Problem Set #5
ECE357 /ECE320
University of Toronto

1) A vector field $\vec{A} = \hat{a}_p(3\cos\phi) - \hat{a}_\phi 2\rho + 5\hat{a}_z$ is expressed in cylindrical coordinates. Find its representation in rectangular coordinate system.

2) Consider the central force, $\vec{F} = \frac{1}{r^2}\hat{a}_r$. Calculate the work done in presence of the above field in moving from $P_1(1,1)$ to $P_2(2,1)$ along the line P_1P_2.

3)

   a. Let $\vec{r}$ be the position vector, pointing from origin to the point of observation P. Show that $\nabla|\vec{r}| = \nabla r = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r} = \hat{a}_r$ where $\hat{a}_r$ is the unit normal in the direction of $\vec{r}$.

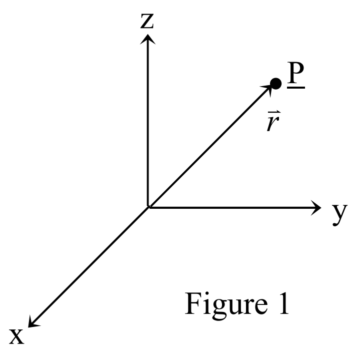

Figure 1

   b. Function $g(r)$ depends only on the distance between the source point (primed coordinates) and the observation point (unprimed coordinates), see Fig. 2. Prove that $\nabla g(r) = \nabla f(R) = \hat{a}_R \frac{\partial f(R)}{\partial R}$. Use this information to calculate the gradient of $g(x, y, z) = f(R) = \sin\left(\pi \frac{R^2}{4}\right)$.

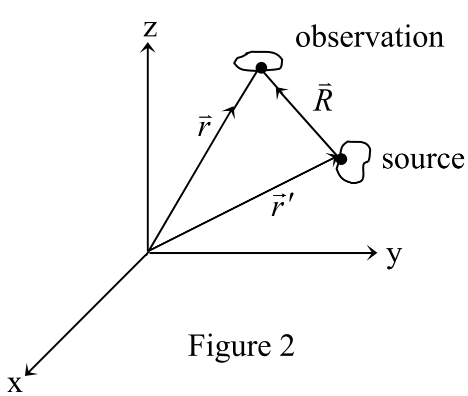

Figure 2

4) Consider a plane wave $\vec{E} = \vec{E}_o e^{-j\vec{k}\cdot\vec{r}+j\omega t}$ propagating in a homogeneous, lossless, source free region for which $\varepsilon > 0$ and $\mu > 0$ and $\vec{E}_o$ is constant.

   a. Show that $\vec{K} \perp \vec{E}$ and $\vec{K} \perp \vec{H}$.
   b. Show that $\vec{K}, \vec{E},$ and $\vec{H}$ form a right hand triplet.
   c. Now suppose in a medium both permittivity and permeability are negative, what is the relation between $\vec{K}, \vec{E},$ and $\vec{H}$. Has the direction of the Poynting vector changed from the case considered in (b).

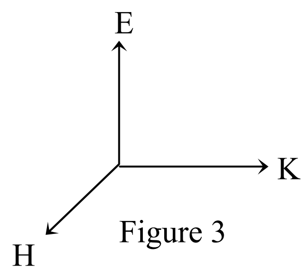

Figure 3

5) Starting with the differential form of the Maxwell's equations, find their integral form formulation.