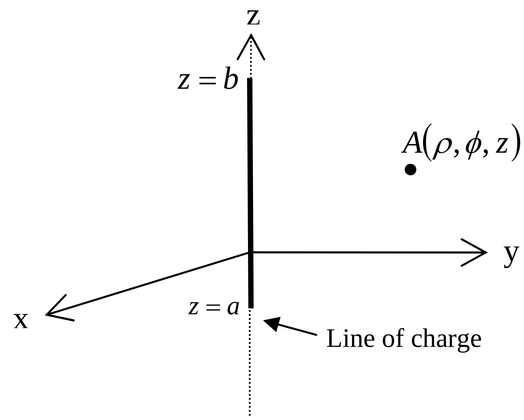
Homework #2 ECE 1228



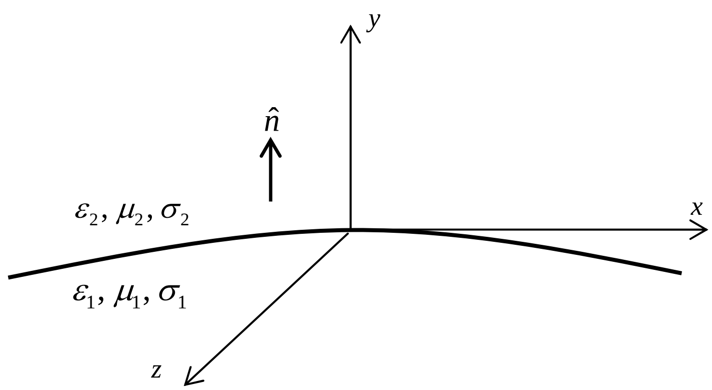
1) An infinitely long straight line charge has a constant charge density ρ_l [C/m].

a) Using the integral formulation for $\vec{E}$ discussed in the class calculate the electric field at an arbitrary point $A(\rho,\phi,z)$.
b) Using the Gauss law calculate the same as in part (a)
C) Now suppose that our uniformly charged line (ρ_l constant) has a finite extension from $z=a$ to $z=b$, find the electric field at the arbitrary point A.

Note: Express your results in cylindrical coordinate system.

2) If gradient of a scalar function ψ in rectangular coordinate system is given by $\vec{\nabla}\psi = \frac{\partial\psi}{\partial x}\hat{a}_x + \frac{\partial\psi}{\partial y}\hat{a}_y + \frac{\partial\psi}{\partial z}\hat{a}_z$, using coordinate transformation and chain rule show that the gradient of ψ in cylindrical coordinate is given by $\vec{\nabla}\psi = \frac{\partial\psi}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial\psi}{\partial \phi}\hat{a}_\phi + \frac{\partial\psi}{\partial z}\hat{a}_z$.

3) In the class note we showed that when there were no sources at the interface between two media and neither of the two media was a perfect conductor ($\sigma_1, \sigma_2 \neq \infty$) the boundary condition on the tangential magnetic field was given by $\hat{n}\times\left(\vec{H}_2-\vec{H}_1\right)=0$. Here, show that when $\vec{J}_i+\vec{J}_c=\vec{J}_{ic}\neq 0$, the B.C. is given by $\hat{n}\times\left(\vec{H}_2-\vec{H}_1\right)=\vec{J}_s$, where, $\vec{J}_s=\lim_{\Delta y\to 0}\vec{J}_{ic}\,\Delta y$.
Note: Used the geometry provided in figure below for your proof.

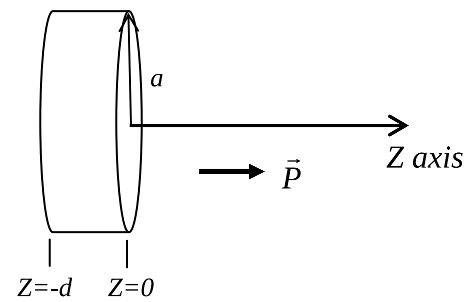


4) Show that when there are source charges present at the boundary between two media ($\rho_{ev}\neq 0$), the B.C. on normal $\vec{D}$ is given by $\hat{n}\cdot\left(\vec{D}_2-\vec{D}_1\right)=\rho_s$, where ρ_s [c/m^2] is the surface charge density.

Note: Use the same geometry (coordinate system) as shown in problem 3.

6) The plane $3x + 2y + z = 12\,[m]$ describes the interface between a dielectric and free space. The origin side of the interface has $\varepsilon_{r1} = 3$ and $\vec{E}_1 = 2\hat{a}_x + 5\hat{a}_z\ [V/m]$. What is $\vec{E}_2$ (the field on the other side of the interface)?

7) A dielectric circular desk of radius a and thickness d is permanently polarized with a dipole moment per unit volume $\vec{P}$ [C/m^2], where $|\vec{P}|$ is constant and parallel to the disk axis (*z-axis* here) as shown in the Figure.



a) Calculate the potential along the disk axis for $z > 0$.
b) Approximate the result obtained in part (a) for the case of $Z >> d$.