Question 1:

A lossless transmission line is 3.3λ long. The standing wave ratio on the line was measured to be 3 and the location of the first voltage maximum was 0.11λ from the load. In the following **use the Smith chart** to do your calculations and find the answers.

a) Locate and give the value for the normalized load impedance on the Smith chart (describe your procedure). [10 pts]

b) Locate and give the value for the line input impedance on the Smith chart (describe your procedure). [7 pts]

c) Locate and give the value for the line input admittance on the Smith chart (describe your procedure). [3 pts]

* $S = 3$, we draw a circle centered at O & tangential to $r = 3$ circle. This circle intersects the $x = 0$ circle at A. The radius of the circle drawn, i.e circle at A. The radius of the $r = 0$ circle (OA') is OA divided by the radius of the $|\Gamma|$, i.e

$$|\Gamma| = \frac{OA}{OA'} = \frac{4}{8} = 1/2$$

Check: $|\Gamma| = \frac{S-1}{1+S} = \frac{3-1}{1+3} = 1/2$

· To find θ_r, we move on the Perimeter of the Simth chart from the Point marked A' a distance 0.11 toward the load. This point is marked as B. since A' is at 0.25 & we move 0.11 then B is at $\boxed{0.25 - 0.11 = 0.14}$

* At Point B we read the angle (θ_r) from the chart. This is about 79°. Then

$\boxed{\Gamma = 0.5e^{j79^\circ}}$ $|\Gamma| = 0.5$ $\boxed{\Gamma = 0.094 + j\,0.491}$

Smith's chart $\theta_r = 79^\circ$

Check: The maximum voltage condition is given by

$$\theta_r - 2B z_M' = -2m\pi \qquad m = 0, \pm 1, \pm 2$$

or

$$\theta_r = -2m\pi + 2B z_M' = -2m\pi + \frac{4\pi}{\lambda} z_M'$$

For first MAX $m = 0 \Rightarrow \theta_r = \frac{4\pi z_M'}{\lambda} = 4\pi \times \frac{0.11\lambda}{\lambda} \Rightarrow$

$\boxed{\theta_r = 0.44\pi = 79.2^\circ} \Rightarrow \boxed{\Gamma = 0.5e^{j79.2^\circ}}$ calculated

* The line oB intersects the $|\Gamma| = 0.5$ circle at B'. The values of r & x associated with B' are the normalized load impedance. From the chart they are

$r = 0.7$ & $x = 0.95$

then

$\boxed{z_l = 0.7 + j0.95}$ Smith's chart

Check: $\Gamma = \frac{z_l - 1}{z_l + 1} \Rightarrow z_l = \frac{1+\Gamma}{1-\Gamma} = \frac{1+0.5e^{j79.2^\circ}}{1-0.5e^{j79.2^\circ}}$

$$\Rightarrow \boxed{z_l = 0.706 + j0.924 = 1.163\, e^{j52.6^\circ}}$$ Calculated

b) To find input impedance we move from B (@0.14) a distance equal to the line length (3.3 λ) toward the generator. This means that as measured from $\frac{z'}{\lambda} = 0$ (the P_{sc}) we move $3.3 + 0.14 = 3.44 = 6 \times 0.5 + 0.44$.

That is to say, go around the smith chart 6 times and then add 0.44. This will put us at point C''. The line OC'', intersects the $|\Gamma|$ circle at C. Read the r & x at C & we have $\boxed{z_i = 0.37 - j0.35}$ Smith chart

Check: $Z_i = R_o \frac{Z_L + jR_o \tan \beta l}{R_o + jZ_L \tan \beta l} \Rightarrow$

