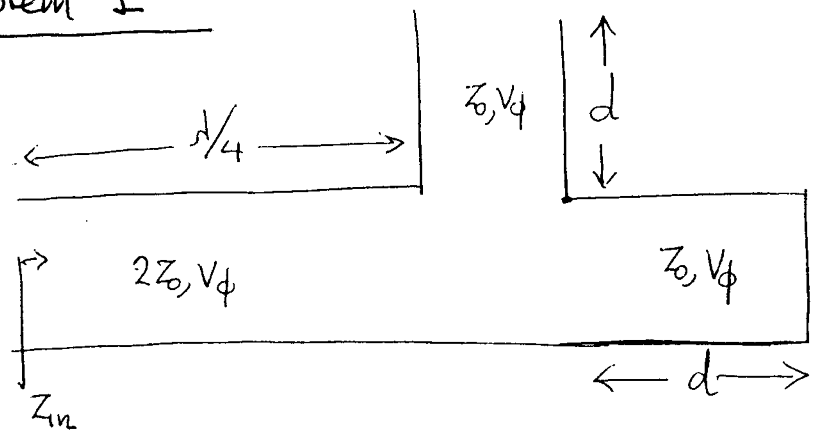
# Problem 1




$\beta d \ll 1$

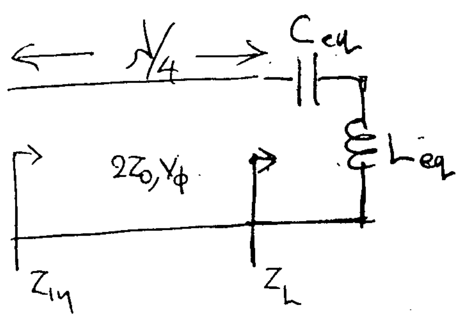
For the shorted-stub:

$$Z_{in} = jZ_0 \tan\beta d \simeq jZ_0\beta d = jZ_0\frac{\omega d}{V_\phi} \equiv j\omega L_{eq} \Rightarrow L_{eq} = \frac{Z_0 d}{V_\phi}$$

For the open-stub:

$$Z_{in} = -jZ_0 \cot\beta d \simeq \frac{-jZ_0}{\beta d} = \frac{-jZ_0 V_\phi}{\omega d} \equiv \frac{-j}{\omega C_{eq}} \Rightarrow C_{eq} = \frac{d}{Z_0 V_\phi}$$

So,



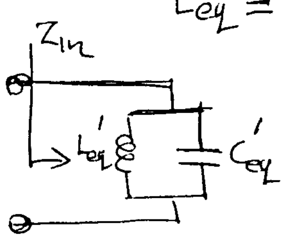

But for an λ/4-transformer: $Z_{in} = \dfrac{(2Z_0)^2}{Z_L}$, i.e., $Y_{in} = \dfrac{Z_L}{4Z_0^2} = \dfrac{j\omega L_{eq} + \frac{1}{j\omega C_{eq}}}{4Z_0^2}$

$$= j\omega\left(\frac{L_{eq}}{4Z_0^2}\right) + \frac{1}{j\omega(4Z_0^2 C_{eq})}$$

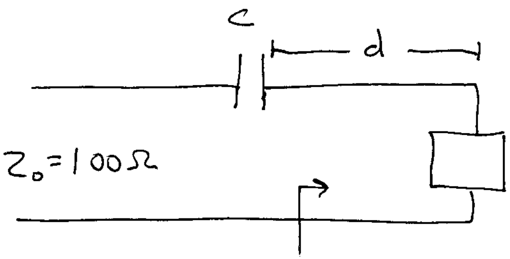
This α parallel combination of $C'_{eq} = \dfrac{L_{eq}}{4Z_0^2} = \dfrac{Z_0 d}{V_\phi}\dfrac{1}{4Z_0^2} = \dfrac{d}{4Z_0 V_\phi} = \dfrac{1}{4}C_{eq}$

$$L'_{eq} = 4Z_0^2 C_{eq} = 4Z_0^2\frac{d}{Z_0 V_\phi} = \frac{4Z_0 d}{V_\phi} = 4L_{eq}$$

Equivalent Circuit:



