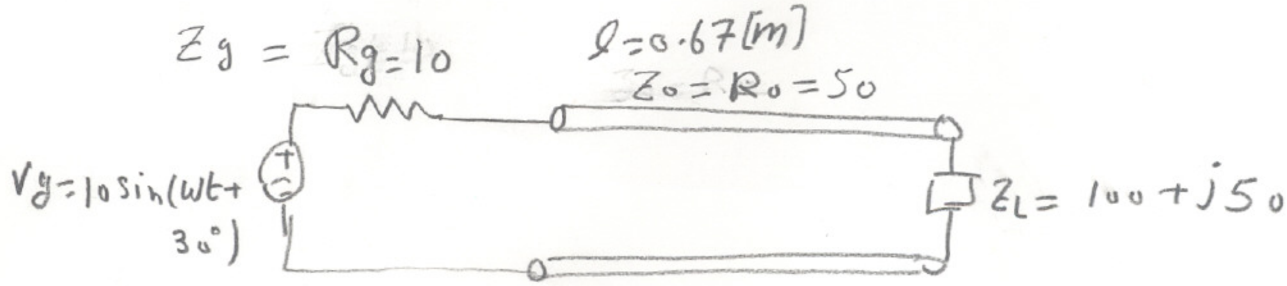
problem 1)



$\nu = 1.05$ GHz

$V_p = 0.7c$

① $V_p = \frac{\omega}{\beta} \Rightarrow$ ② $\beta = \frac{\omega}{V_p} = \frac{2\pi\nu}{0.7c} = \frac{2\pi \times 1.05 \times 10^9}{0.7 \times 3 \times 10^8} = 10\pi$

$V_g = 10\sin(\omega t + 30^\circ) = 10\cos(90^\circ - \omega t - 30^\circ) = 10\cos(60 - \omega t) =$

$10\cos(\omega t - 60) \Rightarrow$ ③ $\boxed{\tilde{V}_g = 10 \angle -60^\circ = 10 \angle -\pi/3}$

④ $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j50 - 50}{100 + j50 + 50} = \boxed{0.4 + j0.2 = 0.447\, e^{j26.565^\circ} = 0.447\, e^{j0.464 (R)}}$

⑤ $\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = \frac{10 - 50}{10 + 50} = \frac{-40}{60} = \boxed{-2/3 = -0.667}$

⑥ $V(z) = \frac{Z_0 V_g}{Z_0 + Z_g} e^{-\gamma z} \left(\frac{1 + \Gamma_L e^{-2\gamma z'}}{1 - \Gamma_g \Gamma_L e^{-2\gamma l}} \right) = \frac{R_0 V_g}{R_0 + R_g} e^{-j\beta z} \left(\frac{1 + \Gamma_L e^{-2j\beta z'}}{1 - \Gamma_L \Gamma_g e^{-2j\beta l}} \right)$

since ⑦ $z + z' = l \Rightarrow \boxed{z' = l - z}$ then

⑧ $V(z) = \frac{R_0 V_g}{R_0 + R_g} e^{-j\beta z} \left(\frac{1 + \Gamma_L e^{-2j\beta l} e^{+2j\beta z}}{1 - \Gamma_g \Gamma_L e^{-2j\beta l}} \right)$ OR Finally

Problem 1)

(9)

$$V(z) = V_g \frac{R_0}{R_0+R_g} \frac{\left(e^{-j\beta z} + \Gamma_L e^{-2j\beta l} e^{+j\beta z}\right)}{\left(1 - \Gamma_g \Gamma_L e^{-2j\beta l}\right)}$$

substitute the values for V_g, R_0, R_g, Γ_L & Γ_g we have

$$V(z) = 10 e^{-j\frac{\pi}{3}} \frac{50}{50+10} \frac{\left(e^{-j\beta z} + 0.447 e^{j0.464} e^{-j2\times 10\pi \times 0.67} e^{+j\beta z}\right)}{\left[1 - \left(\frac{-2}{3}\right) 0.447 e^{j0.464} e^{-j2\times 10\pi\times 0.67}\right]}$$

$$= 8.333 e^{-j\frac{\pi}{3}} \frac{\left[e^{-j\beta z} + 0.447 e^{-j41.633} e^{j\beta z}\right]}{\left[1 + 0.298 e^{-j41.633}\right]} \Rightarrow$$

$$\boxed{V(z) = 10.178 e^{-j1.309} \left[e^{-j\beta z} + 0.447 e^{-j41.633} e^{j\beta z}\right]}$$

$$\boxed{V(z) = 10.178 e^{-j1.309} e^{-j\beta z} + 4.55 e^{-j42.942} e^{j\beta z}}$$