$z_i = \frac{z_l + j\tan \beta l}{1 + j z_l \tan \beta l}$ & $\beta l = \frac{2\pi}{\lambda} l = 2\pi \frac{3.3\lambda}{\lambda} \Rightarrow$

$$\boxed{\beta l = 6.6\pi}$$

$$\boxed{z_i = \frac{(0.706+j0.924)+j\tan(6.6\pi)}{1+j(0.706+j0.924)\tan(6.6\pi)} = 0.379-j0.346 = 0.513\,e^{-j42.^{\circ}4}}$$

calculated

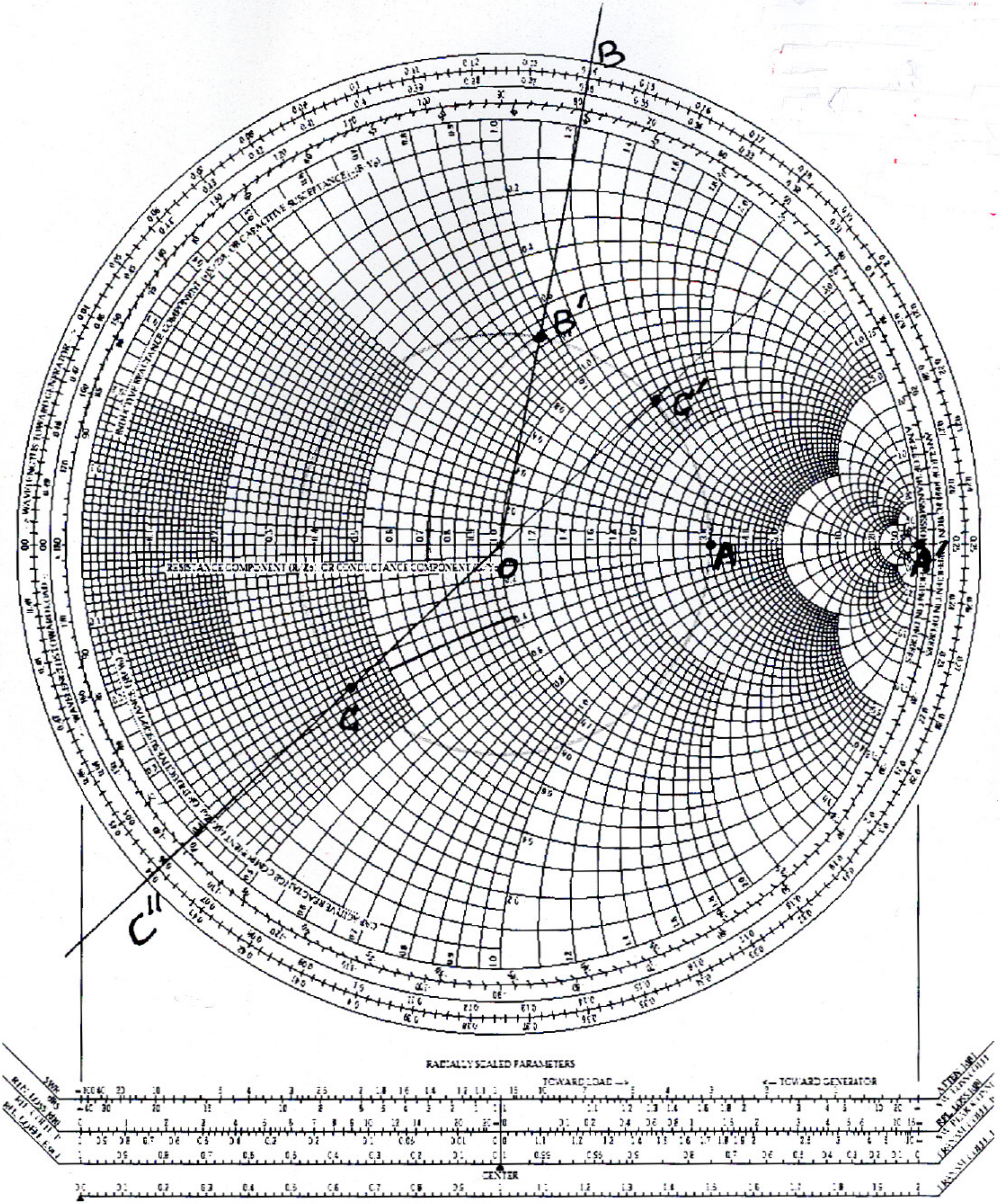
c) to find y_i we find the point c', diametrically across C. We read x & r for this point c'

