HW 6.1, Fall 2010

1) Two identical sources (S_1 and S_2) and two identical detectors (D_1 and D_2) are a distance L apart in vacuum as shown in Fig.1.1. Simultaneously the sources radiate two well-behaved wave packets that are detected by their respective detectors. While the pulse from S_1 propagates through vacuum, the pulse from S_2 travels through vacuum and an one dimensional photonic crystal of length L_{PC}. The detected wave packets as a function of time are shown in Fig. 1.2, where pulse 2 corresponds to D_2 and pulse 1 corresponds to D_1.

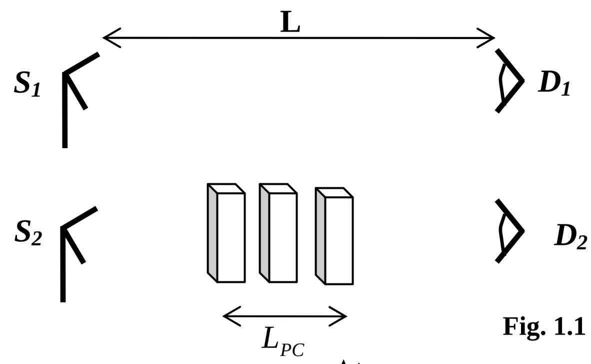


Fig. 1.1

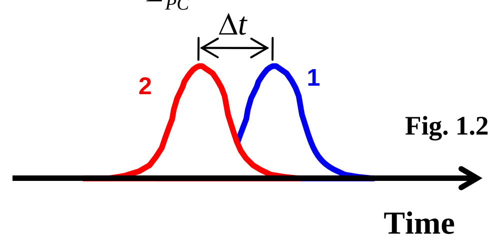


Fig. 1.2

a) Find an expression for the time difference Δt ($\Delta t > 0$) in terms of the group delay, the length of the photonic crystal, etc.

b) Find an expression for the group velocity of the wave packet propagating through the photonic crystal in terms of the time difference Δt, the length of the photonic crystal, etc.

2) Figure shows a dielectric slab (a perfect dielectric), infinite in z- and x-direction, with permittivity and permeability ε_1, μ_1 surrounded by vacuum (ε_0, μ_0). The slab thickness is $2h$ and it supports a propagating Transverse Magnetic mode (TM^Z) traveling in positive z-direction (coordinate system is shown.) Vector potential A_z for the TM^Z even modes of the waveguide in each region is given below. The dispersion relation ($\omega\ vs.\ \beta_z$) for the TM^Z even modes is given by

$$-\frac{\varepsilon_0}{\varepsilon_1} f(\omega,\beta_z)\cot[f(\omega,\beta_z)\,h] = g(\omega,\beta_z)$$

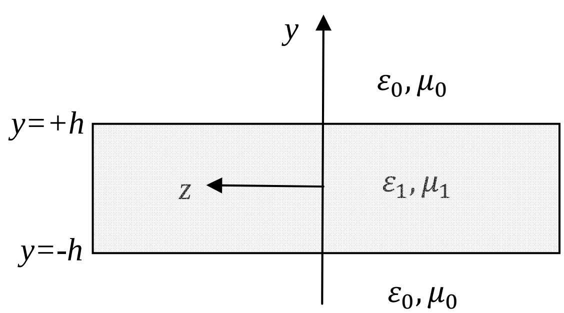
Where $f(\omega,\beta_z)$ and $g(\omega,\beta_z)$ are functions of ω and β_z that you are asked to determine. Show all your work.

For $-h \le y \le h$

$$A_z = A_1 \cos(\beta_{y1}\, y)\, e^{-j\,\beta_z\, z}$$

&

$$\beta_{y1}^2 + \beta_z^2 = \beta_1^2 = \omega^2 \varepsilon_1\, \mu_1$$



For $\boldsymbol{y \geq h}$

$$A_z = A_0 e^{-\alpha_{y0}\, y}\, e^{-j\,\beta_z\, z}$$

&

$$\beta_{y0}^2 + \beta_z^2 = -\alpha_{y0}^2 + \beta_z^2 = \beta_0^2 = \omega^2 \varepsilon_0\, \mu_0$$

For $\boldsymbol{y \leq -h}$

$$A_z = A_0 e^{\alpha_{y0}\, y}\, e^{-j\,\beta_z\, z}$$

&

$$\beta_{y0}^2 + \beta_z^2 = -\alpha_{y0}^2 + \beta_z^2 = \beta_0^2 = \omega^2 \varepsilon_0\, \mu_0$$

3) In class we stated that the eigenvalue equation (the so called master equation) in the case of photonic crystal is given by $\Theta\, \vec{H}(\vec{r}) = (\omega/c)^2\, \vec{H}(\vec{r})$. Here I will ask you to derive this expression: Consider wave propagation in an infinite, linear, isotropic, but inhomogeneous dielectric. There are no magnetic or electric impressed sources ($\vec{J}_i = \vec{M}_i = 0$), no free charges ($\rho_{ev} = 0$), and $\mu_r = 1$. Show that wave equation in this medium – the eigenvalue equation for $\vec{H}$– is given by $\Theta\, \vec{H}(\vec{r}) = (\omega/c)^2\, \vec{H}(\vec{r})$. Determine the operator Θ.