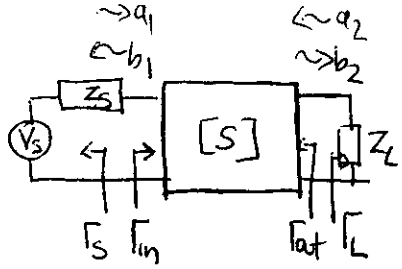
# Homework #9

Problem #1 :




(i) (a) $$\left.\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}\right]$$

Let $\Gamma_L = \frac{a_2}{b_2}$ hence $b_2 = \frac{a_2}{\Gamma_L} = S_{21}a_1 + S_{22}a_2$

$\Rightarrow a_2 = S_{21}\Gamma_L a_1 + S_{22}\Gamma_L a_2 \Rightarrow$

$a_2(1 - S_{22}\Gamma_L) = S_{21}\Gamma_L a_1$

Hence $b_1 = S_{11}a_1 + S_{12}\frac{S_{21}\Gamma_L a_1}{1 - S_{22}\Gamma_L}$ I.e B

$$\boxed{\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}}$$

(b) By symmetry $$\boxed{\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}}$$

(ii) Let $Z_S = Z_0 \Rightarrow \Gamma_S = 0$.

$P_{avs} = \frac{|V_s|^2}{8Z_0}$


$V_L = \frac{V_s Z_{in}}{Z_{in} + Z_0} = \sqrt{Z_0}(a_1 + b_1)$

$= \sqrt{Z_0}(1 + \Gamma_{in})a_1$

$\Rightarrow a_1 = \frac{V_s Z_{in}}{Z_{in} + Z_0}\frac{1}{\sqrt{Z_0}}\frac{1}{(1 + \Gamma_{in})}$

But $1 + \Gamma_{in} = \frac{2Z_{in}}{Z_{in} + Z_0}$

$\Rightarrow a_1 = \frac{V_s \cancel{Z_{in}}}{\cancel{Z_{in} + Z_0}}\frac{1}{\sqrt{Z_0}}\frac{\cancel{Z_{in} + Z_0}}{2\cancel{Z_{in}}} = \frac{V_s}{2\sqrt{Z_0}}$

Hence $P_{avs} = \frac{|V_s|^2}{8Z_0} = \frac{|a_1 2\sqrt{Z_0}|^2}{|V_s|^2 8Z_0} = \frac{1}{2}|a_1|^2$

(iii) $P_{Load} = \frac{1}{2}\left(|b_2|^2 - |a_2|^2\right) = \frac{1}{2}|b_2|^2\left(1 - |\Gamma_L|^2\right)$

But $b_2 = S_{21}a_1 + S_{22}a_2 = S_{21}a_1 + S_{22}b_2\Gamma_L \Rightarrow$

$$b_2\left(1 - S_{22}\Gamma_L\right) = S_{21}a_1 \Rightarrow$$

$$b_2 = \frac{S_{21}}{1 - S_{22}\Gamma_L}\,a_1$$

Hence $P_{Load} = \frac{1}{2}\,\frac{|S_{21}|^2}{|1 - S_{22}\Gamma_L|^2}\left(1 - |\Gamma_L|^2\right)|a_1|^2$

But $a_1 = \frac{V_S}{2Z_0}$, Hence

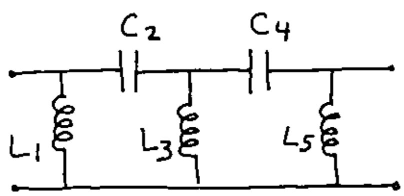
$$P_{Load} = P_{avs}\,\frac{|S_{21}|^2}{|1 - S_{22}\Gamma_L|^2}\left(1 - |\Gamma_L|^2\right), \quad P_{avs} = \frac{|V_S|^2}{8Z_0}$$

(iv) $G_T = \frac{P_{Load}}{P_{avs}} = |S_{21}|^2\,\frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$

(8.9) $f_0 = 1\,GHz$, HIGH PASS, 3dB E.R., $N=5$, $Z_0 = 50\,\Omega$

at $f = 0.6\,GHz$, $\left|\frac{\omega}{\omega_c}\right| - 1 = \frac{1}{.6} - 1 = 0.667$, so from Figure 8.27b, the attenuation for $N=5$ should be about 41 dB. From Table 8.4 the prototype values are,

