Family Name:	Given name:
Student number	Signature

Faculty of Applied Science and Engineering

ECE357 Electromagnetic Fields

First Test, February 16, 2006

Examiners – M. Mojahedi

Only Calculators approved by Registrar allowed
Answer the questions in the spaces provided or on the facing page
A complete paper consists of answers to all questions
For numerical answers specify units

DO NOT REMOVE STAPLE

Do not write in these spaces

1	2	3	TOTAL

$\varepsilon_0 = 8.854 \times 10^{-12}\ [F/m], \quad \mu_0 = 4\pi \times 10^{-7}\ [H/m], \quad c = 3 \times 10^8\ [m/s]$

4.5.4 $\operatorname{csch} z = 1/\sinh z$

4.5.5 $\operatorname{sech} z = 1/\cosh z$

4.5.6 $\coth z = 1/\tanh z$

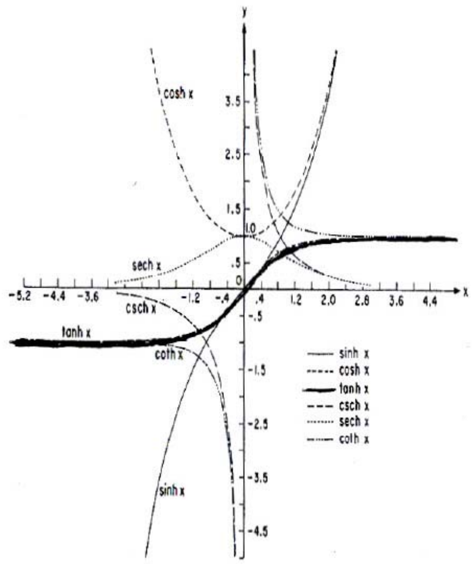


FIGURE 4.6. *Hyperbolic functions.*

Relation to Circular Functions (see 4.3.49 to 4.3.54)

Hyperbolic formulas can be derived from trigonometric identities by replacing z by iz

4.5.7 $\sinh z = -i \sin iz$

4.5.20 $\cosh z - \sinh z = e^{-z}$

Negative Angle Formulas

4.5.21 $\sinh(-z) = -\sinh z$

4.5.22 $\cosh(-z) = \cosh z$

4.5.23 $\tanh(-z) = -\tanh z$

Addition Formulas

4.5.24 $\sinh(z_1+z_2) = \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_2$

4.5.25 $\cosh(z_1+z_2) = \cosh z_1 \cosh z_2 + \sinh z_1 \sinh z_2$

4.5.26 $\tanh(z_1+z_2) = (\tanh z_1 + \tanh z_2)/(1+\tanh z_1 \tanh z_2)$

4.5.27 $\coth(z_1+z_2) = (\coth z_1 \coth z_2 + 1)/(\coth z_2 + \coth z_1)$

Half-Angle Formulas

4.5.28

$$\sinh \frac{z}{2} = \left(\frac{\cosh z - 1}{2}\right)^{\frac{1}{2}}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}, \quad \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Problem 1: On a lossless transmission line with characteristic impedance of 150 Ω the following observations were noted: distance of first voltage minimum from the load = 3 cm; distance of first voltage maximum from the load = 9 cm, standing wave ratio = 3. Determine the value of the load impedance (Z_L). **(Total points: 30)**

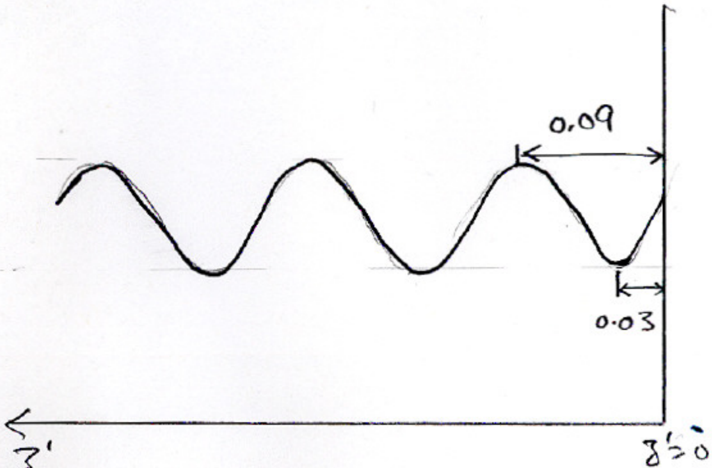
Problem 1)

