

Problem 1)

$$a) \textcircled{1} \nabla \times \vec{E}(\vec{r}, t) = -\frac{\partial \mu \vec{H}}{\partial t} - \vec{M}_i$$

$$\textcircled{2} \nabla \times \vec{H}(\vec{r}, t) = \sigma \vec{E} + \frac{\partial}{\partial t} \epsilon \vec{E} + \vec{J}_i$$

$$\vec{J}_c = \sigma \vec{E}$$

$$\textcircled{3} \nabla \cdot \epsilon \vec{E}(\vec{r}, t) = \rho_v \Rightarrow \nabla \cdot \vec{E} = \frac{\rho_v}{\epsilon}$$

$$\textcircled{4} \nabla \cdot \mu \vec{H}(\vec{r}, t) = \rho_m \Rightarrow \nabla \cdot \vec{H} = \frac{\rho_m}{\mu}$$

$$\text{from (2)} \textcircled{5} \nabla \times \nabla \times \vec{H} = \sigma \nabla \times \vec{E} + \epsilon \frac{\partial}{\partial t} \nabla \times \vec{E} + \nabla \times \vec{J}_i \Rightarrow$$

$$\textcircled{6} \nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = \sigma \left[-\frac{\partial}{\partial t} \mu \vec{H} - \vec{M}_i \right] + \epsilon \frac{\partial}{\partial t} \left[-\frac{\partial}{\partial t} \mu \vec{H} - \vec{M}_i \right] +$$

$$\nabla \times \vec{J}_i$$

using (4) in (6)

$$\nabla \left(\frac{\rho_m}{\mu} \right) - \nabla^2 \vec{H} = -\mu \sigma \frac{\partial}{\partial t} \vec{H} - \sigma \vec{M}_i - \epsilon \mu \frac{\partial^2}{\partial t^2} \vec{H} - \epsilon \frac{\partial}{\partial t} \vec{M}_i + \nabla \times \vec{J}_i \Rightarrow$$

$$\frac{1}{\mu} \nabla(\rho_m) + \sigma \vec{M}_i - \nabla \times \vec{J}_i + \epsilon \frac{\partial}{\partial t} \vec{M}_i + \mu \sigma \frac{\partial}{\partial t} \vec{H} + \epsilon \mu \frac{\partial^2}{\partial t^2} \vec{H} = \nabla^2 \vec{H}$$

where $\vec{H} = \vec{H}(\vec{r}, t)$, $\vec{M}_i = \vec{M}_i(\vec{r}, t)$, $\vec{J}_i = \vec{J}_i(\vec{r}, t)$ & $\rho_m = \rho_m(\vec{r}, t)$

$$b) \left[\frac{1}{\mu} \nabla(\rho_m) + \sigma \vec{M}_i - \nabla \times \vec{J}_i + j\omega \epsilon \vec{M}_i + j\omega \mu \sigma \vec{H} - \omega^2 \epsilon \mu \vec{H} = \nabla^2 \vec{H} \right]$$

where $\rho_m = \rho_m(\vec{r})$, $\vec{M}_i = \vec{M}_i(\vec{r})$, $\vec{J}_i = \vec{J}_i(\vec{r})$, $\vec{H} = \vec{H}(\vec{r})$

Problem 2)

$$a) \quad Z_L = \frac{100 + j150}{75} = 1.33 + j2 = r + jx \quad (\text{normalized load})$$

This is point P on the Smith chart. We draw a circle centered at O, with radius OP.

