Family Name:	Given name:
Student number	Signature

Faculty of Applied Science and Engineering

ECE357 Electromagnetic Fields

Second Test, March 27, 2006

Examiners – M. Mojahedi

Only Calculators approved by Registrar allowed
Answer the questions in the spaces provided or on the facing page
A complete paper consists of answers to all questions
For numerical answers specify units

DO NOT REMOVE STAPLE

Do not write in these spaces

1	2	3	TOTAL

$\varepsilon_0 = 8.854\times10^{-12}\ [F/m], \quad \mu_0 = 4\pi\times10^{-7}\ [H/m], \quad c = 3\times10^{8}\ [m/s]$

Vector Identities

$$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = (\mathbf{B} \times \mathbf{C}) \cdot \mathbf{A} = (\mathbf{C} \times \mathbf{A}) \cdot \mathbf{B}$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$

$$\nabla \cdot (V\mathbf{A}) = \mathbf{A} \cdot \nabla \mathbf{V} + V \nabla \cdot \mathbf{A}$$

$$\nabla \times (V\mathbf{A}) = \nabla V \times \mathbf{A} + V \nabla \times \mathbf{A}$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B}$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A})$$

$$\nabla \cdot \nabla \equiv \nabla^2$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \times (\nabla V) = 0$$

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

$$\int_V (\nabla \cdot \mathbf{A}) dv = \oint_S \mathbf{A} \cdot \mathbf{ds}$$

$$\int_S (\nabla \times \mathbf{A}) \cdot ds = \oint_C \mathbf{A} \cdot \mathbf{d\ell}$$

$$\int_V \nabla \times \mathbf{F} dv = -\oint_S \mathbf{F} \times \mathbf{ds}$$

Gradient, Divergence, Curl, and Laplacian Operations

Cartesian Coordinates (x, y, z)

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{a}}_x + \frac{\partial f}{\partial y}\hat{\mathbf{a}}_y + \frac{\partial f}{\partial z}\hat{\mathbf{a}}_z$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{\mathbf{a}}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\hat{\mathbf{a}}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\hat{\mathbf{a}}_z$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Cylindrical Coordinate (ρ, ϕ, z)

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\mathbf{a}}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\mathbf{a}}_\phi + \frac{\partial f}{\partial z}\hat{\mathbf{a}}_z$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho A_\rho)\right] + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{\mathbf{a}}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\hat{\mathbf{a}}_\phi + \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi}\right]\hat{\mathbf{a}}_z$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinate (r, θ, ϕ)

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{a}}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\mathbf{a}}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\mathbf{a}}_\phi$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\left[\frac{\partial}{\partial r}(r^2 A_r)\right] + \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\theta \sin\theta)\right] + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

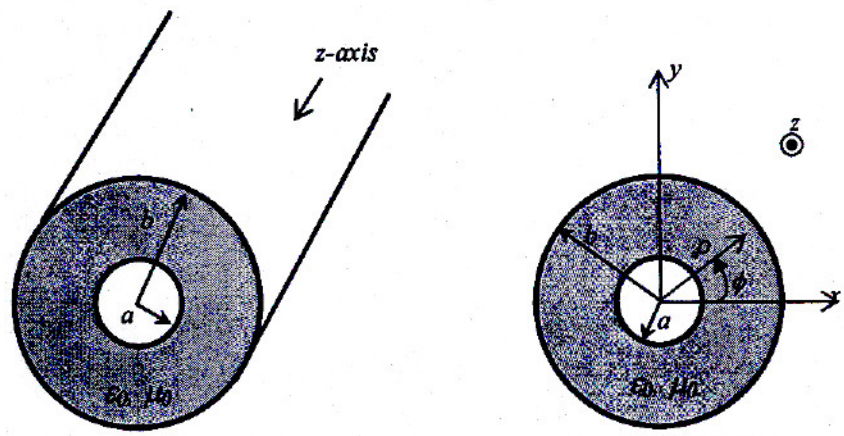
$$\nabla \times \mathbf{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{a}}_r$$

$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\hat{\mathbf{a}}_\phi + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]\hat{\mathbf{a}}_\phi$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Problem 1:
Consider a coaxial line shown in the figure where the conductors (assumed to be perfect conductors) are at $\rho = a$ and $\rho = b$. Assuming a TEM wave propagating in the +z-direction, find the appropriate expressions for the $\vec{E}(\rho, z)$ and $\vec{H}(\rho, z)$ inside the guide, $a \le \rho \le b$. **[35 Pts]**

Hint 1: Due to azimuthal symmetry you can assume that there is no ϕ-variation, i.e. $\partial/\partial\phi = 0$.
Hint 2: If there are any additive or multiplicative constants in your expressions for $\vec{E}$ and $\vec{H}$ you do not need to evaluate them.