$R_0 = 150\,\Omega$

$S = 3$

First $z'_{max} = 0.09\,m$

First $z'_{min} = 0.03\,m$



* since $|\Gamma| = \frac{S-1}{S+1} \Rightarrow \boxed{|\Gamma| = \frac{3-1}{3+1} = 1/2}$

* The distance between consecutive maximum & minimum in a TL is $\lambda/4 \Rightarrow$ $\frac{\lambda}{4} = 0.09 - 0.03 \Rightarrow$

$$\boxed{\lambda = 0.24\ (m)}$$

* location of Voltage maximums are given by

$\theta_\Gamma - 2\beta z'_{max} = -2n\pi$ first maximum is at $n=0 \Rightarrow$

$\theta_\Gamma - 2\beta z'_{max} = 0 \Rightarrow \theta_\Gamma - 2\frac{2\pi}{\lambda} z'_{max} = 0 \Rightarrow$

$\theta_\Gamma = \frac{4\pi}{\lambda} z'_{max} = \frac{4\pi}{0.24} \times 0.09 = 4.712\ (\text{radian}) = 270^\circ$

* or similarly the location of first minimum is given by

$\theta_\Gamma - 2\beta z'_{min} = -\pi \Rightarrow \theta_\Gamma = \frac{4\pi}{\lambda} z'_{min} - \pi = \frac{4\pi}{0.24} \times 0.03 - \pi \Rightarrow$

$\theta_\Gamma = -1.5708\ (\text{rad}) = 4.712\ (\text{rad}) = 270^\circ$

$$\boxed{\begin{gathered}\text{* from above } \Gamma = |\Gamma| e^{j\theta_\Gamma} = 0.5 e^{j270^\circ} = 0 - j0.5 \ \& \\ Z_L = R_0 \frac{1+\Gamma}{1-\Gamma} = 150 \frac{1-j0.5}{1+j0.5} = 90 - j120 = 150 e^{-j53.13^\circ}\end{gathered}}$$

Problem 2: In class we showed that the input impedance [$Z_i(l)$] for a lossless transmission line terminated on a short is identically zero at $l = \lambda/2, \lambda, 3\lambda/2, 2\lambda, ...$ Prove that for a short circuited lossy transmission line the input impedance is never identically zero at $l = \lambda/2, \lambda, 3\lambda/2, 2\lambda, ...$ **(Total points: 35)**

Problem 2)

For a lossy Transmission line

$$Z_i(l) = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$$ For short ckt

$$Z_i(l) = Z_0 \tanh(\gamma l) = Z_0 \tanh[(\alpha + j\beta)l] =$$

$$Z_0 \frac{\sinh(\alpha l + j\beta l)}{\cosh(\alpha l + j\beta l)} =$$

$$Z_0 \frac{\sinh(\alpha l)\cosh(j\beta l) + \cosh(\alpha l)\sinh(j\beta l)}{\cosh(\alpha l)\cosh(j\beta l) + \sinh(\alpha l)\sinh(j\beta l)} =$$

$$Z_0 \frac{\sinh(\alpha l)\cos(\beta l) + j\cosh(\alpha l)\sin(\beta l)}{\cosh(\alpha l)\cos(\beta l) + j\sinh(\alpha l)\sin(\beta l)}$$

Now at $l = n\frac{\lambda}{2}$ $n = 1, 2, 3, 4 \ldots$ then $\beta l = \frac{2\pi}{\lambda} l = \frac{2\pi}{\lambda} n \frac{\lambda}{2} \Rightarrow$

