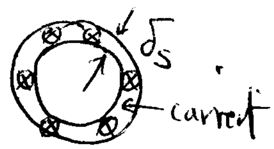


$\ell = 100\,m$ $\quad$ $\sigma = 5.813 \times 10^7$ for copper.
$a = 1\,mm$
$f = 3\,GHz$

$$R_{dc} = \frac{\ell}{\sigma(\pi a^2)} = \frac{100}{(5.813 \times 10^7)(\pi)(0.001)^2} \cong 0.55\,\Omega$$

Skin-depth $\quad \delta_s = \sqrt{\frac{2}{\omega \mu_0 \sigma}} = \sqrt{\frac{2}{(2\pi \times 3 \times 10^9)(4\pi \times 10^{-7})(5.813 \times 10^7)}} = 1.205\,\mu m$

The current essentially is concentrated within a shell of thickness δ_s



. Hence

$$R_{ac} = \frac{\ell}{\sigma(2\pi a)\delta_s} = \frac{100}{(5.813 \times 10^7)(2\pi)(0.001)(1.205 \times 10^{-6})} = 227\,\Omega$$

I.e the R_{ac} is more than 400 Hundred times greater than the D.C. resistance $R_{d.c.}$

Problem ③ :

- For the TEM-mode:

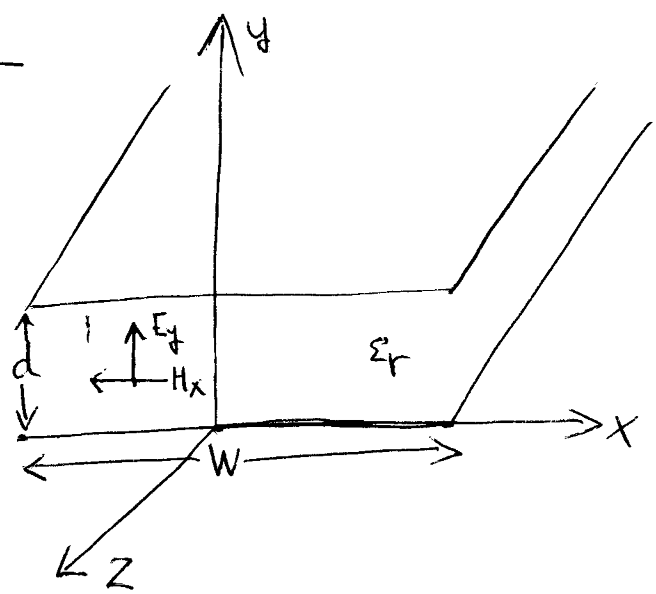
$$V_\phi = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times10^8}{\sqrt{2.5}} = 1.897\times10^8 \text{ m/s}$$ p. 18

But $V_\phi = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{C V_\phi^2} = \frac{1}{(200\times10^{-12})(1.897\times10^8)} \simeq 139 \text{ nH/m}$ p. 59

Therefore, $Z_0 = \sqrt{\frac{L}{C}} = 26.4\ \Omega.$

Attenuation Constant $\alpha = \frac{R}{2Z_0} = \frac{2}{2(26.4)} \cong 0.038$ NP/m.

Problem ④:



$\bar{E} = \hat{y} E_0 e^{-\gamma z}$

For a forward-wave: $\bar{H} = \frac{1}{\eta}(\hat{z}\times\bar{E}) = -\hat{x}\frac{E_0}{\eta} e^{-\gamma z}$, $\eta = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}}$

Problem 4, Cont.

At $z=0$, the voltage between the plates is:

$$V_0 = +\int_0^d E_y\,dy = +E_0 d \Rightarrow E_0 = +V_0/d.$$

On the other hand, the current is given at $z=0$ by:

$$I_0 = \oint \bar{H}\cdot d\bar{l} = \frac{E_0}{\eta} w \Rightarrow \frac{E_0}{\eta} = I_0/w$$

Hence,

$$E_y = +\frac{V_0}{d} e^{-\gamma z}$$

$$H_x = -\frac{I_0}{w} e^{-\gamma z}$$

Compute L, C, R, G @ $(z=0)$, $S =$ cross-section $w \times d$ at $z=0$

$$L = \frac{\mu_0}{|I_0|^2}\int_S \bar{H}\cdot\bar{H}^* ds = \frac{\mu_0}{|I_0|^2}\frac{I_0}{w}\frac{I_0^*}{w} wd = \mu_0 \frac{d}{w} \quad H/m$$

$$C = \frac{\varepsilon'}{|V_0|^2}\int_S \bar{E}\cdot\bar{E}^* ds = \frac{\varepsilon'}{|V_0|^2}\frac{V_0}{d}\frac{V_0^*}{d} wd = \varepsilon' \frac{w}{d} \quad F/m \quad \left(\varepsilon = \varepsilon_r(\varepsilon' - j\varepsilon'')\right).$$

$$R = \frac{R_s}{|I_0|^2}\int_{C_1+C_2\ (=\text{plates})} \bar{H}\cdot\bar{H}^* dl = \frac{R_s}{|I_0|^2}\left(\frac{|I_0|^2}{w^2}w + \frac{|I_0|^2}{w^2}w\right) = 2\frac{R_s}{w}\left(R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}}\right) \Omega/m$$

$$= \frac{1}{\sigma\delta_s}$$

$$G = \frac{\omega\varepsilon''}{|V_0|^2}\int_S \bar{E}\cdot\bar{E}^* ds = \frac{\omega\varepsilon''}{|V_0|^2}\frac{|V_0|^2}{d^2} wd = \frac{\omega\varepsilon'' w}{d} \quad \text{Siemens}/m$$

Note that $$Z_0 \triangleq \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu_0 d}{w}\frac{d}{w\varepsilon'}} = \frac{d}{w}\sqrt{\frac{\mu}{\varepsilon'}} = -V_0/I_0$$

whereas $$V_\phi = \frac{1}{\sqrt{LC}} = \sqrt{\frac{w}{\mu_0 \cancel{d}}\frac{\cancel{d}}{\varepsilon' \cancel{w}}} = \frac{1}{\sqrt{\mu_0\varepsilon'}}$$