$g_1 = 3.4817$
$g_2 = 0.7618$
$g_3 = 4.5381$
$g_4 = 0.7618$
$g_5 = 3.4817$




Impedance and frequency scaling using (8.70):

$$L_1 = \frac{Z_0}{\omega_c g_1} = 2.28\, nH \quad ✓$$

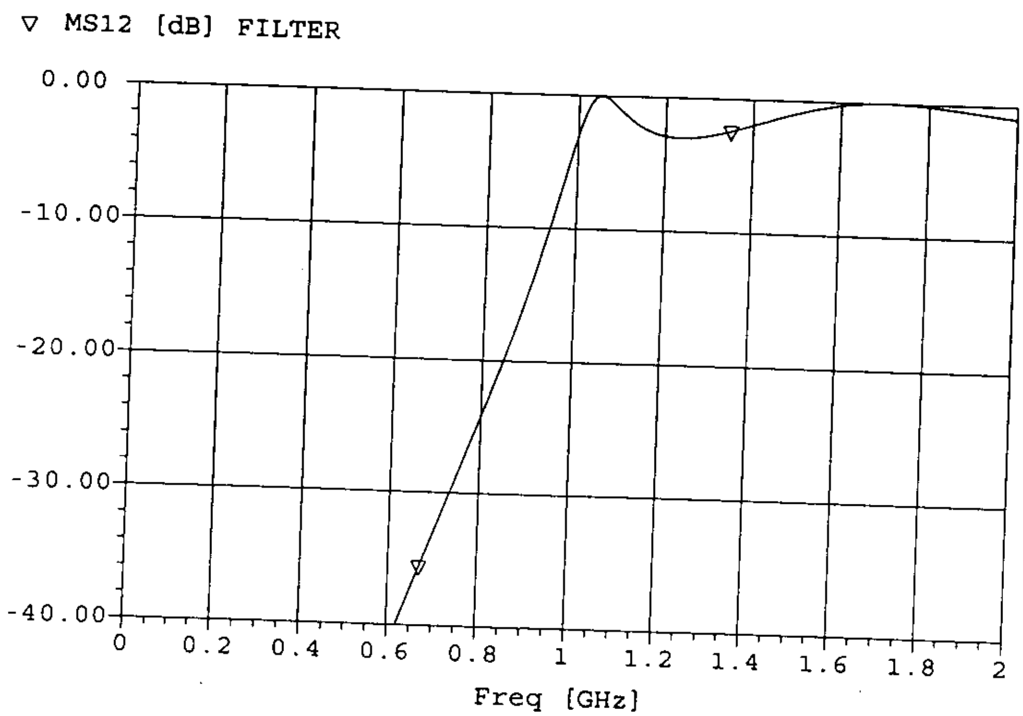
$$C_2 = \frac{1}{Z_0 \omega_c g_2} = 4.18\, pF \quad ✓$$

$$L_3 = \frac{Z_0}{\omega_c g_3} = 1.75\, nH \quad ✓$$

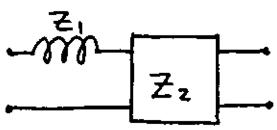
$$C_4 = \frac{1}{Z_0 \omega_c g_4} = 4.18\, pF \quad ✓$$

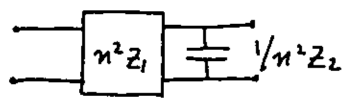
$$L_5 = \frac{Z_0}{\omega_c g_5} = 2.28\, nH \quad ✓$$

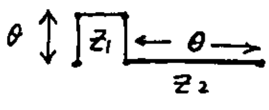
The calculated filter response is shown below. Note that the insertion loss at $f = 0.6$ GHz is just a bit more than 40dB.