$\boxed{\beta l = n\pi \quad n = 1, 2, 3 \ldots}$ use this in above

$$Z_i\left(l = n\frac{\lambda}{2}\right) = Z_0 \frac{\sinh(\alpha l)\cos(n\pi) + j\cosh(\alpha l)\cancel{\sin(n\pi)}^{0}}{\cosh(\alpha l)\cos(n\pi) + j\sinh(\alpha l)\cancel{\sin(n\pi)}_{0}}$$

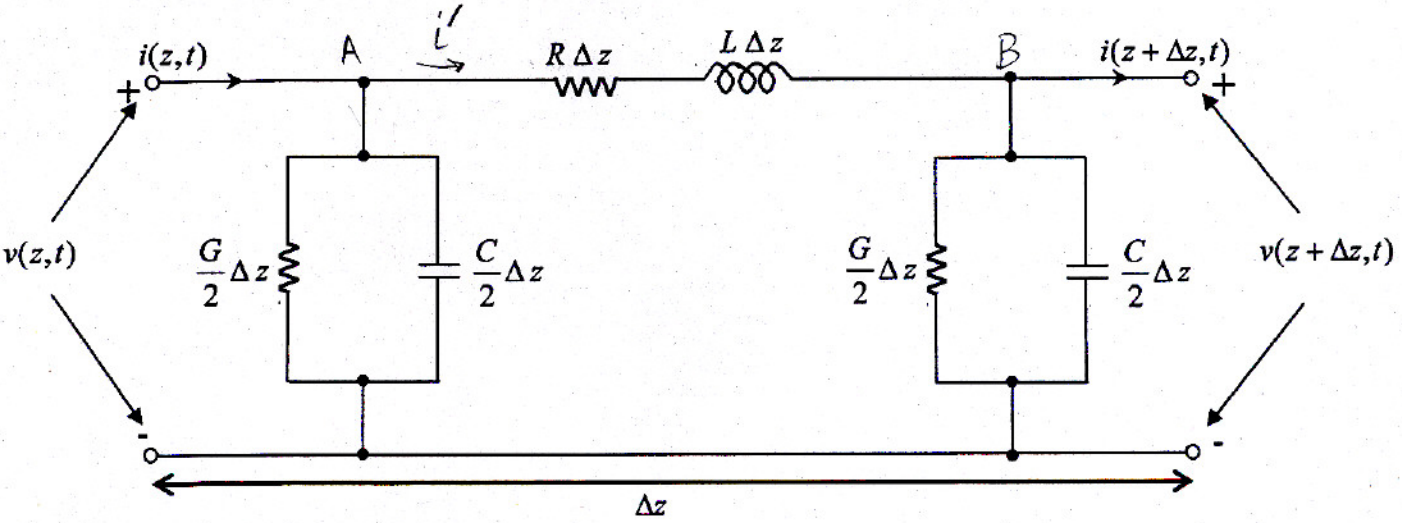
$= Z_0 \tanh(\alpha l)$

for $n = 1, 2, 3$

A Plot of $\tanh(\alpha l)$ shows that $\tanh(\alpha l) = 0$ only when $\alpha l = 0 \Rightarrow$ $l = 0$, otherwise $\tanh(\alpha l) \neq 0$.

In other words, $Z_i\left(l = n\frac{\lambda}{2}\right) \neq 0$ for $n = 1, 2, 3, \ldots$

Problem 3: A transmission line is modeled by an equivalent circuit shown below. For this transmission line, find the general transmission line equations (the so-called telegrapher's equations) for instantaneous voltage and current, i.e., as a function of time (t) and space (z). Show all your work. **(Total points: 35)**



KCL at node A:

$$i(z,t) = i' + v(z,t)\frac{G}{2}\Delta z + \frac{C}{2}\Delta z \frac{\partial}{\partial t} v(z,t) \qquad ①$$

KCL at node B!

$$i' = i(z+\Delta z,t) + \frac{G}{2}\Delta z\, v(z+\Delta z,t) + \frac{C}{2}\Delta z \frac{\partial}{\partial t} v(z+\Delta z,t) \qquad ②$$

