Problem 1)

a) ① $\nabla \times \vec{E}(\vec{r},t) = -\frac{\partial \mu \vec{H}}{\partial t} - \vec{M_i}$

② $\nabla \times \vec{H}(\vec{r},t) = \sigma \vec{E} + \frac{\partial}{\partial t} \varepsilon \vec{E} + \vec{J_i}$ $\qquad\qquad \vec{J_c} = \sigma \vec{E}$

③ $\nabla \cdot \varepsilon \vec{E}(\vec{r},t) = \rho_{ev} \Rightarrow \nabla \cdot \vec{E} = \frac{\rho_{ev}}{\varepsilon}$

④ $\nabla \cdot \mu \vec{H}(\vec{r},t) = \rho_{mv} \Rightarrow \nabla \cdot \vec{H} = \frac{\rho_{mv}}{\mu}$

from (2) ⑤ $\nabla \times \nabla \times \vec{H} = \sigma \nabla \times \vec{E} + \varepsilon \frac{\partial}{\partial t} \nabla \times \vec{E} + \nabla \times \vec{J_i} \Rightarrow$

⑥ $\nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = \sigma\left[-\frac{\partial}{\partial t} \mu \vec{H} - M_i\right] + \varepsilon \frac{\partial}{\partial t}\left[-\frac{\partial}{\partial t} \mu \vec{H} - \vec{M_i}\right] +$

$\nabla \times \vec{J_i}$

using (4) in (6)

$$\nabla\left(\frac{\rho_{mv}}{\mu}\right) - \nabla^2 \vec{H} = -\mu\sigma \frac{\partial}{\partial t} \vec{H} - \sigma \vec{M_i} - \varepsilon\mu \frac{\partial^2}{\partial t^2} \vec{H} - \varepsilon \frac{\partial}{\partial t} \vec{M_i} + \nabla \times \vec{J_i} \Rightarrow$$

$$\boxed{\frac{1}{\mu} \nabla(\rho_{mv}) + \sigma \vec{M_i} - \nabla \times \vec{J_i} + \varepsilon \frac{\partial}{\partial t} \vec{M_i} + \mu\sigma \frac{\partial}{\partial t} \vec{H} + \varepsilon\mu \frac{\partial^2}{\partial t^2} \vec{H} = \nabla^2 \vec{H}}$$

where $\vec{H} = \vec{H}(r,t)$, $\vec{M_i} = M_i(\vec{r},t)$, $\vec{J_i} = J_i(\vec{r},t)$ & $\rho_{mv} = \rho_{mv}(\vec{r},t)$

b) $$\boxed{\frac{1}{\mu} \nabla(\rho_{mv}) + \sigma \vec{M_i} - \nabla \times \vec{J_i} + j\omega\varepsilon \vec{M_i} + j\omega\mu\sigma \vec{H} - \omega^2 \varepsilon\mu \vec{H} = \nabla^2 \vec{H}}$$

where $\rho_{mv} = \rho_{mv}(\vec{r})$, $\vec{M_i} = M_i(\vec{r})$, $\vec{J_i} = \vec{J_i}(\vec{r})$, $\vec{H} = \vec{H}(\vec{r})$

Problem 2)

1

a) $z_\ell = \frac{100 + j150}{75} = 1.33 + j2 = r + jx$ (normalized load)

This is point P on the smith chart. We draw a circle centered at O, with radius OP.

* Measuring the length $OP = 2.1''$ & $OQ = 3.2''$ we get

$$\Gamma = \frac{\overline{OP}}{\overline{OQ}} = \frac{2.1''}{3.2''} = 0.66$$

* The θ_Γ is found from $\theta_\Gamma = (0.25 - 0.195) \times 4\pi$

$$= 0.69 \text{ (rad)} = 39.6° \simeq 40°$$

check $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j150 - 75}{100 + j150 + 75} = 0.506 + j0.424 = 0.66 \angle 39.9°$

b) The circle with radius OP cuts the $x = 0$ line at A & B points. The r-circle going through A-point is the standing wave ratio. This is $r = 5$ circle $\Rightarrow$

$$S' = 5$$

Check: $S = \frac{|\Gamma| + 1}{1 - |\Gamma|} = \frac{0.66 + 1}{1 - 0.66} = 4.88 \simeq 5$

c) The load addmitance is obtained by locating point P', diametrically across point P on the Γ circle (circle of radius OP centered at O). Reading this value on the chart we have