Coaxial Cable **Coaxial Cable End View**

We start with

① $\nabla \times \vec{E} = j\omega\varepsilon \vec{E}$ using $\nabla\times$ for cylindrical coordinate we have

$$② \left(\frac{1}{\rho}\frac{\partial}{\partial\phi}\cancelto{0}{H_z} - \frac{\partial}{\partial z}H_\phi\right)\hat{a}_\rho + \left(\frac{\partial}{\partial z}H_\rho - \frac{\partial}{\partial\rho}\cancelto{0}{H_z}\right)\hat{a}_\phi +$$

$$\frac{1}{\rho}\left(\frac{\partial}{\partial\rho}\rho H_\phi - \cancelto{0}{\frac{\partial}{\partial\phi}H_\rho}\right)\hat{a}_z = j\omega\varepsilon\left[E_\rho\hat{a}_\rho + E_\phi\hat{a}_\phi + \cancelto{0}{E_z}\hat{a}_z\right]$$

where since $TEM^z \Rightarrow H_z = E_z = 0$ & since azimuthal symmetry

$\frac{\partial}{\partial\phi}H_\rho = 0$ then (2) $\Rightarrow$

$$③ -\frac{\partial}{\partial z}H_\phi\hat{a}_\rho + \frac{\partial}{\partial z}H_\rho\hat{a}_\phi + \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho H_\phi\hat{a}_z = j\omega\varepsilon E_\rho\hat{a}_\rho + j\omega\varepsilon E_\phi\hat{a}_\phi$$

From (3) it is clear that

(4) $\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho H_\phi = 0 \Rightarrow$ (5) $\boxed{H_\phi = \frac{g(z)}{\rho}}$

(5)

* From $\nabla\times\vec{E} = -j\omega\mu\vec{H}$ we have

(6) $\left(\frac{1}{\rho}\frac{\partial}{\partial\phi}\cancel{E_z}^{0} - \frac{\partial}{\partial z}E_\phi\right)\hat{a_\rho} + \left(\frac{\partial}{\partial z}E_\rho - \frac{\partial}{\partial\rho}\cancel{E_z}^{0}\right)\hat{a_\phi} +$

$\frac{1}{\rho}\left[\frac{\partial}{\partial\rho}\rho E_\phi - \cancel{\frac{\partial}{\partial\phi}E_\rho}_{0}\right]\hat{a_z} = -j\omega\mu\left[H_\rho\hat{a_\rho} + H_\phi\hat{a_\phi} + \cancel{H_z}^{0}\hat{a_z}\right]$

where again $E_z = H_z = 0$ & $\frac{\partial}{\partial\phi}E_\rho = 0 \Rightarrow$

(7) $\underbrace{-\frac{\partial}{\partial z}E_\phi\hat{a_\rho}} + \underbrace{\frac{\partial}{\partial z}E_\rho\hat{a_\phi}} + \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho E_\phi\hat{a_z} = \underbrace{-j\omega\mu H_\rho\hat{a_\rho}}\ \underbrace{-j\omega\mu H_\phi\hat{a_\phi}}$

(7) $\Rightarrow$ $\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho E_\phi = 0 \Rightarrow \boxed{E_\phi = \frac{f(z)}{\rho}}$ (8)

*Now, at the boundary $\rho = a$ & $\rho = b$ we must have tangential $\vec{E}$ to be zero

Perfect dielectric – perfect conductor interface, then

$E_\phi(\rho = a) = E_\phi(\rho = b) = 0 \Rightarrow \frac{f(z)}{a} = \frac{f(z)}{b} = 0 \Rightarrow f(z) = 0 \Rightarrow$

(9)