KVL of inner loop:

$$v(z,t) = R\Delta z\, i' + L\Delta z \frac{\partial}{\partial t} i' + v(z+\Delta z,t) \qquad ③$$

use ② in ① to eliminate i':

$$i(z,t) - i(z+\Delta t,t) = \frac{G}{2}\Delta z\, v(z,t) + \frac{C}{2}\Delta z \frac{\partial}{\partial t} v(z,t) + \frac{G}{2}\Delta z\, v(z+\Delta z,t) + \frac{C}{2}\Delta z \frac{\partial}{\partial t} v(z+\Delta z,t)$$

Divide by Δz and take the limit as $\Delta z \to 0$:

$$\lim_{\Delta z\to 0} \frac{i(z,t)-i(z+\Delta z,t)}{\Delta z} = -\frac{\partial i(z,t)}{\partial z} = \lim_{\Delta z\to 0}\left[\frac{G}{2} v(z,t) + \frac{C}{2}\frac{\partial}{\partial t} v(z,t)\right] \qquad ④$$

$$+ \lim_{\Delta z\to 0}\left[\frac{G}{2} v(z+\Delta z,t) + \frac{C}{2}\frac{\partial}{\partial t} v(z+\Delta z,t)\right]$$

Note that $\lim_{\Delta z\to 0} v(z+\Delta z,t) \neq v(z,t)$; Physically this would mean that nodes A and B have the same voltage and R, L contribute zero voltage drop.

Sub ③ into ④ to eliminate $v(z+\Delta z,t)$:

$$-\frac{\partial i(z,t)}{\partial z} = \frac{G}{2} v(z,t) + \frac{C}{2}\frac{\partial}{\partial t} v(z,t) + \lim_{\Delta z\to 0}\left[\frac{G}{2} v(z,t) - \cancelto{0}{\frac{G}{2} R\Delta z\, i'} - \cancelto{0}{\frac{G}{2} L\Delta z \frac{\partial}{\partial t} i'} + \frac{C}{2}\frac{\partial}{\partial t} v(z,t) - \cancelto{0}{\frac{C}{2} R\Delta z \frac{\partial i'}{\partial t}} - \cancelto{0}{\frac{C}{2} L\Delta z \frac{\partial i'}{\partial t}}\right]$$

these terms go to zero in $O((\Delta z)^2)$.

Finally,

$$\boxed{-\frac{\partial i(z,t)}{\partial z} = G\, v(z,t) + C\frac{\partial}{\partial t} v(z,t)}$$

Now divide ③ by Δz and take limit as $\Delta z \to 0$:

$$\lim_{\Delta z\to 0} \frac{v(z,t) - v(z+\Delta z,t)}{\Delta z} = \lim_{\Delta z\to 0}\left[R i' + L\frac{\partial i'}{\partial t}\right] \qquad ⑤$$

Since some current must drain through the shunt G, C's, it must be that

$$\lim_{\Delta z\to 0} i' \neq i(z,t)$$
$$\neq i(z+\Delta z,t)$$

So sub ① into ⑤ to remove i' in favour of $i(z,t)$:

$$-\frac{\partial v(z,t)}{\partial z} = \lim_{\Delta z\to 0}\left[R i(z,t) - \cancelto{0}{R\frac{G}{2}\Delta z\, v(z,t)} - \cancelto{0}{R\frac{C}{2}\Delta z \frac{\partial}{\partial t} v(z,t)} + L\frac{\partial i(z,t)}{\partial t} - \cancelto{0}{L\frac{G}{2}\Delta z \frac{\partial}{\partial t} v(z,t)} - \cancelto{0}{L\frac{C}{2}\Delta z \frac{\partial^2}{\partial t^2} v(z,t)}\right]$$

$$\boxed{-\frac{\partial v(z,t)}{\partial z} = R\, i(z,t) + L\frac{\partial i(z,t)}{\partial t}}$$