At c' $r=1.5$ & $x=1.3$ $\Rightarrow$ $\boxed{y_i = 1.5+j1.3}$

Smithchart

Check: from $z_i = 0.379 - j0.346 \Rightarrow$

$$\boxed{y_i = \frac{1}{z_i} = 1.439+j1.314}$$ calculated


B
B'
C'
A
A'
O
C
C''
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
RADIALLY SCALED PARAMETERS
TOWARD LOAD →
← TOWARD GENERATOR
CENTER

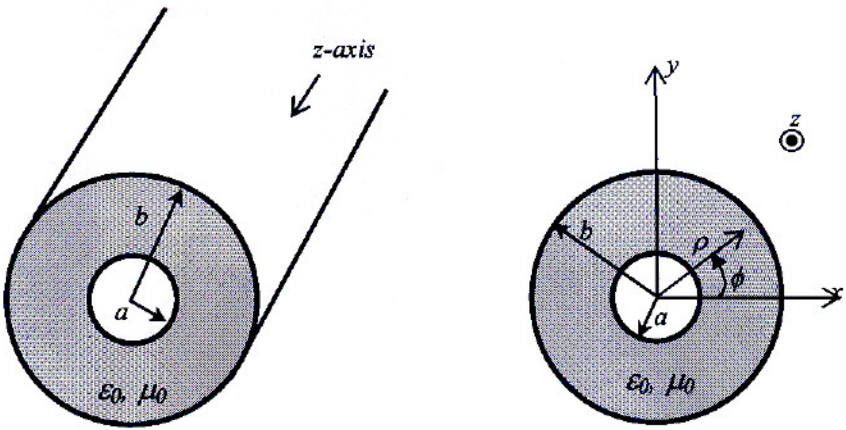
Question 2:

Consider a coaxial line shown in the figure below where the conductors are at $\rho=a$ and $\rho=b$ and the material between the two conductors is vacuum.

a) Assuming a TEM wave propagation along the z-axis, show that the magnetic field has the following from: $\vec{H} = H_\phi(\rho, z)\,\hat{a}_\phi = \dfrac{g(z)}{\rho}\hat{a}_\phi$, where $g(z)$ is a non-zero scalar function of z only. [15 pts]

b) If magnetic field has the form given in part (a), what is the direction of the electric field? Justify your answer [5 pts]

Hint: Because of the cylindrical symmetry you can assume that $\vec{E}$ and $\vec{H}$ are independent of ϕ, i.e., $\dfrac{\partial}{\partial \phi} = 0$



Coaxial Cable

Coaxial Cable End View

Question 2)

a) we start with Ampere's law

$\nabla \times \vec{H} = j\omega\varepsilon_0 \vec{E} \Rightarrow$ (1)

(2) $\left[\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right]\hat{a}_\rho + \left[\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right]\hat{a}_\phi + \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho H_\phi) - \frac{\partial}{\partial \phi} H_\rho\right]\hat{a}_z = j\omega\varepsilon_0\left[E_\rho \hat{a}_\rho + E_\phi \hat{a}_\phi + E_z \hat{a}_z\right]$

since we have a TEM with respect to z-axis then

(3) $H_z = E_z = 0$ also from cylindrical symmetry

(4) $\frac{\partial}{\partial \phi} = 0$ then (2) $\Rightarrow$

(5) $-\frac{\partial H_\phi}{\partial z}\hat{a}_\rho + \frac{\partial H_\rho}{\partial z}\hat{a}_\phi + \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho H_\phi)\hat{a}_z = j\omega\varepsilon_0\left[E_\rho \hat{a}_\rho + E_\phi \hat{a}_\phi\right]$

From (5) it is clear that

(6) $\frac{1}{\rho}\frac{\partial}{\partial \rho}\rho H_\phi = 0$ where $H_\phi = H_\phi(\rho, z)$

For $\frac{\partial}{\partial \rho}\rho H_\phi(\rho,z) = 0$ we must have

(7) $H_\phi(\rho, z) = \frac{g(z)}{\rho}$ where $g(z)$ is a scaler function of z

then, In general $\vec{H} = H\rho(\rho,z)\hat{a\rho} + H\phi(\rho,\phi)\hat{a\phi}$

but for (7) to be true & for us to have a TEM then

$H\rho(\rho,z)$ must be zero. $\Rightarrow$

(8) $$\boxed{\vec{H} = H\phi(\rho,z)\hat{a\phi} = \frac{g(z)}{\rho}\hat{a\phi}}$$

b) For $\vec{H}$ given by (8) & TEM along z-axis, then

$\vec{E}$ must point along $\hat{a\rho}$ $\Rightarrow$

$$\vec{E} = E_\rho(\rho,z)\hat{a\rho}$$

Question 3:

