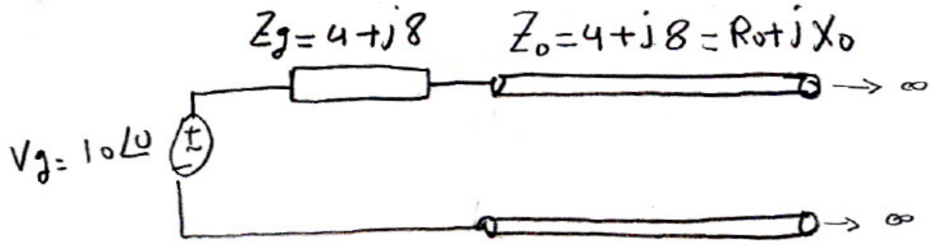
Question 1:

A sinusoidal voltage source $V_g = 10\angle 0^0$ with internal impedance of $Z_g = 4 + j8\ \ \Omega$ is connected to an *infinitely long general transmission line* with characteristic impedance of $Z_0 = 4 + j8\ \Omega$. The transmission line resistance and conductance were measured to be $R = 3.2\ \ \Omega/\mathrm{m}$ and $G = 5\times 10^{-4}\ \ \mathrm{S/m}$, respectively.

a) Find a numerical value for the attenuation constant in Np/m and dB/m. (12 Pts)

b) What is the numerical value for the *magnitude of the voltage* wave 2 meters away from the source? (As always, source is at $z = 0$) (12 pts)

c) Let $P(z)$ be the time averaged power. What is the numerical value for the ratio $P(z_2)/P(z_1)$ where $z_2 = 2$ m and $z_1 = 0$ m. (11pts)



note line is matched to source

$R = 3.2$ [Ω/m]

$G = 5\times10^{-4}$ [S/m]

a) For an infinitly long TL we have shown (in class & HW & lectures) that

① $\alpha = \frac{1}{2R_0}\left(R + G|Z_0|^2\right)$ [NP/m] $\Rightarrow$

② $\alpha = \frac{1}{2\times4}\left(3.2 + 5\times10^{-4}|4+j8|^2\right) = 0.405$ [NP/m]

$$\underset{\text{NP/m}}{\frac{1}{0.405}} = \underset{\text{dB/m}}{\frac{8.69}{\alpha(dB)}} \Rightarrow \alpha(dB) = 3.519 \text{ [dB/m]}$$

b) $V(z) = V_0^+ e^{-\gamma z} + \underbrace{\cancel{V_0^- e^{+\gamma z}}}_{0}$ since line is infintely long $= V_0^+ e^{-\gamma z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$

Recall $V_0^+ = V_g \frac{Z_0}{Z_0+Z_g} \frac{1}{(1 - \underbrace{\cancel{\Gamma_L \Gamma_g e^{-2\gamma l}}}_{0})}$ since $\Gamma_L = \Gamma_g = 0$ then

$$V_0^+ = V_g \frac{Z_0}{Z_0+Z_g} = 10\angle 0 \frac{\cancel{Z_0}}{2\cancel{Z_0}} = 5\angle 0$$

then

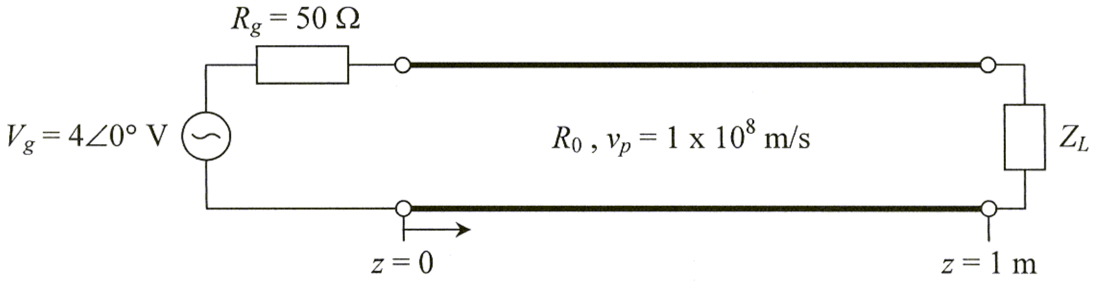
$$V(z) = 5e^{-0.405z} e^{-j\beta z} \Rightarrow |V(z=2)| = 5e^{-0.405\times2} = 2.224 \text{ [V]}$$

