1) Consider an infinitely periodic one dimensional photonic crystal (1DPC) shown in Fig. 1.1 where n_i and n_j are the indices of refractions (in general complex) associated

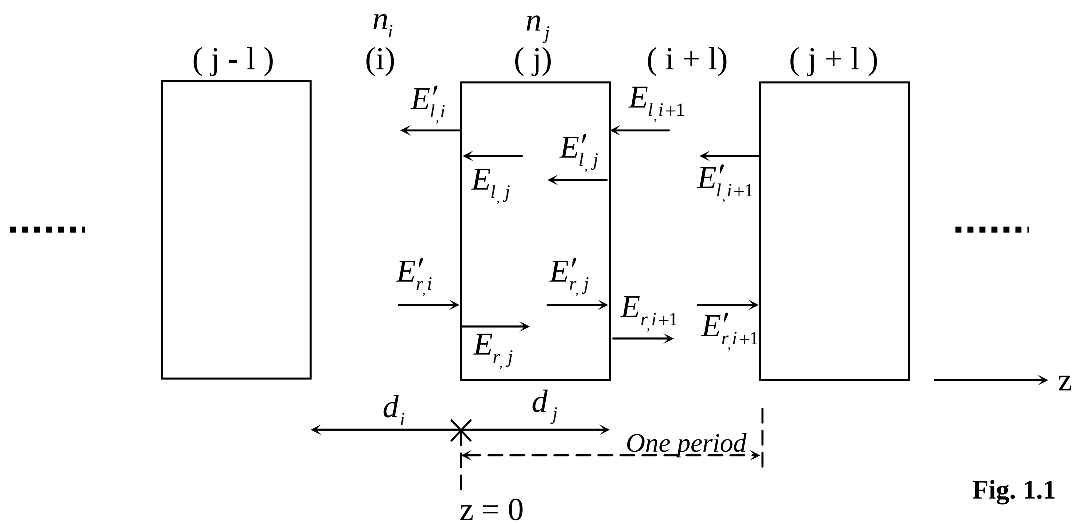

Fig. 1.1

with the regions *i* and *j* having thicknesses d_i and d_j. The one period transfer matrix ($\underline{M}$) relates the fields according to

$$\begin{pmatrix} E'_{l,i} \\ E'_{r,i} \end{pmatrix} = \underline{M} \begin{pmatrix} E'_{l,i+1} \\ E'_{r,i+1} \end{pmatrix}$$, where $\underline{M} = g \begin{pmatrix} a & b \\ \hat{b} & \hat{a} \end{pmatrix}$, $c = \dfrac{1}{1-\rho_{i,j}^2}$, and $\rho_{i,j}$ is the Fresnel reflection coefficient. Give the expressions for $a, \hat{a}, b, \hat{b}$ in terms of β_i, β_j and $\rho_{i,j}$ where $\beta_i = \dfrac{\omega}{c} n_i\, d_i \cos\theta_i$, $\beta_j = \dfrac{\omega}{c} n_j\, d_j \cos\theta_j$, and θ_i or θ_j are the incident angles. (Note that β_i and β_j are the phase constants in regions *i* and *j*)

2) Consider a truncated (finite length) one dimensional photonic crystal shown in Fig. 1.1, in which there are *N* dielectric slab of index n_j and length d_j. Find the transmission and reflection functions for this structure as a function of λ_1, λ_2, *a*, *b*, *g*, and β_i, where λ_1 and λ_2 are the eigenvalues of the one period matrix ($\underline{M}$) given in problem 1 and *a*, *b*,

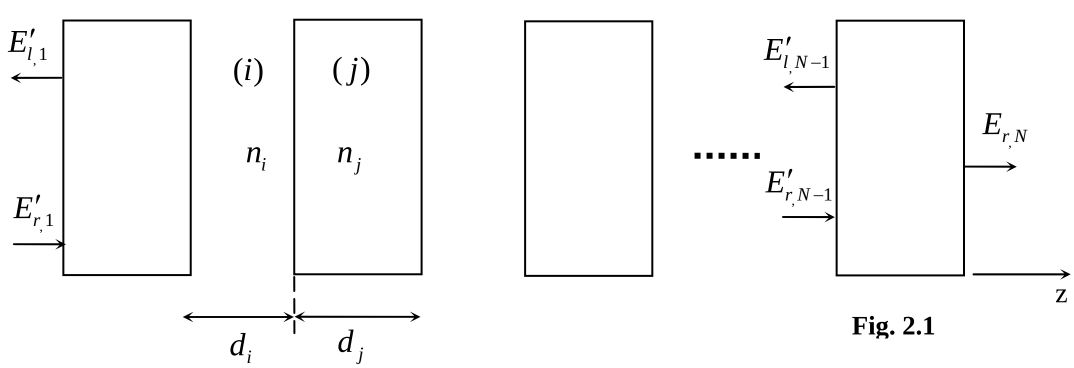

Fig. 2.1

c, and β_i are also defined in the same problem.

3) Use the expression for transmission function obtained in problem 2 and the values and instructions in the table below to plot the following at normal incidence:
a) Transmission magnitude and phase as a function of frequency for the case *N=3*.
b) The group delay as a function of frequency for the cases *N=1, 2, 3, 4*.
c) The group velocity as a function of frequency for the cases *N=1, 2, 3*.

Table 6.1

$n_i = 1$ (this is air), $n_j = 3.4 - j\,0.002$ (this is Eccostock)
$d_i = 1.76$ [cm], $d_j = 1.33$ [cm], $L_{PC} = (N-1)(d_i + d_j) + d_j$
Frequency range for all plots: 20 [GHz] to 23 [GHz]
Use linear scale for transmission magnitude (not dB) and express the transmission phase in Degrees
Plot the group delay in nanosecond
Plot the group velocity in units of V_g / c, where c is the speed of light in vacuum

4) Now suppose that n_i and n_j are real numbers

a) What is the general form of the $\underline{M}$ matrix?

b) Show that $\underline{M}$ is uni-modular. What does this mean in terms of Bloch theorem?

c) Is $\underline{M}$ unitary? (Justify your answer)