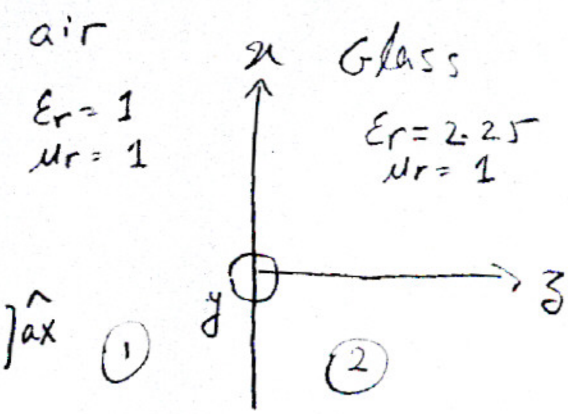
A linearly polarized beam of yellow light with wavelength of 0.6 μm is normally incident in air upon a glass surface. The glass surface is situated at $z=0$ and is infinite in the x- and y- directions. The relative permittivity of the glass is 2.25 and its relative permeability is 1, and it can be considered as a perfect dielectric. Calculate the followings:

a) The location of the first maximum for the electric filed magnitude in the air side ($z<0$). Show all your work. (8 pts)

b) The standing wave ratio. (4 pts)

c) What percentage of the incident time-averaged power is transmitted into the glass? Show all your work. (8 pts)

* The locations of $|E|_{max}$ or $|E|_{min}$ are found from the followings.



$$E_1 = \vec{E_i} + \vec{E_r} = [E_{i0}\, e^{-j\beta_1 z} + \Gamma E_{i0}\, e^{+j\beta_1 z}]\hat{a}_x$$

$$\Rightarrow |\vec{E_1}|^2 = \vec{E_1}\cdot\vec{E_1}^* = (E_{i0}\, e^{-j\beta_1 z} + \Gamma E_{i0}\, e^{+j\beta_1 z})(E_{i0}^*\, e^{+j\beta_1 z} + \Gamma^* E_{i0}^*\, e^{-j\beta_1 z})$$

$$|\vec{E_1}|^2 = |E_{i0}|^2 + |E_{i0}|^2\,\Gamma^*\, e^{-2j\beta_1 z} + |E_{i0}|^2\,\Gamma\, e^{+2j\beta_1 z} + |E_{i0}|^2\,|\Gamma|^2$$

$$= |E_{i0}|^2\{1 + |\Gamma|^2 + 2\,\mathrm{Re}[\Gamma\, e^{+2j\beta_1 z}]\}$$

$$= |E_{i0}|^2\{1 + |\Gamma|^2 + 2\,\mathrm{Re}[|\Gamma|\, e^{j\theta_r}\, e^{2j\beta_1 z}]\} \Rightarrow$$

$$|\vec{E_1}|^2 = |E_{i0}|^2\{1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta_1 z + \theta_r)\}$$

we see $|\vec{E_1}|$ is max when

Max condition $\boxed{2\beta_1 z_{max} + \theta_r = \pm 2m\pi}$ $m = 0, 1, 2$

or $2 \times \frac{2\pi}{\lambda_1} z_{max} + \theta_r = \pm 2m\pi$

$$z_{max} = \frac{\pm 2m\cancel{\pi}}{\cancel{4}\pi}\,\lambda_1 - \frac{\theta_r}{4\pi}\lambda_1$$

$$\boxed{z_{max} = \pm\frac{m}{2}\lambda_1 - \frac{\theta_r}{4\pi}\lambda_1} \qquad m = 0, 1, 2,$$