$\boxed{E_\phi = 0}$

* From (9) & the fact we are dealing with TEM z, then it is

obvious that (10) $\boxed{\vec{E} = E_\rho(\rho, z)\hat{a_\rho} \quad [E_\phi = E_z = 0]}$ &

(11) $\boxed{\vec{H} = H_\phi(\rho, z)\hat{a_\phi} \quad [H_\rho = H_z = 0]}$

with (12) $\boxed{H_\phi(\rho, z) = \frac{g(z)}{\rho}}$

* But what functional form $E_\rho(\rho, z)$ has? We go back to Eq. (3) & consider the $\hat{a}_\rho$ component given below

(13) $-\frac{\partial}{\partial z} H_\phi(\rho, z) = j\omega\varepsilon E_\rho(\rho, z)$ use (12 in (13) $\Rightarrow$

$$-\frac{\partial}{\partial z}\frac{g(z)}{\rho} = j\omega\varepsilon E_\rho(\rho, z) \Rightarrow -\frac{1}{\rho}\frac{\partial}{\partial z} = j\omega\varepsilon E_\rho(\rho, z) \Rightarrow$$

This is to say that in general $\boxed{E_\rho(\rho, z) = \frac{h(z)}{\rho}}$ (14)

* In summary we have $\boxed{\vec{E} = E_\rho(\rho, z)\hat{a}_\rho = \frac{h(z)}{\rho}\hat{a}_\rho}$ (15) &

$\boxed{\vec{H} = H_\phi(\rho, z)\hat{a}_\phi = \frac{g(z)}{\rho}\hat{a}_\phi}$ (16)

where $h(z)$ & $g(z)$ are functions of z only.

* with the help of (15) & (16) we go back to (3) & (4) & consider the terms with E_ρ & H_ϕ

(3) $\Rightarrow$ $-\frac{\partial}{\partial z}H_\phi = j\omega\varepsilon E_\rho \Rightarrow \boxed{-\frac{\partial}{\partial z}\frac{g(z)}{\rho} = j\omega\varepsilon\frac{h(z)}{\rho}}$ (17) &

(4) $\Rightarrow$ $\frac{\partial}{\partial z}E_\rho = -j\omega\mu H_\phi \Rightarrow \boxed{\frac{\partial}{\partial z}\frac{h(z)}{\rho} = -j\omega\mu\frac{g(z)}{\rho}}$ (18)

(17) $\Rightarrow$ (19) $\frac{d g(z)}{dz} = -j\omega\varepsilon h(z)$ &

(18) $\Rightarrow$ (20) $\frac{d}{dz}h(z) = -j\omega\mu g(z)$

$\Rightarrow \frac{d^2}{dz^2}g(z) = (-j\omega\varepsilon)(-j\omega\mu)g(z) \Rightarrow$

$$\frac{d^2}{dz^2}g(z) = -\omega^2\mu\varepsilon\, g(z) = -k^2 g(z) \Rightarrow \boxed{g(z) = Ae^{-jkz}}$$

where A is a constant (21)

* similarly $\boxed{h(z) = Be^{-jkz} \text{ where } B \text{ is a constant}}$ (22)

finally $\boxed{\vec{E} = E_\rho(\rho, z)\hat{a}_\rho = \frac{h(z)}{\rho}\hat{a}_\rho = B\frac{e^{-jkz}}{\rho}\hat{a}_\rho}$ and

$\boxed{\vec{H} = H_\phi(\rho, z)\hat{a}_\phi = \frac{g(z)}{\rho}\hat{a}_\phi = A\frac{e^{-jkz}}{\rho}\hat{a}_\phi}$

Problem 2:

Consider an unbounded simple medium (linear, homogeneous and isotropic) which is source free ($\rho_{ev} = \rho_{mv} = 0, \vec{M}_i = \vec{J}_i = 0$) but lossy ($\sigma \neq 0$). For such a medium

a) Derive the instantaneous wave equation for the electric field $\vec{\mathcal{E}}(\vec{r},t)$. **[8 pts]**

b) Now, assuming no conduction losses, give the wave equation for $\vec{E}(\vec{r})$ in time harmonic form. **[7 pts]**

c) We suggest that the solution to the wave equation in part (b) is a plane wave of the form $\vec{E}(\vec{r}) = \vec{E}_0 \exp(-j\vec{k}\cdot\vec{r})$, where $\vec{E}_0$ is a constant and $\vec{k} = k_x\hat{a}_x + k_y\hat{a}_y + k_z\hat{a}_z$.

