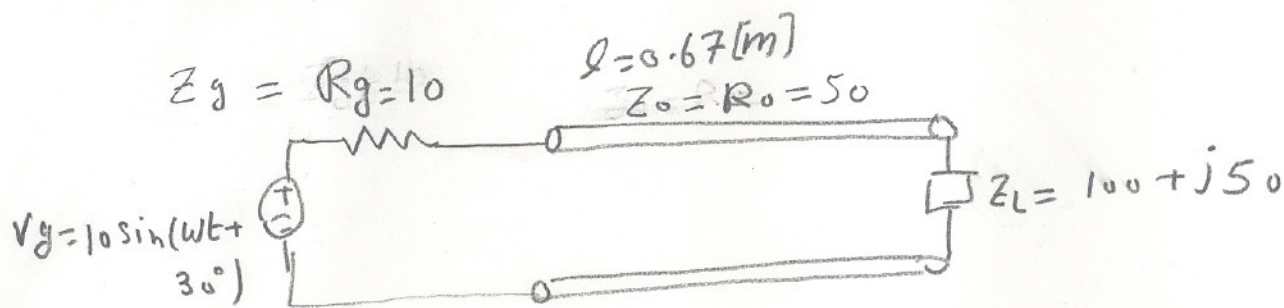


problem 1)



$$v = 1.05 \text{ GHz}$$

$$V_p = 0.7 \text{ V}$$

$$\textcircled{1} V_p = \frac{\omega}{\beta} \Rightarrow \beta = \frac{\omega}{V_p} = \frac{2\pi v}{0.7 \text{ V}} = \frac{2\pi \times 1.05 \times 10^9}{0.7 \times 3 \times 10^8} = 10 \text{ rad/m}$$

$$V_g = 10 \sin(\omega t + 30^\circ) = 10 \cos(90^\circ - \omega t - 30^\circ) = 10 \cos(60 - \omega t) =$$

$$10 \cos(\omega t - 60^\circ) \Rightarrow \textcircled{3} \boxed{\tilde{V}_g = 10 \angle -60^\circ = 10 \angle -\pi/3}$$

$$\textcircled{4} \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j50 - 50}{100 + j50 + 50} = \frac{50 + j50}{150 + j50} = \frac{0.4 + j0.2}{0.4 + j0.2} = 0.447 e^{j26.565^\circ}$$

$$\textcircled{5} \Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = \frac{10 - 50}{10 + 50} = \frac{-40}{60} = -2/3 = -0.667$$

$$\textcircled{6} V(z) = \frac{Z_0 V_g}{Z_0 + Z_g} e^{-\gamma z} \left(\frac{1 + \Gamma_L e^{-2\gamma z'}}{1 - \Gamma_g \Gamma_L e^{-2\gamma l}} \right) = \frac{R_0 V_g}{R_0 + R_g} e^{-j\beta z} \left(\frac{1 + \Gamma_L e^{-2j\beta z'}}{1 - \Gamma_g \Gamma_L e^{-2j\beta l}} \right)$$

$$\text{since } \textcircled{7} z + z' = l \Rightarrow \boxed{z' = l - z} \text{ then}$$

$$\textcircled{8} V(z) = \frac{R_0 V_g}{R_0 + R_g} e^{-j\beta z} \left(\frac{1 + \Gamma_L e^{-2j\beta l + 2j\beta z}}{1 - \Gamma_g \Gamma_L e^{-2j\beta l}} \right) \text{ OR Finally}$$

Problem 1)

(9)

$$V(z) = V_g \frac{R_o}{R_o + R_g} \frac{(e^{-j\beta z} + \Gamma_L e^{-2j\beta L} + j\beta z)}{(1 - \Gamma_g \Gamma_L e^{2j\beta L})}$$

substitute the values for V_g , R_o , R_g , Γ_L & Γ_g we have

$$V(z) = 10 e^{-j\frac{\pi}{3}} \frac{50}{50 + 10} \frac{(e^{-j\beta z} + 0.447 e^{j0.464} e^{-j2 \times 10\pi \times 0.67} + j\beta z)}{[1 - (-\frac{2}{3}) 0.447 e^{j0.464} e^{-j2 \times 10\pi \times 0.67}]}$$

$$= 8.333 e^{-j\frac{\pi}{3}} \frac{[e^{-j\beta z} + 0.447 e^{-j41.633} e^{j\beta z}]}{[1 + 0.298 e^{-j41.633}]} \Rightarrow$$

$$V(z) = 10.178 e^{-j1.309} [e^{-j\beta z} + 0.447 e^{-j41.633} e^{j\beta z}]$$

$$V(z) = 10.178 e^{-j1.309} e^{-j\beta z} + 4.55 e^{-j42.942} e^{j\beta z} \quad \text{OR}$$

$$V(z) = 10.178 e^{-j75^\circ} e^{-j\beta z} + 4.55 e^{-j300.4^\circ} e^{j\beta z}$$