OR

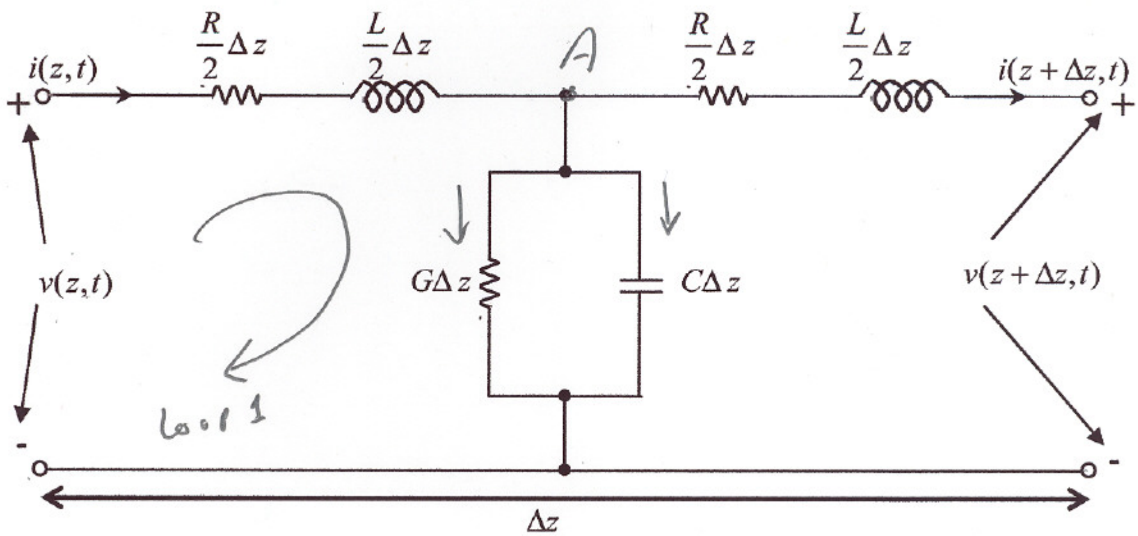
$$\boxed{V(z) = 10.178 e^{-j75^\circ} e^{-j\beta z} + 4.55 e^{-j300.4^\circ} e^{+j\beta z}}$$

$$\boxed{V(z,t) = 10.178 \cos(\omega t - \beta z - 75^\circ) + 4.55 \cos(\omega t + \beta z - 300.4^\circ)}$$

where $\beta = 10\pi z$ $\omega = 2\pi \times 1.05 \times 10^9 = 6.597 \times 10^9$ Hz

Problem 2)

Problem 2) A transmission line is modeled by an equivalent circuit shown below. For this transmission line find the general transmission line equations (the so called telegrapher's equation) in time (t) and space (z). Show all your work. **(Total point 33)**



KVL at loop 1

$$\Delta z \frac{R}{2} i(z,t) + \Delta z \frac{L}{2} \frac{\partial i(z,t)}{\partial t} + v(z+\frac{\Delta z}{2},t) = v(z,t)$$

$$\frac{v(z+\frac{\Delta z}{2},t) - v(z,t)}{\Delta z/2} = -R\,i(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\lim_{\frac{\Delta z}{2}\to 0} \left[\frac{v(z+\frac{\Delta z}{2},t) - v(z,t)}{\Delta z/2} \right] = \lim_{\frac{\Delta z}{2}\to 0} \left\{ -R\,i(z,t) - L\frac{\partial}{\partial t} i(z,t) \right\}$$

$$\boxed{\frac{\partial v(z,t)}{\partial z} = -R\,i(z,t) - L\frac{\partial}{\partial t} i(z,t)}$$