Derive the condition that the components of the vector $\vec{k}$ must satisfy. **[15 pts]**

a) (1) $\nabla \times \vec{\mathcal{E}} = -\mu \frac{\partial}{\partial t}\vec{\mathcal{H}}$ & (2) $\nabla \times \vec{\mathcal{H}} = \sigma\vec{\mathcal{E}} + \varepsilon \frac{\partial}{\partial t}\vec{\mathcal{E}}$

$(1) \Rightarrow$ (3) $\nabla \times \nabla \times \vec{\mathcal{E}} = -\mu \frac{\partial}{\partial t}\nabla \times \vec{\mathcal{H}} \Rightarrow$ (4) $\nabla(\nabla\cdot\vec{\mathcal{E}}) - \nabla^2\vec{\mathcal{E}} = -\mu\frac{\partial}{\partial t}\left[\sigma\vec{\mathcal{E}} + \varepsilon\frac{\partial}{\partial t}\vec{\mathcal{E}}\right]$

since $\rho_v = 0 \Rightarrow \nabla\cdot\vec{\mathcal{E}} = 0$ then $(4) \Rightarrow -\nabla^2\vec{\mathcal{E}} = -\mu\sigma\frac{\partial}{\partial t}\vec{\mathcal{E}} - \mu\varepsilon\frac{\partial^2}{\partial t^2}\vec{\mathcal{E}}$ OR

(5) $$\boxed{\nabla^2\vec{\mathcal{E}} - \mu\varepsilon\frac{\partial^2}{\partial t^2}\vec{\mathcal{E}} - \mu\sigma\frac{\partial}{\partial t}\vec{\mathcal{E}} = 0}$$

b) with $\sigma = 0$ then $(5) \Rightarrow$ (6) $\nabla^2\vec{\mathcal{E}} - \mu\varepsilon\frac{\partial^2}{\partial t^2}\vec{\mathcal{E}} = 0$ which in

time harmonic $\frac{\partial}{\partial t} \to j\omega$ & $\frac{\partial^2}{\partial t^2} \to -\omega^2$ hence

$(6) \Rightarrow$ (7) $$\boxed{\nabla^2\vec{E}(\vec{r}) + \omega^2\mu\varepsilon\vec{E}(\vec{r}) = 0}$$

c) If (8) $\vec{E}(r) = \vec{E}_0 e^{-j\vec{k}\cdot\vec{r}}$ is solution of (7) then it must satisfy it.

Pluging (8) in (7) we have

$$\nabla^2\vec{E}_0 e^{-j\vec{k}\cdot\vec{r}} + \omega^2\mu\varepsilon\vec{E}_0 e^{-j\vec{k}\cdot\vec{r}} = 0 \Rightarrow$$

$$\nabla^2\vec{E}_0 e^{-j(k_x x + k_y y + k_z z)} + \omega^2\mu\varepsilon\vec{E}_0 e^{-j\vec{k}\cdot\vec{r}} = 0 \Rightarrow$$

$$\vec{E}_0 \nabla^2 e^{-j(k_x x + k_y y + k_z z)} + \omega^2 \mu \varepsilon e^{-j\vec{k}\cdot\vec{r}} = 0 \Rightarrow$$

$$\vec{E}_0 \{ (-jk_x)(-jk_x) + (-jk_y)(-jk_y) + (-jk_z)(-jk_z) \} e^{-j(k_x x + k_y y + k_z z)}$$

$$+ \omega^2 \mu \varepsilon e^{-j\vec{k}\cdot\vec{r}} = 0 \Rightarrow$$

$$\vec{E}_0 \{ -k_x^2 - k_y^2 - k_z^2] e^{-j\vec{k}\cdot\vec{r}} + \omega^2 \mu \varepsilon e^{-j\vec{k}\cdot\vec{r}} = 0 \Rightarrow$$

$$(-k_x^2 - k_y^2 - k_z^2) \vec{E}_0 e^{-j\vec{k}\cdot\vec{r}} + \omega^2 \mu \varepsilon e^{-j\vec{k}\cdot\vec{r}} = 0 \Rightarrow$$