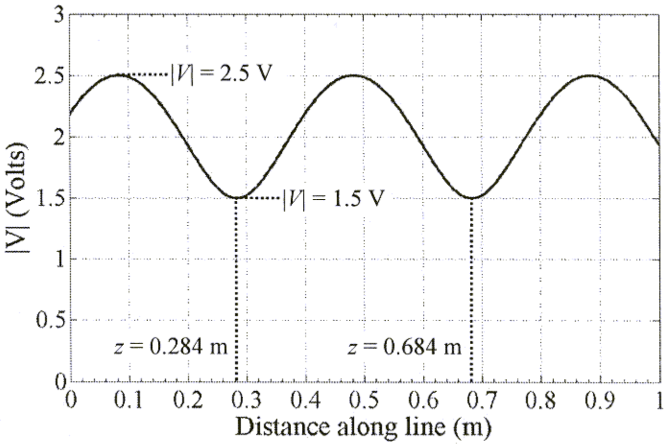
c) we have $P(z) = \frac{(V_0^+)^2}{2|Z_0|^2} R_0 e^{-2\alpha z} \Rightarrow \frac{P(z=z_2)}{P(z=z_1)} = \frac{e^{-2\alpha z_2}}{e^{-2\alpha z_1}} = \frac{e^{-2\alpha\times2}}{e^{-0}} \Rightarrow$

$$\frac{P(z=z_2)}{P(z=z_1)} = e^{-4\times0.405} = 0.198$$

Question 2

In the figure below, an unknown load, Z_L, is attached to an $R_0 = 50\ \Omega$ lossless transmission line. Also shown is the measured voltage standing wave pattern for this transmission line circuit.





(a) Determine the standing wave ratio, S, the wavelength, λ, and the frequency of the source, f.

$$S = \frac{|V_{max}|}{|V_{min}|} = \frac{2.5}{1.5} = 1.\bar{6} = \underline{\underline{1.67}}$$

$$\lambda = 2 \times (0.684\,m - 0.284\,m) = \underline{\underline{0.8\ m}}$$

$$f = \frac{v_p}{\lambda} = \frac{1\times10^8\ m/s}{0.8\,m} = 1.25\times10^8\ Hz = \underline{\underline{125\ MHz}}$$

(b) Find the magnitude *and* phase of the reflection coefficient at the load.

* The magnitude is found from the standing wave ratio value

$$|\Gamma_L| = \frac{S-1}{S+1} = \frac{1}{4}$$

* The phase is found from the position of the first minimum from the load.

$$\phi_{\Gamma_L} - 2\beta z'_{min} = \pm\pi$$

→ with $z'_{min} = 1m - 0.684m = 0.316\,m$

→ For $+\pi$, $\phi_{\Gamma_L} = \pi + 2\left(\frac{2\pi}{\lambda}\right) z'_{min} = \pi + 4\pi\left(\frac{0.316m}{0.8m}\right)$

$= 2.58\pi = 464.4°$ or $104.4°$.

→ For $-\pi$, $\phi_{\Gamma_L} = -\pi + 4\pi\left(\frac{0.316m}{0.8m}\right) = 0.58\pi = 104.4°$

→ So it doesn't matter whether you choose $\pm\pi$.

$$\therefore \quad \Gamma_L = \frac{1}{4}\angle 104.4° = -0.062 + j\,0.242$$

(c) Find the unknown load impedance, Z_L.

Since $\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$ we can solve for Z_L.

So $(Z_L + Z_o)\Gamma_L = Z_L - Z_o$

$$Z_L(\Gamma_L - 1) = Z_o(-1 - \Gamma_L)$$

$$\therefore \quad Z_L = Z_o \frac{1+\Gamma_L}{1-\Gamma_L} = (50)\frac{1 + 0.25\angle 104.4°}{1 - 0.25\angle 104.4°}$$

$$= 39.5 + 20.4j$$

* So we can see this load is a resistance in series with an inductance.

(d) Determine the expression for the instantaneous voltage on the line.

* The general expression for the total phasor voltage on the line is

$$\bar{V}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

→ Since the source is matched to the line ($R_g = R_0 = 50\Omega$),

$$V_0^+ = V_g \frac{R_0}{R_0 + R_g} = \frac{V_g}{2} = 2\angle 0^\circ \text{ V}.$$

* To find out what V_0^- is we must evaluate $\bar{V}(z)$ at the load:

$$\bar{V}(z=\ell) = \underbrace{V_0^+ e^{-j\beta(\ell)}}_{\text{incident wave at the load}} + \underbrace{V_0^- e^{+j\beta(\ell)}}_{\text{reflected wave at the load}}$$