First max is at $m=0$ $\Rightarrow$ $\Leftarrow$

$$\boxed{z_{max}^{(1)} = -\frac{\theta_r}{4\pi}\lambda_1}$$

Now $\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\sqrt{\frac{1}{2.25}}-\sqrt{\frac{1}{1}}}{\sqrt{\frac{1}{2.25}}+\sqrt{1}} = \boxed{-0.2}$

or $\Gamma = \frac{\sqrt{\frac{\mu}{\varepsilon_2}}-\sqrt{\frac{\mu}{\varepsilon_1}}}{\sqrt{\frac{\mu}{\varepsilon_2}}+\sqrt{\frac{\mu}{\varepsilon_1}}} = \frac{\frac{1}{\sqrt{\varepsilon_2}}-\frac{1}{\sqrt{\varepsilon_1}}}{\frac{1}{\sqrt{\varepsilon_2}}+\frac{1}{\sqrt{\varepsilon_1}}} = \frac{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}} = \frac{n_1-n_2}{n_1+n_2} \Rightarrow$

$$\Gamma = \frac{1-\sqrt{2.25}}{1+\sqrt{2.25}} = \boxed{-0.2}$$

$$\boxed{\Gamma = 0.2e^{\pm j\pi} \Rightarrow \theta_r = \pm\pi}$$

then $z_{max}^{(1)} = -\frac{\cancel{\pi}}{4\cancel{\pi}}\lambda_1 = -\frac{\lambda_1}{4} = \frac{-0.6}{4} = \boxed{-0.15\,\mu m}$

From the glass interface

b) $S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.2}{1-0.2} = \boxed{1.5}$

c) We calculate the ratio $\frac{\text{Power transmitted}}{\text{power incident}}$

$$\langle \vec{P_i} \rangle = \frac{1}{2} Re[\vec{E_i} \times \vec{H_i}^*]$$

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$$\vec{H_i} = \frac{\vec{E_i}}{\eta_1} \hat{ay} = \frac{E_{io}}{\eta_1} e^{-j\beta_1 z} \hat{ay}$$

$$\langle \vec{P_i} \rangle = \frac{1}{2} Re\left[E_{io} e^{-j\beta_1 z} \hat{ax} \times \frac{E_{io}^*}{\eta_1^*} e^{+j\beta_1 z} \hat{ay} \right]$$

$$\boxed{\langle \vec{P_i} \rangle = \frac{1}{2} |E_{io}|^2 \frac{1}{\eta_1} \hat{az}} \quad \text{since } \eta_1^* = \eta_1$$

$$\langle \vec{P_t} \rangle = \frac{1}{2} Re[\vec{E_t} \times \vec{H_t}^*]$$

$$\vec{E_t} = \tau E_{io} e^{-j\beta_2 z} \hat{ax}$$

$$\vec{H_t} = \frac{\tau E_{io}}{\eta_2} e^{-j\beta_2 z} \hat{ay}$$

$$\langle \vec{P_t} \rangle = \frac{1}{2} Re\left[\tau E_{io} e^{-j\beta_2 z} \frac{\tau^* E_{io}^*}{\eta_2^*} e^{+j\beta_2 z} \hat{ax} \times \hat{ay} \right]$$

$$\boxed{\langle \vec{P_t} \rangle = \frac{1}{2} \tau^2 \frac{|E_{io}|^2}{\eta_2} \hat{az}} \quad \text{where } \tau^* = \tau \text{ \& } \eta_2^* = \eta_2$$

$$\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\sqrt{\frac{1}{2.25}}}{\sqrt{\frac{1}{1}} + \sqrt{\frac{1}{2.25}}}$$

$$\boxed{\tau = 0.8} \quad \text{also}$$

$$\tau = 1 + \Gamma = 1 - 0.2 = 0.8$$

then

$$\frac{|\langle \vec{P_t} \rangle|}{|\langle \vec{P_i} \rangle|} = \frac{\frac{1}{2} \tau^2 \frac{|E_{io}|^2}{\eta_2}}{\frac{1}{2} \frac{|E_{io}|^2}{\eta_1}} \Rightarrow$$

$$\frac{|\langle \vec{P_t} \rangle|}{|\langle \vec{P_i} \rangle|} = \frac{\eta_1}{\eta_2} \tau^2 = \frac{1}{\frac{1}{\sqrt{2.25}}} \times 0.8^2 = \sqrt{2.25} \times 0.8^2 = 0.96$$