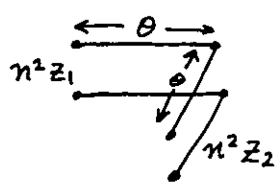


8.12




$Y_s = \frac{j}{n^2 Z_2} \tan\theta$

$Z_s = j Z_1 \tan\theta$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & jZ_1 \tan\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & jZ_2 \sin\theta \\ \frac{j}{Z_2}\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta - \frac{Z_1}{Z_2}\frac{\sin^2\theta}{\cos\theta} & j(Z_1+Z_2)\sin\theta \\ \frac{j}{Z_2}\sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jn^2 Z_1 \sin\theta \\ \frac{j\sin\theta}{n^2 Z_1} & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{j\tan\theta}{n^2 Z_2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta - \frac{Z_1}{Z_2}\frac{\sin^2\theta}{\cos\theta} & jn^2 Z_1 \sin\theta \\ \frac{j}{n^2}\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)\sin\theta & \cos\theta \end{bmatrix}$$

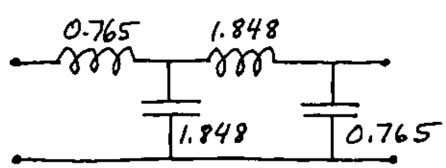
So these two matrices are equal if,

$$Z_1 + Z_2 = n^2 Z_1$$

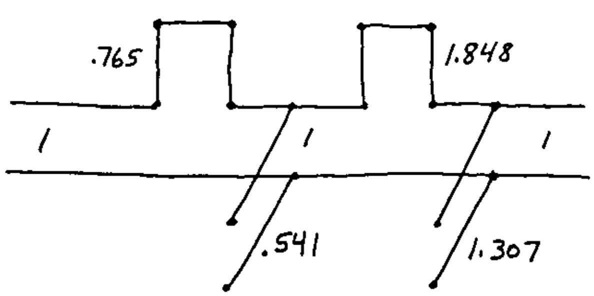
or, $\quad n^2 = 1 + Z_2/Z_1$ ✓

8.14 $f_0 = 8 GHz$, $N=4$, L.P., M.F., $Z_0 = 50\Omega$

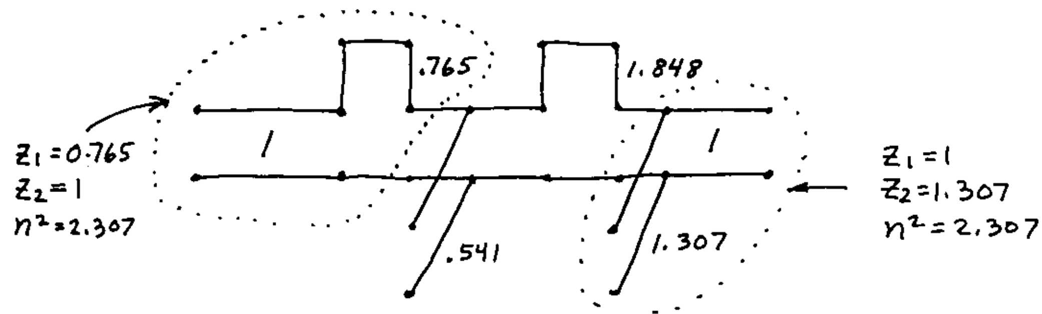
From Table 8.3 the L.P. prototype is,



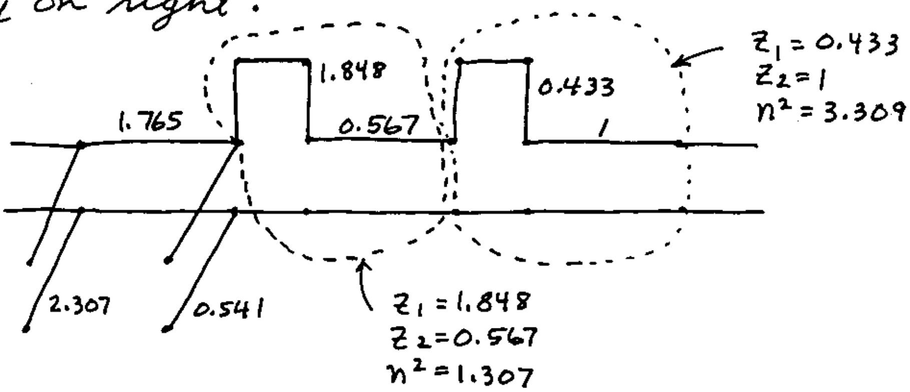

applying Richard's transform:



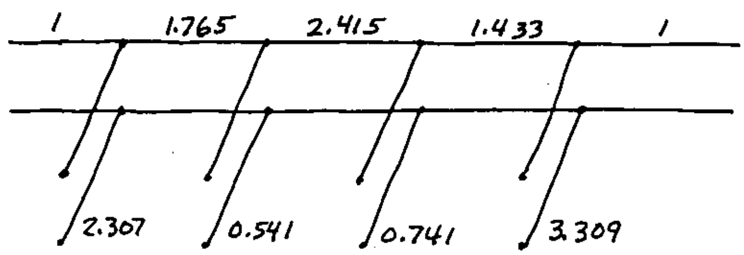
add unit elements:



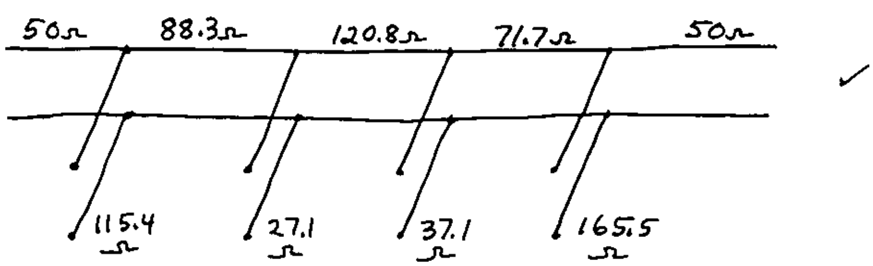

Use the second Kuroda identity on left; first Kuroda identity on right:



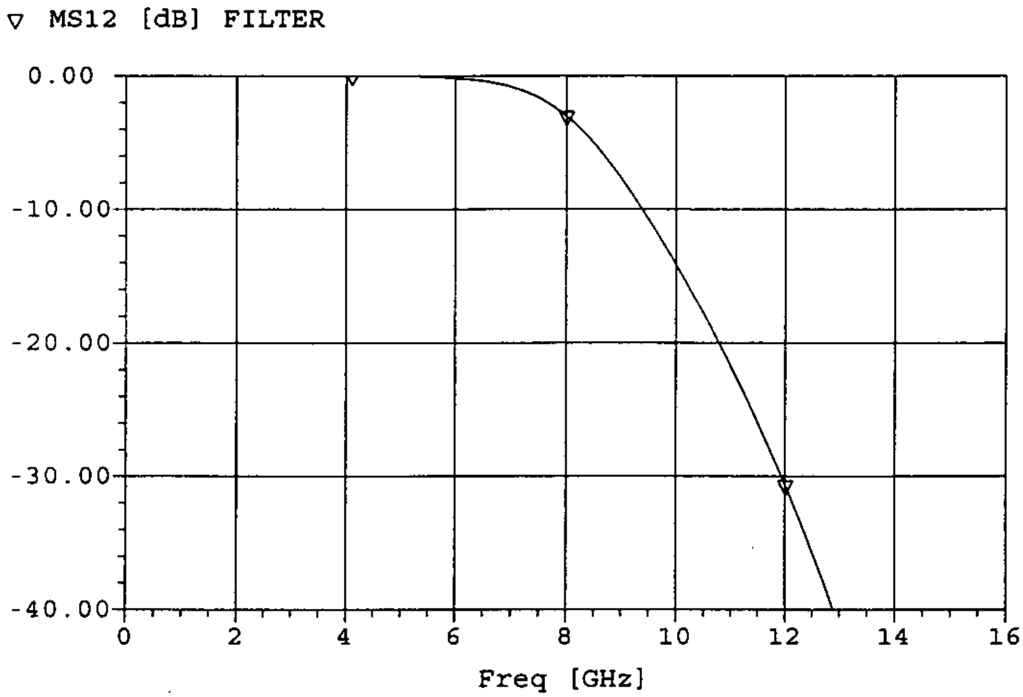

Now use the second Kuroda identity twice:




Scale to 50Ω:




all lines are λ/8 long at 8GHz. The calculated filter response is shown on the following page.


▽ MS12 [dB] FILTER
0.00
-10.00
-20.00
-30.00
-40.00
0
2
4
6
8
10
12
14
16
Freq [GHz]