$y = g + jb = 0.22 - j0.36 \Rightarrow Y = Y_0 y = \frac{1}{75}(0.22 - j0.36)$

$$= 0.003 - j0.005 \; \left[\frac{1}{\Omega}\right]$$

Check: $Y_L = \frac{1}{Z_L} = \frac{1}{100+j150} = 0.003 - j0.005$

d) Input impedance at 0.4λ away from the load is found by moving away from the load (Point P at $\frac{\Delta z'}{\lambda} = 0.195$) a distance 0.4 toward generator. In other words, we move $0.4+0.195=0.595$ toward generator. This is equivalent to $0.595-0.5=0.095$ on the chart circumference and is marked by line O O'. Line O O' intersect the Γ-circle at R. We read the values for r & x at R ⇒

$$z_{in} = r_{in} + jx_{in} = 0.3 + j0.65 \Rightarrow$$

$$Z_{in} = 75(0.3+j0.65) = 22.5 + j48.75 \ [\Omega]$$
$$= 53.7\angle 65.2^\circ$$

Test $Z = R_0 \frac{Z_L + jR_0 \tan(\beta l)}{R_0 + jZ_L \tan(\beta l)} = 75 \frac{(100+j150)+j75\tan(144^\circ)}{75+j(100+j150)\tan(144^\circ)}$

$\beta l = \frac{2\pi}{\lambda} l = \frac{2\pi}{\lambda} 0.4\lambda = 0.8\pi = 144^\circ$ $= 21.97 + j47.6$