* Measuring the length $OP = 2.1''$ & $OQ = 3.2''$ we get

$$\Gamma = \frac{OP}{OQ} = \frac{2.1''}{3.2''} = 0.66$$

* The θ_r is found from $\theta_r = (0.25 - 0.195) \times 4\pi$
 $= 0.69 \text{ (rad)} = 39.6^\circ \approx 40^\circ$

check $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j150 - 75}{100 + j150 + 75} = 0.506 + j0.424 = 0.66 \angle 39.9^\circ$

b) The circle with radius OP cuts the $x=0$ line at A & B

points. The r-circle going through A-point is the standing wave ratio. This is $r=5$ circle $\Rightarrow$

$$S' = 5$$

check: $S = \frac{|\Gamma| + 1}{1 - |\Gamma|} = \frac{0.66 + 1}{1 - 0.66} = 4.88 \approx 5$

c) The load admittance is obtained by locating point P', diametrically across point P on the Γ -circle (circle of radius OP centered at O). Reading this value on the chart we have

$$Y = g + jb = 0.22 - j0.36 \Rightarrow Y = Y_0 Y = \frac{1}{75} (0.22 - j0.36)$$

$$= 0.003 - j0.005 \left[\frac{1}{\Omega} \right]$$

Check: $\underline{Y}_L = \frac{1}{Z_L} = \frac{1}{100 + j150} = 0.003 - j0.005$

2

d) Input impedance at 0.4λ away from the load is found by moving away from the load (Point P at $\frac{\Delta\phi'}{\lambda} = 0.195$) a distance 0.4 toward generator. In other words, we move $0.4 + 0.195 = 0.595$ toward generator. This is equivalent to $0.595 - 0.5 = 0.095$ on the chart circumference and is marked by line $O O'$. Line $O O'$ intersect the r -circle at R. We read the values for r & x at R $\Rightarrow$

$$Z_{in} = r_{in} + jX_{in} = 0.3 + j0.65 \Rightarrow$$

$$Z_{in} = 75(0.3 + j0.65) = 22.5 + j48.75 \text{ } [\Omega] \\ = 53.7 \angle 65.2^\circ$$

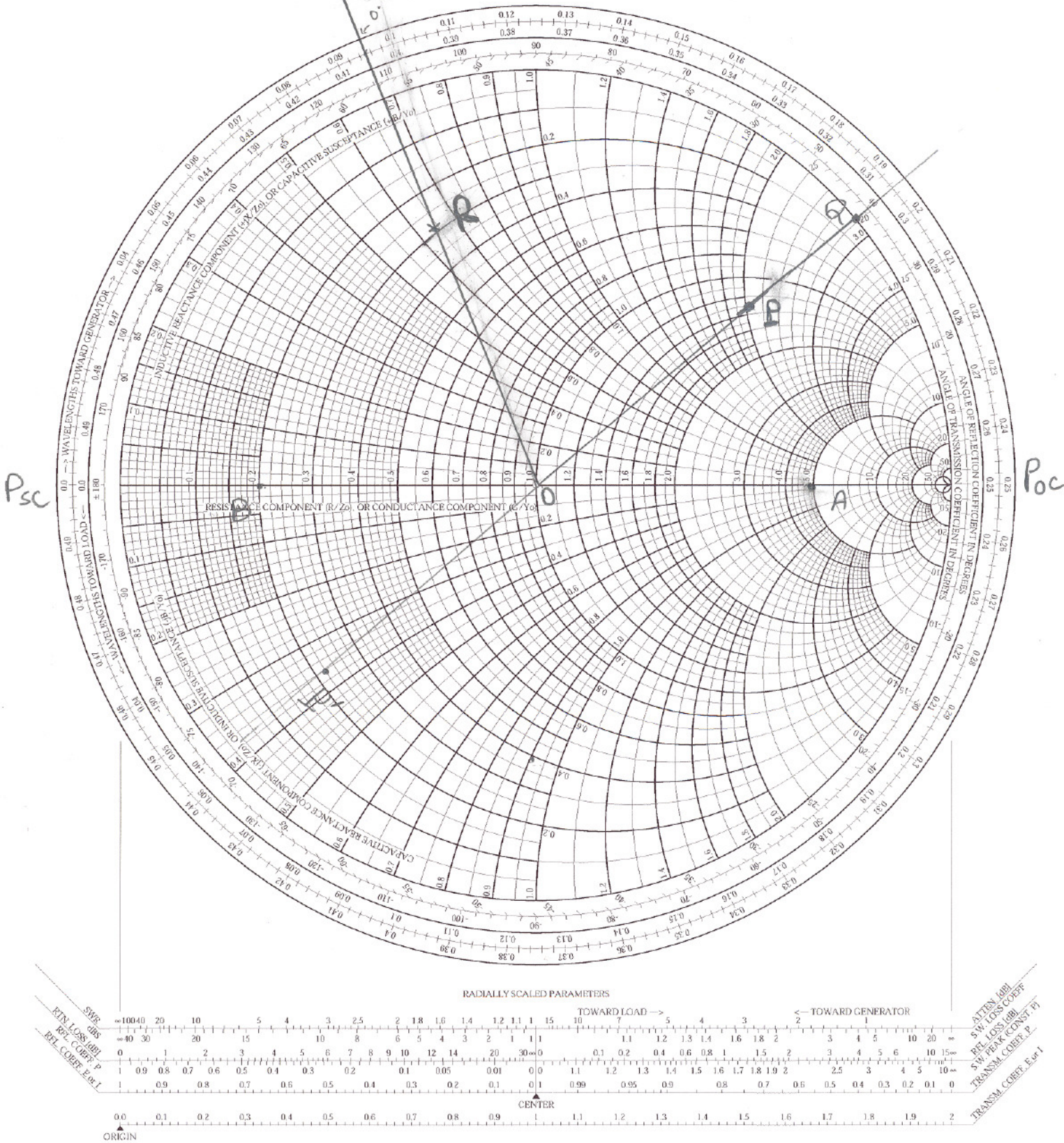
Test $Z = R_0 \frac{Z_L + jR_0 \tan(\beta l)}{R_0 + jZ_L \tan(\beta l)} = 75 \frac{(100 + j150) + j75 \tan(144^\circ)}{75 + j(100 + j150) \tan(144^\circ)}$