$\therefore$ Power transmitted $= 96\%$

Question 4)

A uniform plane wave propagating along the +z-axis through a medium with $\varepsilon' = 8\varepsilon_0$ and $\mu = 2\mu_0$, has an electric field given by

$$\overline{E}(z,t) = 10e^{-z/200}\left[\cos\left(10^8 t - \beta z\right)\mathbf{a}_x - \sin\left(10^8 t - \beta z\right)\mathbf{a}_y\right] \text{ V/m}$$

(a) Describe the polarization of this wave. [4 marks]

(b) Determine if this medium is a good dielectric or a good conductor. Justify your answer. [5 marks]

Hint: Recall that, in general $\alpha = \omega\sqrt{\mu\varepsilon'}\sqrt{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon'}\right)^2} - 1\right]}$

(b) Calculate the phase constant, β. State whether your value is exact or approximate. [3 marks]

(d) Find the instantaneous magnetic field. [8 marks]

(a) $\bar{E}(z,t) = 10e^{-z/200}\left[\cos(10^8 t - \beta z)\hat{a}_x - \cos(10^8 t - \beta z - \frac{\pi}{2})\hat{a}_y\right]$

$\therefore \bar{E}(z) = 10e^{-z/200}\left[e^{-j\beta z}\hat{a}_x - e^{-j\beta z}e^{-j\frac{\pi}{2}}\hat{a}_y\right]$

$= 10e^{-z/200}\left[\hat{a}_x + j\hat{a}_y\right]e^{-j\beta z}$

3 marks for statement of left-hand or (CCW) circularly polarized wave

1 mark for some sign of justification

$\Rightarrow$ This describes a left-handed circularly polarized wave.

(b) From $\bar{E}$, $\alpha = \frac{1}{200} = 0.005$ Np/m $= \omega\sqrt{\mu\varepsilon'}\sqrt{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon'}\right)^2} - 1\right]}$

Solving for $\frac{\sigma}{\omega\varepsilon'}$: $2\left(\frac{\alpha}{\omega\sqrt{\mu\varepsilon'}}\right)^2 = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon'}\right)^2} - 1$

$\therefore \frac{\sigma}{\omega\varepsilon'} = \sqrt{\left[1 + 2\left(\frac{\alpha}{\omega\sqrt{\mu\varepsilon'}}\right)^2\right]^2 - 1} = 0.0075$

3 marks for solving for tan delta

Since $\frac{\sigma}{\omega\varepsilon'} << 1$ and specifically $\frac{\sigma}{\omega\varepsilon'} < \frac{1}{100}$, this is a low-loss dielectric.

2 marks for concluding it is a low-loss dielectric

(c) For low-loss dielectrics, $\beta = \omega\sqrt{\mu\varepsilon'} = \underline{1.33 \text{ rad/m}}$

2 marks for value of beta

$\Rightarrow$ This is an approximation, the exact value would be given by $\beta = \omega\sqrt{\mu\varepsilon'}\sqrt{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon'}\right)^2} + 1\right]} = 1.3343$ rad/m

1 mark for statement of approximation

(d) For low-loss dielectrics, $\eta \simeq \sqrt{\frac{\mu}{\epsilon'}} = 188\,\Omega$.

$$\therefore \bar{H}(z) = \frac{1}{\eta}\hat{a}_z \times \bar{E}(z) = \frac{1}{\eta}\hat{a}_z \times \left[10e^{-z/200} e^{-j\beta z}(\hat{a}_x + j\hat{a}_y)\right]$$

$$= \frac{10e^{-z/200}e^{-j\beta z}}{\eta}\left[\hat{a}_y - j\hat{a}_x\right]$$

$$= 0.053\, e^{-z/200} e^{-j\beta z}\left[-j\hat{a}_x + \hat{a}_y\right]$$

$$\bar{H}(z,t) = Re\left[\bar{H}(z)\, e^{j\omega t}\right]$$

$$= 0.053\, e^{-z/200}\, Re\left[e^{j\omega t} e^{-j\beta z} e^{-j\frac{\pi}{2}}\hat{a}_x + e^{j\omega t} e^{-j\beta z}\hat{a}_y\right]$$

$$= 0.053\, e^{-z/200}\left[\cos(\omega t - \beta z - \tfrac{\pi}{2})\hat{a}_x + \cos(\omega t - \beta z)\hat{a}_y\right]$$

$$= 0.053\, e^{-z/200}\left[\sin(\omega t - \beta z)\hat{a}_x + \cos(\omega t - \beta z)\hat{a}_y\right] \quad A/m$$