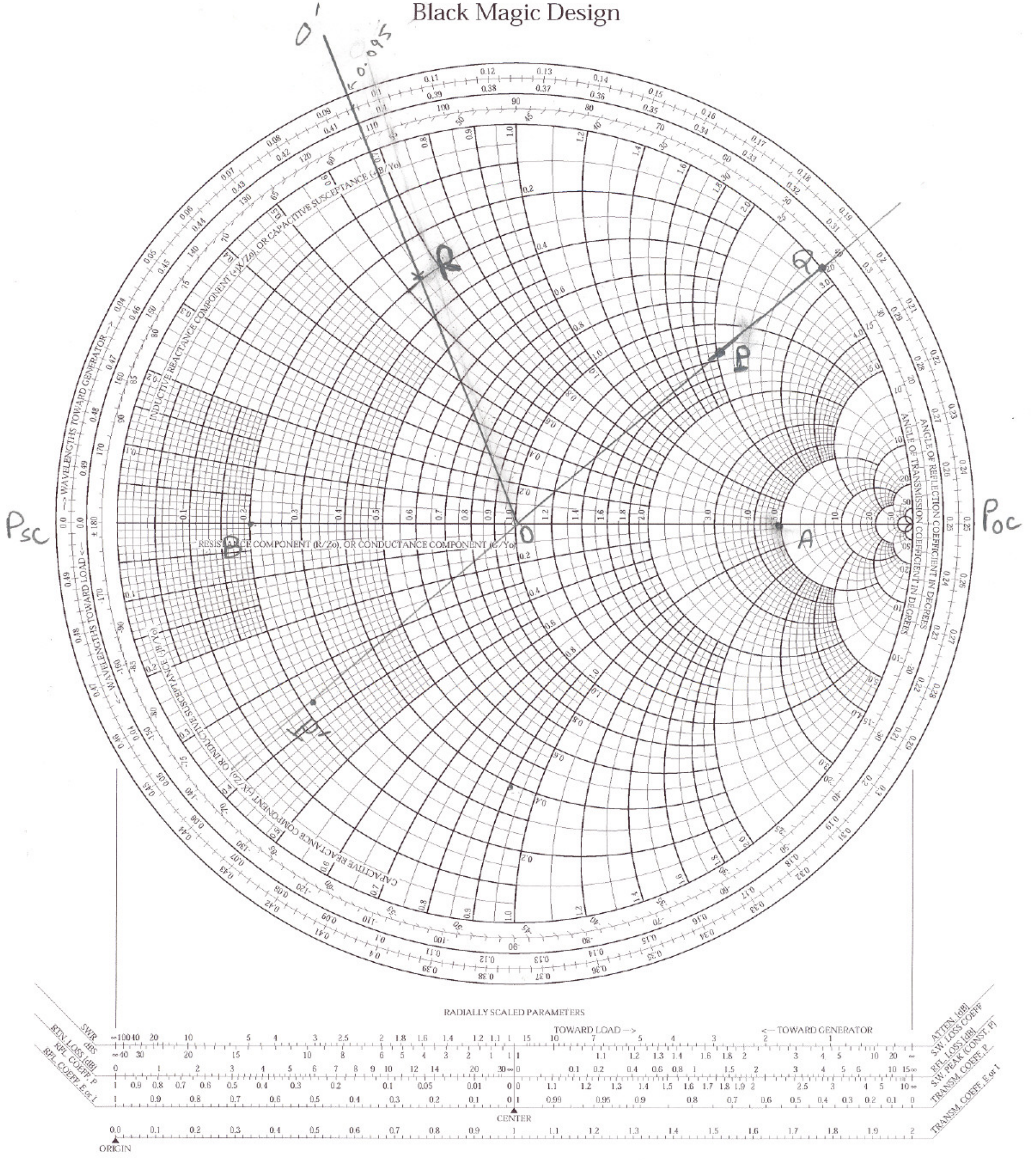
$= 52.4\angle 65.2$

e) The first voltage maximum occurs at A (in going from P to R we cross the OP_{oc} at A where the voltage is maximum.) From the chart circumference this is at $(0.25-0.195)\lambda$

$= 0.055\lambda$ from the load

The Complete Smith Chart

Black Magic Design



prob 3;

*we want to show that solution to $-\nabla^2\psi(r)=\phi(r)$ (1) is given by $\psi(r)=\frac{1}{4\pi}\iiint \frac{\phi(r')}{|\vec{r}-\vec{r}'|}\,dr'^3$. (2)

In homework we have shown that $-\nabla^2\frac{1}{R}=4\pi\,\delta^3(\vec{R})$, (3)

where $R=|\vec{R}|$ (4). Here, $\vec{R}=\vec{r}-\vec{r}'$ (5) & hence $|\vec{R}|=R=|\vec{r}-\vec{r}'|$ (6)

in otherwords (3) $\Rightarrow$ $\boxed{-\nabla^2\frac{1}{|\vec{r}-\vec{r}'|}=4\pi\,\delta^3(\vec{r}-\vec{r}')}$ (7)

(8)

From (2) $\Rightarrow \nabla^2\psi(r)=\nabla^2\iiint\frac{\phi(r')}{4\pi|\vec{r}-\vec{r}'|}\,dr'^3=\iiint\nabla^2\left(\frac{1}{4\pi}\frac{\phi(r')}{|\vec{r}-\vec{r}'|}\right)dr'^3$

$=\iiint\phi(r')\,\nabla^2\left(\frac{1}{4\pi}\frac{1}{|\vec{r}-\vec{r}'|}\right)dr'^3$ From (7) $\Rightarrow$

(9) $\nabla^2\psi(r)=\iiint\phi(r')\,(-1)\,\delta^3(\vec{r}-\vec{r}')\,dr'^3$

$=-\phi(r)\Rightarrow$

(10) $\nabla^2\psi(r)=-\phi(r)$, hence we see that if $\psi(r)$ is given by (2) it will satisfy (10) which is the Poisson equation