# Assignment #1

## ECE357/ECE320
## University of Toronto

1. Show that $e^{j\theta} = \cos\theta + j\sin\theta$.

2. The current through a circuit is found to be $\vec{I} = 6\angle\text{-}40°$. What is the expression for the time dependent current with $\upsilon = 60$ Hz?

3. Prove that $\text{Re}\{A\} = \dfrac{A + A^*}{2}$, where $A \in C^2$.

4. Prove that for a sinusoidal function $a(z,t)$ where $z$ is the space coordinate and $t$ is the time, there exists a phasor $\vec{A}(z)$ such that

   a. $\dfrac{\partial}{\partial t}a(z,t) = \dfrac{\partial}{\partial t}\text{Re}\left\{\vec{A}(z)e^{j\omega t}\right\}$

   b. $\dfrac{\partial}{\partial t}a(z,t) = \text{Re}\left\{j\omega\vec{A}(z)e^{j\omega t}\right\}$.

5. Show that $\dfrac{\partial^2}{\partial t^2}a(z,t) = \text{Re}\left\{-\omega^2\vec{A}(z)e^{j\omega t}\right\}$.

6. Prove that if $\text{Re}\left\{\vec{A}(z)e^{j\omega t}\right\} = \text{Re}\left\{\vec{B}(z)e^{j\omega t}\right\}$, then $\vec{A}(z) = \vec{B}(z)$. This means that the $\text{Re}\{\ \}$ operator can be removed on phasors of the same frequency.

7. Show that $\text{Re}\left\{\sum_{n=1}^{M}\vec{A_n}(z)\right\} = \sum_{n=1}^{M}\text{Re}\left\{\vec{A_n}(z)\right\}$, i.e. that the real part of the sum of phasors is the sum of the real parts.

8. Show that $\text{Re}\left\{\vec{A}(z)\right\}\text{Re}\left\{\vec{B}(z)\right\} = \dfrac{\text{Re}\left\{\vec{A}(z)\vec{B}(z)\right\}}{2} + \dfrac{\text{Re}\left\{\vec{A}(z)\vec{B}^*(z)\right\}}{2}$.

9. Show that $\left\langle \text{Re}\left\{\vec{A}(z)e^{j\omega t}\right\}\text{Re}\left\{\vec{B}(z)e^{j\omega t}\right\}\right\rangle_{timeavg} = \dfrac{1}{2}\text{Re}\left\{\vec{A}(z)\vec{B}^*(z)\right\}$.

10. The voltage across and current through a device are given by $v(t) = V_m\cos(\omega t + \theta_v)$ and $i(t) = I_m\cos(\omega t + \theta_i)$. What is the average power dissipated in the device?

11. A voltage source is connected to resistor R as shown in Fig. 1. The source produces the voltage waveform shown if Fig. 2.

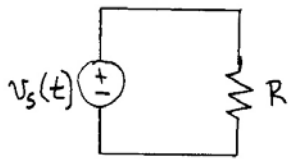


Figure 1.

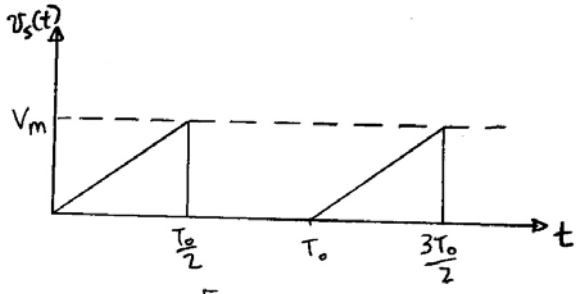


Figure 2.

a. Find the instantaneous power delivered to R.

b. What is the average power delivered to R?

12. Show that the RMS (or effective value) for a sinusoidal voltage $v(t) = V_m \cos(\omega t + \theta)$ is $\frac{V_m}{\sqrt{2}}$.