$$\beta l = \frac{2\pi}{\lambda} l = \frac{2\pi}{\lambda} 0.4\lambda = 0.8\pi = 144^\circ \\ = 21.97 + j47.6 \\ = 52.4 \angle 65.2$$

e) The first voltage maximum occurs at A (in going from P to R we cross the $O P_{oc}$ at A where the voltage is maximum.) From the chart circumference this is at $(0.25 - 0.195)\lambda$
 $= 0.055\lambda$ from the load

The Complete Smith Chart

Black Magic Design



prob 3,

* We want to show that solution to $-\nabla^2 \psi(r) = \phi(r)$ is given by $\psi(r) = \frac{1}{4\pi} \iiint \frac{\phi(r')}{|\vec{r} - \vec{r}'|} d^3r'$ (2)

In homework we have shown that $-\nabla^2 \frac{1}{R} = 4\pi \delta^3(\vec{R})$, (3)

where $R = |\vec{R}|$. Here, $\vec{R} = \vec{r} - \vec{r}'$ & hence $|\vec{R}| = R = |\vec{r} - \vec{r}'|$ (4)

In other words (3) $\Rightarrow$ $-\nabla^2 \frac{1}{|\vec{r} - \vec{r}'|} = 4\pi \delta^3(\vec{r} - \vec{r}') \quad (7)$

From (2) $\Rightarrow \nabla^2 \psi(r) = \nabla^2 \iiint \frac{\phi(r')}{4\pi |\vec{r} - \vec{r}'|} d^3r' = \iiint \nabla^2 \left(\frac{1}{4\pi |\vec{r} - \vec{r}'|} \right) \phi(r') d^3r'$ (8)

$= \iiint \phi(r') \nabla^2 \left(\frac{1}{4\pi |\vec{r} - \vec{r}'|} \right) d^3r' \quad \text{From (7)} \Rightarrow$

$\nabla^2 \psi(r) = \iiint \phi(r') (-1) \delta^3(\vec{r} - \vec{r}') d^3r' = -\phi(r) \Rightarrow$ (9)

(10) $\nabla^2 \psi(r) = -\phi(r)$, hence we see that if $\psi(r)$ is given by (2) it will satisfy (10) which is the Poisson equation