Homework #5 ECE 1228

1) For the geometry shown in Fig. 1, obtain the TM ($E\|$) Fresnel reflection and transmission coefficients. Express your results in terms of the propagation constant k_{1z} and k_{2z}, (i.e., the projection of $\vec{k}_1$ and $\vec{k}_2$ along z-direction.) Note that the interface is at *Z=0* plane.

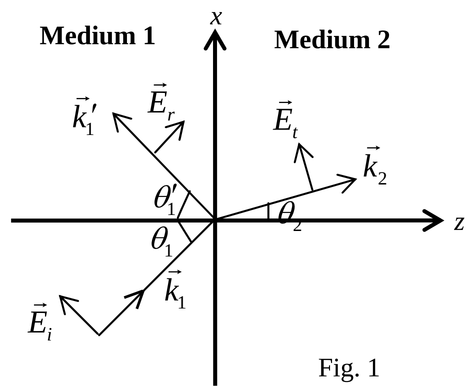


Fig. 1

2)
a) Give the TE transmission function [$T^{TE}(\omega)$] for a slab of length *d* with permittivity and permeability ε_2, μ_2, surrounded by medium characterized by ε_1 and μ_1 as shown in Fig. 2. [Make sure you provide the expressions for the terms appearing in the transmission function $T^{TE}(\omega)$.]
b) Suppose medium (*II*) is a meta-material with $\varepsilon_2 = -\varepsilon_1$ and $\mu_2 = -\mu_1$, where $\varepsilon_1 > 0$, and $\mu_1 > 0$. What is the transmission function [$T^{TE}(\omega)$] in this case. Express your results in terms of the propagation constant in medium (I), i.e. k_{1z}.
c) Now consider a source located at *Z=0* generating a uniform plane wave, and for simplicity suppose a one-dimensional propagation. What is the field at the second interface (*Z=2d*). What is the meaning of your results?

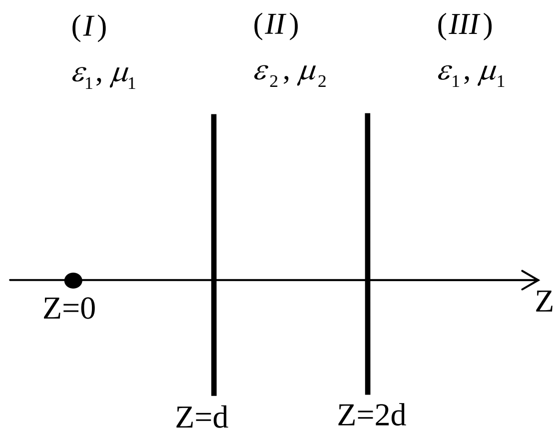
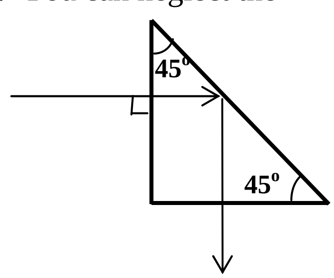


Fig. 2

3) Show that Bessel function and Hankel function in their large argument asymptotic forms respectively represent standing and traveling waves in the radial direction.

4) A linearly polarized wave is incident on an isosceles right triangle (prism) of glass, and it exits as shown in the figure. Assuming that the dielectric constant of the prism is 2.25, find the ratio of the exited power density S_e to that of the incident S_i. You can neglect the multiple reflections within the glass prism in your analysis.



5) Using the non-existence of magnetic monopole and Faraday's law

(a) Define the vector and scalar vector potentials $\vec{A}(r,t)$ and $V(r,t)$.

(b) Let $\vec{J} = \vec{J}_i + \vec{J}_c$ be the current [A/m^2] and ρ be the charge [C/m^3] densities. Assuming a simple medium and Lorentz gauge, derive the decoupled non-homogeneous wave equations for $\vec{A}(r,t)$ and $V(r,t)$.

(c) Replace the Lorentz gauge of part (b) with the Coulomb gauge, and obtain the non-homogeneous differential equations for $\vec{A}(r,t)$ and $V(r,t)$.

(d) What fundamental theorem allows us to use different gauges in parts (b) and (c)? (Justify your answer.)

Note: From the problem statement it should be clear that I want the results for the instantaneous fields and not in the form of time harmonic fields