Homework #6 ECE1228

1) In previous homework you found the inhomogeneous wave equation for $V(r,t)$ and $\vec{A}(r,t)$ for a simple medium (linear, isotropic, and homogeneous), subject to Lorentz gauge. Here, obtain the differential equations governing the dynamical behavior of $V(r,t)$ and $\vec{A}(r,t)$, when ε and μ depend on position (assume Lorentz gauge). Simplify your results as much as possible.

2) For the auxiliary vector potentials $\vec{A}$ and $\vec{F}$ with $A_x = A_y = F_x = F_y = 0$, $A_z \neq 0$, $F_z \neq 0$, $\partial/\partial x \neq 0$, and $\partial/\partial y \neq 0$ show that

a) The electromagnetic mode is TEM^z (what are the conditions that $A_z(x,y,z)$ and $F_z(x,y,z)$ must satisfy.)

b) Find the expressions for the electric and magnetic field components that are not zero.

c) What is the wave impedance?

3) We have stated that the boundary condition for a perfect conductor is such that there is no electric field or charge distribution inside of the conductor. Here we will study the dynamics of this process. Start with continuity equation $\nabla \cdot \vec{J} = -\partial\rho/\partial t$, where $\vec{J}$ is the current density [A/m^2] and ρ is the charge density [C/m^3]. Show that a charge (charge density) placed inside a conductor will decay in an exponential manner.

4) The constitutive relation for superconductors in weak magnetic fields can be macroscopically characterized by the first London equation

$$\frac{\partial \vec{J}_{Sup}}{\partial t} = \alpha \vec{E}$$

and the second London equation

$$(\nabla \times \vec{J}_{sup}) = -\alpha_1 \vec{B}$$

where $\vec{J}_{sup}$ stands for the superconducting current, $\alpha = n_s q^2 / m$ and $\alpha_1 \simeq \alpha$, with n_s, m and q denoting, respectively, the number density, the effective mass, and the charge of the Cooper pairs responsible for the superconductivity in a charged Boson fluid model.

(a) From the first London equation, derive and equation for $\dot{\vec{B}} = \partial \vec{B}/\partial t$ by using the static Maxwell equation $\nabla \times \vec{H} = \vec{J}_{Sup}$ without the displacement current. Show that

$$\nabla^2 \dot{\vec{B}} = \mu_o \alpha \dot{\vec{B}}$$

(b) From the second London equation and the Ampere's law stated above derive an equation for $\vec{B}$.

(c) What are the penetration depths in the (a) and (b) cases? Justify your answer.

Remark: from above analysis we see that both the current and magnetic field are confined to a thin layer of the order of the penetration depth which is very small. The exclusion of static magnetic field in a superconductor is known as the Meissner effect experimentally discovered in 1933.

5) Consider an elemental oscillating electric dipole oriented along the z-axis as shown in the figure. The electric current varies in time according to $i(t) = I\cos(\omega t)$ [I is constant]. The magnetic current $i_m(t)$ is zero. Assuming the dipole length L is very small, calculate the following:

a) The electric and magnetic field components $E_r, E_\theta, E_\phi, H_r, H_\theta, H_\phi$ at any arbitrary point (note the fields are to be expressed in spherical coordinates. Do not make the near filed or far filed approximation at this point.) **(25 pts)**

b) The approximate electric and magnetic fields for the near field region ($\beta r << 1$) **(10 pts)**

c) Comparing the electric field components under the near filed condition to the fields of an electric dipole under electrostatic condition what is your conclusion. Justify your answer. **(10 pts)**