## Problem ②:




$f = 3\,GHz$

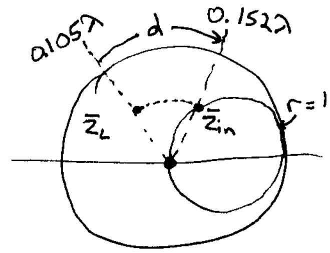
$\lambda = \frac{c}{f}$

$= 10\,cm$

$Z_o = 100\,\Omega$

$Z_L = \cancel{70+j50}\ \Omega$
$= 70 + j50\,\Omega$

$\bar{Z}_{in} = 1 + j\,X_{in}$




$\bar{Z}_L = Z_L/_{100} = 0.7 + j0.5$

$d = 0.047\lambda$
$= (0.047)(10cm)$
$= \underline{\underline{0.47\ cm}}$

Moving towards the generator by a distance 'd' we cross the $Re(Z) = 1$ circle.

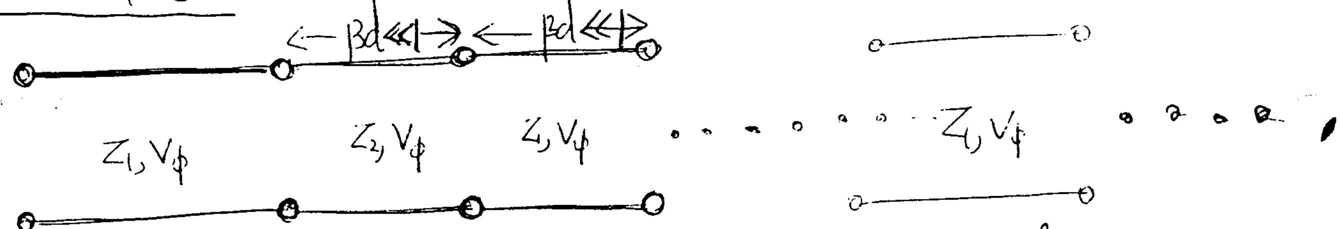
Hence, the value of $\bar{X}_{in} = 0.7$

$$\Rightarrow X_{in} = (\bar{X}_{in})(100) = 70\ \Omega$$

Then, $X_{in} = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_{in}} = \frac{1}{(2\pi)(3\times10^9)(70)} = 0.758\ pF$

$\therefore$ For $d = 0.47\,cm$ and $C = 0.758\,pF$ a match is achieved.

## Problem 3:




HW#3, Problem 2

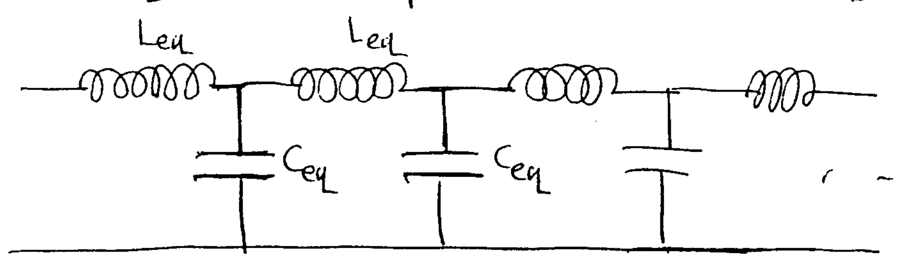
For a short-section of a high-impedance T-line:

$$L_{eq} = \frac{Z_1 d}{V_\phi} \quad \text{in series.}$$

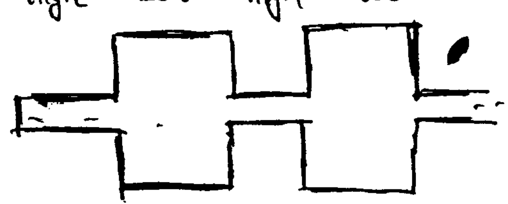
For a short-section of a Low-impedance line:

$$C_{eq} = \frac{d}{Z_2 V_\phi} \quad \text{in shunt}$$

Hence the equivalent Lumped element circuit is:




Microstrip realization




We note that this is the circuit of an equivalent transmission-line with per-unit quantities:

$$\left.\begin{aligned} L' &= L_{eq}/d = Z_1/V_\phi \\ C' &= C_{eq}/d = 1/Z_2 V_\phi \end{aligned}\right\} \quad \text{The limit } \beta d \ll 1 \text{ implies that } d \ll \lambda$$

So I can define an equivalent Characteristic-impedace $Z_0' = \sqrt{L'/C'}$

$$= \sqrt{Z_1 Z_2}$$

Hence, if the circuit is terminated to $Z_0'$, the equivalent line will be matched and no-reflections shall be incurred.