Dispersive Effects

- **Equivalent representations of dispersive effects:**

$$\omega(k) = \omega(k_0) + \left.\frac{d\omega}{dk}\right|_{k_0}(k-k_0) + \frac{1}{2}\left.\frac{d^2\omega}{dk^2}\right|_{k_0}(k-k_0)^2 + \ldots$$

$$= v_p\, k_0 + v_g(k-k_0) + (1/2)\,\beta(k-k_0)^2 + \ldots$$

$$k(\omega) = k(\omega_0) + \left.\frac{dk}{d\omega}\right|_{\omega_0}(\omega-\omega_0) + \frac{1}{2}\left.\frac{d^2k}{d\omega^2}\right|_{\omega_0}(\omega-\omega_0)^2 + \ldots$$

$$= v_p^{-1}\omega_0 + v_g^{-1}(\omega-\omega_0) + (1/2)\,\psi(\omega-\omega_0)^2 + \ldots$$

$$k(\omega) = \frac{\omega}{c}\, n_p(\omega) = \frac{1}{c}\left\{\omega_o\, n_p\big|_{\omega_0} + n_g\big|_{\omega_0}(\omega-\omega_0) + \frac{1}{2}\left[2\frac{dn_p}{d\omega} + \omega\frac{d^2n_p}{d\omega^2}\right](\omega-\omega_0)^2 + \ldots\right\}$$

$$v_p = \omega/k = c/n_p$$

$$\psi = \frac{d^2k}{d\omega^2} = \frac{-\beta}{v_g^3} = -\frac{1}{v_g^2}\left(\frac{d}{d\omega}v_g\right) = GVD$$

$$v_g = \frac{d\omega}{dk} = \frac{c}{n_g(\omega)} = \frac{c}{n_p + \omega\, dn_p/d\omega}$$

$$n_g = n_p + \omega\, dn_p(\omega)/d\omega$$

Generalization for Lumped and Distributed Systems

$$\phi(\omega)=\phi(\omega_0)+\left.\frac{\partial\phi}{\partial\omega}\right|_{\omega_0}(\omega-\omega_0)+\frac{1}{2}\left.\frac{\partial^2\phi}{\partial\omega^2}\right|_{\omega_0}(\omega-\omega_0)^2+\ldots$$

$$\phi(\omega)=-\left\{\tau_p\omega_0+\tau_g(\omega-\omega_0)+(1/2)\,GDD\,(\omega-\omega_0)^2+\ldots\right\}$$

$$\tau_p=-\phi/\omega\,\Big|_{\omega_0}$$ **Phase Delay**

$$\tau_g=-\partial\phi/\partial\omega\,\Big|_{\omega_0}$$ **Group Delay**

$T(\omega)=|T(\omega)|\,e^{j\phi(\omega)}$ **is the transfer function**

$$GDD=-\partial^2\phi/\partial\omega^2\,\Big|_{\omega_0}$$ **Group Delay Dispersion**

- **Relations between phase and group delays and phase and group velocities for spatially extended system of physical length *L*:**

$$v_p=\frac{L}{\tau_p}=\frac{c}{n_p(\omega)} \qquad v_g=\frac{L}{\tau_g}=\frac{c}{n_g(\omega)}$$

$$f_i(t)=f_e(t)\cos(\omega_0 t)\longrightarrow\boxed{T(\omega)}\longrightarrow f_o(t)=f_e(t-\tau_g)\cos(\omega_0 t-\omega_0\tau_p)$$

$$T(\omega)=|T(\omega)|\exp\left[j\,\phi(\omega)\right]$$

$T(\omega)$ ∴ Transfer Function