2 marks for value of eta (exact or approx)

3 marks for correct evaluation of cross-product

3 marks for correct instantaneous expression

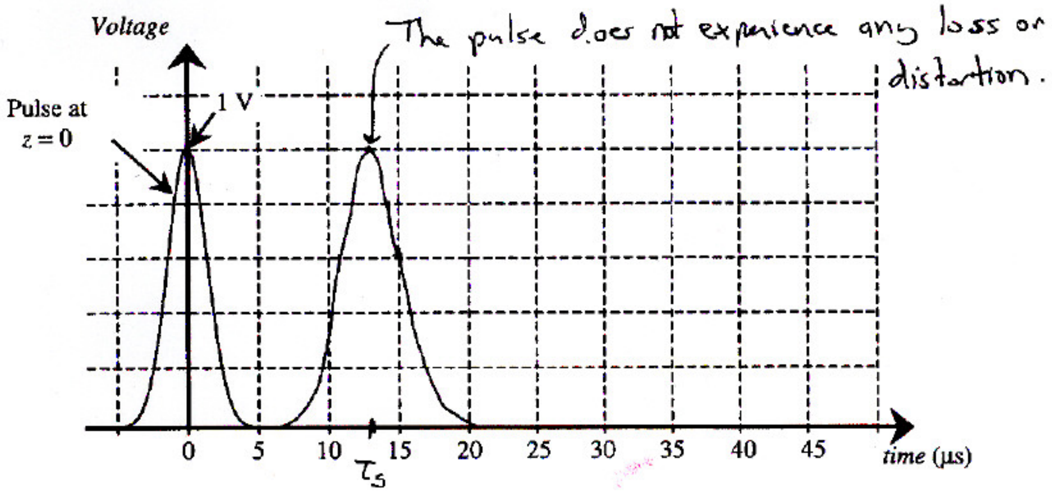
Question 5)

The ω-β diagram for a dispersive medium is shown to the right. From this diagram find

(a) The phase velocity (v_p), phase index (n_p), group velocity (v_g), and group index (n_g), at point **A**. [8 marks]

(b) The phase velocity (v_p), phase index (n_p), group velocity (v_g), and group index, n_g, at point **B** [8 marks]

(c) Suppose a *narrowband* Gaussian pulse centered around $\omega_0 = 3 \times 10^9$ rad/s is traveling along the z-axis through this medium. Estimate the group delay of this pulse after it has traveled to $z = 1$ km, and sketch on the figure below the shape of the pulse envelope as a function of time at this position. [4 marks]



(a) $v_p = \dfrac{\omega}{\beta} = \dfrac{3\times10^9}{40} = \underline{\underline{75\times10^6 \text{ m/s}}}$, $n_p = \dfrac{c}{v_p} = \underline{\underline{4}}$

$v_g = \dfrac{d\omega}{d\beta} = v_p = \underline{\underline{75\times10^6 \text{ m/s}}}$, $n_g = \dfrac{c}{v_g} = \underline{\underline{4}}$

2 marks each
(8 total marks)

(b) $v_p = \dfrac{\omega}{\beta} = \dfrac{5\times10^9}{180} = \underline{\underline{27.8\times10^6 \text{ m/s}}}$, $n_p = \dfrac{c}{v_p} = \underline{\underline{10.8}}$

$v_g = \dfrac{d\omega}{d\beta} = \underline{\underline{0 \text{ m/s}}}$, $n_g = \dfrac{c}{v_g} = \underline{\underline{\infty}}$

2 marks each
(8 total marks)

(c) $\tau_g = \dfrac{L}{v_g} = \dfrac{1\times10^3}{75\times10^6} = \underline{\underline{13.3\ \mu s}}$

2 marks for group delay

2 marks the sketch of a non-distorted pulse