KCL at node A

(5) $$i(z,t) = G\Delta z\, v\left(z+\frac{\Delta z}{2},t\right) + C\Delta z \frac{\partial}{\partial t} v\left(z+\frac{\Delta z}{2},t\right) + i(z+\Delta z,t) \Rightarrow$$

(6) $$\frac{i(z,t)-i(z+\Delta z,t)}{\Delta z} = G v\left(z+\frac{\Delta z}{2},t\right) + C\frac{\partial}{\partial t} v\left(z+\frac{\Delta z}{2},t\right)$$

(7) $$\lim_{\Delta z\to 0} \frac{i(z,t)-i(z+\Delta z,t)}{\Delta z} = \lim_{\Delta z \to 0} \left[G v\left(z+\frac{\Delta z}{2},t\right) + C\frac{\partial}{\partial t} v\left(z+\frac{\Delta z}{2},t\right)\right]$$

(8) $$-\frac{\partial}{\partial z} i(z,t) = \lim_{\Delta z\to 0} \left[G v\left(z+\frac{\Delta z}{2},t\right) + C\frac{\partial}{\partial t} v\left(z+\frac{\Delta z}{2},t\right)\right],$$

but physically, $\lim_{\Delta z\to 0} v\left(z+\frac{\Delta z}{2},t\right) \neq v(z,t)$ since otherwise there will not be a voltage drop from input to point A.

Use $V\left(z+\frac{\Delta z}{2},t\right)$ from (1) in (8) $\Rightarrow$

$$-\frac{\partial}{\partial z} i(z,t) = \lim_{\Delta z\to 0}\left\{G v(z,t) - G\Delta z \frac{R}{2} i(z,t) - G\Delta z\frac{L}{2}\frac{\partial}{\partial t} i(z,t) + C\frac{\partial}{\partial t}\left[v(z,t) - \Delta z\frac{R}{2} i(z,t) - \Delta z \frac{L}{2}\frac{\partial}{\partial t} i(z,t)\right]\right\}$$

$$\Rightarrow -\frac{\partial}{\partial z} i(z,t) = \lim_{\Delta z\to 0}\left\{G v(z,t) - \cancelto{0}{G\Delta z\frac{R}{2} i(z,t)} - \cancelto{0}{G\Delta z\frac{L}{2}\frac{\partial}{\partial t} i(z,t)} + C\frac{\partial}{\partial t} v(z,t) - \cancelto{0}{C\frac{R}{2}\Delta z\frac{\partial}{\partial t} i(z,t)} - \cancelto{0}{C\frac{L}{2}\Delta z\frac{\partial^2}{\partial t^2} i(z,t)}\right\} \Rightarrow$$

$$\boxed{\frac{\partial i(z,t)}{\partial z} = -G v(z,t) - C\frac{\partial}{\partial t} v(z,t)}$$

problem 3)

a) For this configuration $\Gamma_L = \dfrac{V_0^-}{V_0^+}$ (1)

b) we start with

$$V = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad (2)$$

$$I = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z} \quad (3)$$

At load ($z = 0$) & $V_L = I_L Z_L$ & (4)

(5) $V_L = V_0^+ + V_0^-$

(6) $I_L = \dfrac{V_0^+}{Z_0} - \dfrac{V_0^-}{Z_0}$

From (4) $\Rightarrow$ (7) $Z_L = \dfrac{V_L}{I_4}$ use (5) & (6) in (7) $\Rightarrow$

(8) $$Z_L = \frac{V_0^+ + V_0^-}{\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}} = \frac{Z_0 (V_0^+ + V_0^-)}{V_0^+ - V_0^-} = \frac{Z_0 \cancel{V_0^+} (1 + V_0^-/V_0^+)}{\cancel{V_0^+} (1 - V_0^-/V_0^+)} \Rightarrow$$

(9) $$Z_L = \frac{Z_0 (1 + \Gamma_L)}{(1 - \Gamma_L)} \Rightarrow$$ (10) $$Z_L - Z_L \Gamma_L = Z_0 + Z_0 \Gamma_L \Rightarrow$$

$$Z_L - Z_0 = \Gamma_L (Z_L + Z_0) \Rightarrow$$ (11) $$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Eq (11) is the same we found in class with $z = 0$ at the source & $z = l$ at the load.