We must have

$$\boxed{k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \varepsilon}$$

Problem 3:

A dielectric slab of polystyrene ($\varepsilon = 2.56\,\varepsilon_0$, $\mu = \mu_0$, $\sigma = 0$) of height $2h$ is bounded above and below by free space, as shown in the figure. Assuming the electric field within the slab is given by

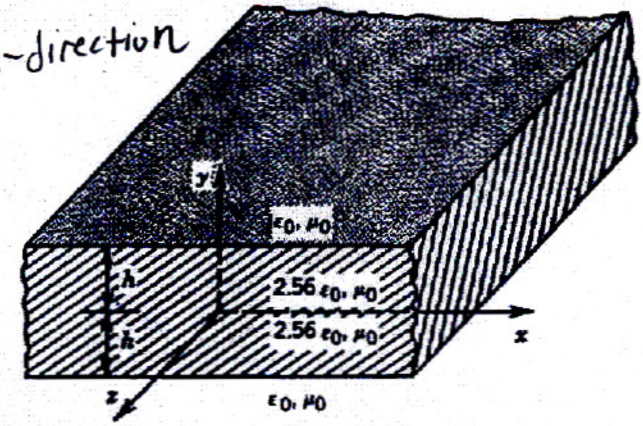
$$\vec{\mathcal{E}}(\vec{r},t) = (\hat{a}_y 5 + \hat{a}_z 10)\cos(\omega t - \beta x)$$

where $\beta = \omega\sqrt{\mu_0\,\varepsilon}$, determine

a) The corresponding instantaneous magnetic field [$\vec{\mathcal{H}}(\vec{r},t)$] within the slab. **[12 Pts]**

b) The instantaneous electric [$\vec{\mathcal{E}}(\vec{r},t)$] and magnetic fields [$\vec{\mathcal{H}}(\vec{r},t)$] in free space just above the slab. **[18 Pts]**

c) The instantaneous electric [$\vec{\mathcal{E}}(\vec{r},t)$] and magnetic fields [$\vec{\mathcal{H}}(\vec{r},t)$] in free space just below the slab. **[5 Pts]**



a)

* Note that this wave travels along +x-direction

* In time harmonic form

① $\boxed{\vec{E} = (5\hat{a}_y + 10\hat{a}_z)e^{-j\beta x} = E_y\hat{a}_y + E_z\hat{a}_z}$

$\nabla\times\vec{E} = -j\omega\mu\vec{H} \Rightarrow \vec{H} = \frac{1}{-j\omega\mu}\nabla\times\vec{E} \Rightarrow$

② $\vec{H} = \frac{1}{-j\omega\mu}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & E_z \end{vmatrix} = \frac{1}{-j\omega\mu}\left\{\hat{a}_x\left(\frac{\partial}{\partial y}E_z - \frac{\partial}{\partial z}E_y\right) - \hat{a}_y\left(\frac{\partial}{\partial x}E_z\right) + \hat{a}_z\left(\frac{\partial}{\partial x}E_y\right)\right\} \Rightarrow$

③ $\vec{H} = \frac{1}{-j\omega\mu}\left\{\hat{a}_x(0-0) - \hat{a}_y(-10j\beta)e^{-j\beta x} + \hat{a}_z(-5j\beta)e^{-j\beta x}\right\}$

$= \frac{1}{-j\omega\mu}\left\{10j\beta e^{-j\beta x}\hat{a}_y - 5j\beta e^{-j\beta x}\hat{a}_z\right\} = -\frac{10\beta}{\omega\mu}e^{-j\beta x}\hat{a}_y + \frac{5\beta}{\omega\mu}e^{-j\beta x}\hat{a}_z$

④ Recall that $\frac{\beta}{\omega\mu} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\mu} = \sqrt{\frac{\varepsilon}{\mu}} = 1/\eta \Rightarrow$

⑤ $\boxed{\vec{H} = -\frac{10}{\eta}e^{-j\beta x}\hat{a}_y + \frac{5}{\eta}e^{-j\beta x}\hat{a}_z = H_y\hat{a}_y + H_z\hat{a}_z}$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\ \frac{1}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{2.56}} = 235.625 \Rightarrow 1/\eta = 4.244\times10^{-3}\ [\Omega]$$