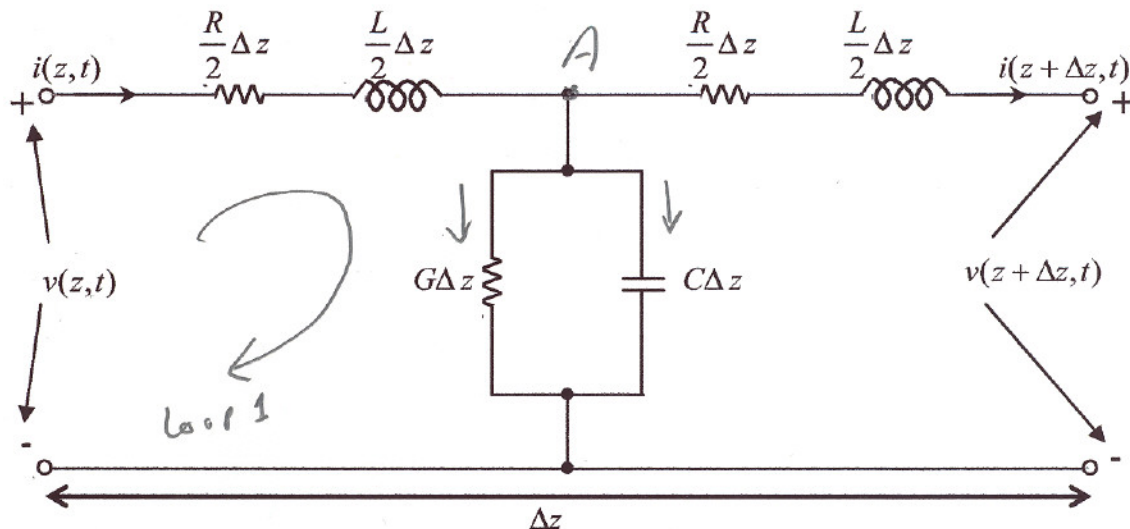
$$V(z, t) = 10.178 \cos(\omega t - \beta z - 75^\circ) + 4.55 \cos(\omega t + \beta z - 300.4^\circ)$$

where $\beta = 10 \pi \text{ rad/m}$ $\omega = 2\pi \times 1.05 \times 10^9 = 6.597 \times 10^9 \text{ Hz}$

Problem 2)

1

Problem 2) A transmission line is modeled by an equivalent circuit shown below. For this transmission line find the general transmission line equations (the so called telegrapher's equation) in time (t) and space (z). Show all your work. (Total point 33)



KVL at loop 1

$$\Delta z \frac{R}{2} i(z, t) + \Delta z \frac{L}{2} \frac{\partial i(z, t)}{\partial t} + v(z + \frac{\Delta z}{2}, t) - v(z, t) = 0$$

$$\frac{v(z + \frac{\Delta z}{2}, t) - v(z, t)}{\Delta z/2} = -R i(z, t) - L \frac{\partial i(z, t)}{\partial t}$$

$$\lim_{\Delta z \rightarrow 0} \left[\frac{v(z + \frac{\Delta z}{2}, t) - v(z, t)}{\Delta z/2} \right] = \lim_{\Delta z \rightarrow 0} \left\{ -R i(z, t) - L \frac{\partial i(z, t)}{\partial t} \right\}$$

$\Delta z \rightarrow 0$

$$\boxed{\frac{\partial v(z, t)}{\partial z} = -R i(z, t) - L \frac{\partial i(z, t)}{\partial t}}$$

KCL at node A

$$i(z, t) = G \Delta z v(z + \frac{\Delta z}{2}, t) + C \Delta z \frac{\partial}{\partial t} v(z + \frac{\Delta z}{2}, t) \\ + i(z + \Delta z, t) \Rightarrow$$

$$- G v(z + \frac{\Delta z}{2}, t) - C \frac{\partial}{\partial t} v(z + \frac{\Delta z}{2}, t)$$

$$= \frac{i(z + \Delta z, t) - i(z, t)}{\Delta z}$$

$$\lim_{\Delta z \rightarrow 0} \left[- G v(z + \frac{\Delta z}{2}, t) - C \frac{\partial}{\partial t} v(z + \frac{\Delta z}{2}, t) \right] =$$

$$\lim_{\Delta z \rightarrow 0} \frac{i(z + \Delta z, t) - i(z, t)}{\Delta z} \Rightarrow$$

$$\boxed{\frac{\partial i(z, t)}{\partial z} = - G v(z, t) - C \frac{\partial}{\partial t} v(z, t)}$$

problem 3)

a) For this configuration

$$\Gamma_L = \frac{V_0^-}{V_0^+} \quad (1)$$

b) We start with

$$V = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad (2)$$

$$I = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z} \quad (3)$$

At load ($z=0$) & $V_L = I_L Z_L$ & (4)

$$(5) \quad V_L = V_0^+ + V_0^-$$

$$(6) \quad I_L = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

From (4) $\Rightarrow Z_L = \frac{V_L}{I_L}$ use (5) & (6) in (7) $\Rightarrow$

$$(8) \quad Z_L = \frac{V_0^+ + V_0^-}{\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}} = \frac{Z_0 (V_0^+ + V_0^-)}{V_0^+ - V_0^-} = \frac{Z_0 V_0^+ (1 + V_0^-/V_0^+)}{V_0^+ (1 - V_0^-/V_0^+)} \Rightarrow$$

$$(9) \quad Z_L = \frac{Z_0 (1 + \Gamma_L)}{(1 - \Gamma_L)} \Rightarrow (10) \quad Z_L - Z_L \Gamma_L = Z_0 + Z_0 \Gamma_L \Rightarrow$$

$$Z_L - Z_0 = \Gamma_L (Z_L + Z_0) \Rightarrow (11) \quad \boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Eq (11) is the same we found in class with $z=0$ at the source & $z=l$ at the load.