→ We know that $\Gamma_L = \dfrac{V_{ref \text{ at load}}}{V_{inc \text{ at load}}} = \dfrac{V_0^- e^{j\beta\ell}}{V_0^+ e^{-j\beta\ell}} = \dfrac{V_0^-}{V_0^+} e^{2j\beta\ell}$

$$\therefore V_0^- = V_0^+ \Gamma_L e^{-2j\beta\ell} = V_0^+ |\Gamma_L| e^{j\phi_{\Gamma_L}} e^{-2j\beta\ell} = V_0^+ |\Gamma_L| e^{j(\phi_{\Gamma_L} - 2\beta\ell)}$$

$$\therefore \bar{V}(z) = V_0^+ e^{-j\beta z} + V_0^+ |\Gamma_L| e^{j(\phi_{\Gamma_L} - 2\beta\ell)} e^{+j\beta z}$$

$$= V_0^+ e^{-j\beta z} + V_0^+ |\Gamma_L| e^{j(\phi_{\Gamma_L} - 2\beta\ell + \beta z)}$$

* The instantaneous voltage is then

$$v(t,z) = Re\left[\bar{V}(z) e^{j\omega t}\right] = Re\left[V_0^+ e^{j(\omega t - \beta z)} + V_0^+ |\Gamma_L| e^{j(\omega t + \phi_{\Gamma_L} - 2\beta + \beta z)}\right]$$

$$= V_0^+ \cos(\omega t - \beta z) + V_0^+ |\Gamma_L| \cos(\omega t + \beta z + \phi_{\Gamma_L} - 2\beta)$$

→ For this circuit, $\phi_{\Gamma_L} = 0.58\pi$, $2\beta = 2\left(\frac{2\pi}{\lambda}\right) = 2\left(\frac{2\pi}{0.8}\right) = 5\pi = \pi$

$$\therefore \boxed{v(t,z) = 2\cos(\omega t - \beta z) + 0.5\cos(\omega t + \beta z - 0.42\pi)}$$

→ For $z' \Rightarrow z = \ell - z'$ this becomes

$$v(t,z') = 2\sin(\omega t + \beta z') + 0.5\sin(\omega t - \beta z' + 0.58\pi)$$

Question 3:

A researcher has measured the input impedance of a lossless transmission line of the length $l = 57$ cm, under the open condition, to be $-j121.24\ \Omega$. The input impedance of the same line under the short condition is $j40.42\ \Omega$. She also knows that the line length is between 3λ and 3.25λ. Answer the following questions:

a) What are the numerical values for the Z_0, α, and β. (20 pts)

b) For the above line terminated with Z_L=3+j5, what is the numerical value of the input impedance? (10 pts)

$Z_{in}^{op} = -j\,121.24\,\Omega, \quad Z_{in}^{sh} = j\,40.42\,\Omega \qquad 3\lambda < l < \frac{13}{4}\lambda$

$l = 0.57\ m$

a)

$Z_0 = \sqrt{Z_{in}^{op} Z_{in}^{sh}} = \sqrt{(-j\,121.24)(j\,40.42)} \Rightarrow$

$$\boxed{Z_0 = 70\,\Omega \Rightarrow R_0 = 70\,\Omega \;\&\; X_0 = 0}$$

For General TL we have $\tanh \gamma l = \sqrt{\frac{Z_{in}^{sh}}{Z_{in}^{op}}}$ $\Rightarrow$ for lossless line

$j\tan(\beta l) = \sqrt{\frac{j\,40.42}{-j\,121.24}} \Rightarrow \beta l = \tan^{-1}\left[\frac{1}{j}\sqrt{\frac{j\,40.42}{-j\,121.24}}\right] + n\pi$

$$\boxed{\beta l = 0.524 + n\pi}$$

when $n = 0, \pm 1, \pm 2, \ldots$

since $3\lambda < l < \frac{13}{4}\lambda \Rightarrow 3\cancel{\lambda}\frac{2\pi}{\cancel{\lambda}} < \beta l < \frac{13}{4}\cancel{\lambda}\frac{2\pi}{\cancel{\lambda}} \Rightarrow$

$6\pi < \beta l < 6.5\pi \Rightarrow \quad 6\pi < 0.524 + n\pi < 6.5\pi \Rightarrow$

$18.850 < 0.524 + n\pi < 20.420 \Rightarrow \boxed{n = 6}$ then

$$\boxed{\beta = \frac{0.524 + n\pi}{l} = \frac{0.524 + 6\pi}{0.57} = 33.989 \text{ rad/m} \quad \beta l = 19.373}$$

b) $Z_{in} = R_0 \frac{Z_L + jR_0 \tan\beta l}{R_0 + jZ_L \tan\beta l} = 70\frac{(3+j5) + j70\tan(19.373)}{70 + j(3+j5)\tan(19.373)} \Rightarrow$

$$\boxed{Z_{in} = 4.35 + j\,47.24 = 47.44\,e^{j84.74^\circ} = 47.44\,e^{j1.48(\text{rad})}}$$