(6)

$$(5) \Rightarrow \boxed{\vec{H} = 4.244\times10^{-3}\left[-10\, e^{-j\beta x}\,\hat{a_y} + 5\, e^{-j\beta x}\,\hat{a_z}\right]}$$

$$\Rightarrow$$

(7)

$$\boxed{\begin{aligned}\vec{\mathcal{H}}(\vec{r},t) &= 4.244\times10^{-3}\left[-10\cos(\omega t-\beta x)\hat{a_y} + 5\cos(\omega t-\beta x)\hat{a_z}\right] \\ &= -4.244\times10^{-2}\cos(\omega t-\beta x)\hat{a_y} + 2.122\times10^{-2}\cos(\omega t-\beta x)\hat{a_z}\end{aligned}}$$

b) At the interface between two perfect dielectrics the tangential $\vec{E}$ & normal $\vec{D}$ are continuous (9)

(8) $$E_z^{FS}(y=h^+) = E_z^{SL}(y=h^+) \Rightarrow \boxed{E_z^{FS}(y=h^+) = 10\, e^{-j\beta x}}$$

(10) $$D_y^{F\text{-}S}(y=h^+) = D_y^{SL}(y=h^+) \Rightarrow \cancel{\varepsilon_0} E_y^{FS}(y=h^+) = \cancel{\varepsilon_0}\varepsilon_{r,Ly} E_y^{SL}(y=h^+) \Rightarrow$$

$$\boxed{E_y^{FS}(y=h^+) = 2.56\times5\, e^{-j\beta x} = 12.8\, e^{-j\beta x}}$$

(11)

then

(11) $$\boxed{\vec{E}^{FS}(y=h^+) = E_y^{FS}\hat{a_y} + E_z^{FS}\hat{a_z} = 12.8\, e^{-j\beta x}\hat{a_y} + 10\, e^{-j\beta x}\hat{a_z}}$$

OR

(12) $$\boxed{\vec{\mathcal{E}}^{FS}(y=h^+,t) = 12.8\cos(\omega t-\beta x)\hat{a_y} + 10\cos(\omega t-\beta x)\hat{a_z}}$$

At the interface between two perfect dielectric the tangential $\vec{H}$ & normal $\vec{B}$ are continuous :-

$$H_z^{FS}(y=h^+) = H_z^{SL}(y=h^+) \Rightarrow H_z^{FS}(y=h^+) = 4.244\times10^{-3}\times5\, e^{-j\beta x}$$

$$= 2.122\times10^{-2}\, e^{-j\beta x}$$

(13) $B_y^{FS}(y=h^+) = B_y^{SL}(y=h^+) \Rightarrow \mu_0 H_y^{FS}(y=h^+) = \mu_0 H_y^{SL}(y=h^+) \Rightarrow$

(14) $H_y^{FS}(y=h^+) = 4.244\times10^{-3}\times(-10)e^{-j\beta x} = -4.244\times10^{-2}e^{-j\beta x}$

(15) $\boxed{\vec{H}^{FS}(y=h^+) = H_y\hat{a_y} + H_z\hat{a_z} = -4.244\times10^{-2}e^{-j\beta x}\hat{a_y} + 2.122\times10^{-2}e^{-j\beta x}\hat{a_z}}$

OR

(16) $\boxed{\vec{\mathcal{H}}^{FS}(y=h^+,t) = -4.244\times10^{-2}\cos(\omega t-\beta x)\hat{a_y} + 2.122\times10^{-2}\cos(\omega t-\beta x)\hat{a_z}}$

c) By symmetry the $\vec{\mathcal{E}}$ & $\vec{\mathcal{H}}$ at $y=h^-$ are the same as $\vec{\mathcal{E}}$ & $\vec{\mathcal{H}}$ at $y=h^+$

hence

(17) $\boxed{\vec{\mathcal{E}}^{FS}(y=h^-) = 12.8\cos(\omega t-\beta x)\hat{a_y} + 10\cos(\omega t-\beta x)\hat{a_z}}$

&

(18) $\boxed{\vec{\mathcal{H}}^{FS}(y=h^-) = -4.244\times10^{-2}\cos(\omega t-\beta x)\hat{a_y} + 2.122\times10^{-2}\cos(\omega t-\beta x)\hat{a_z}}$