Family Name:	Given name:
Student number	Signature

Faculty of Applied Science and Engineering

ECE357 Electromagnetic Fields

First Test, February 4, 2005

Examiners – M. Mojahedi

Only Calculators approved by Registrar allowed
Answer the questions in the spaces provided or on the facing page
A complete paper consists of answers to all questions
For numerical answers specify units

DO NOT REMOVE STAPLE

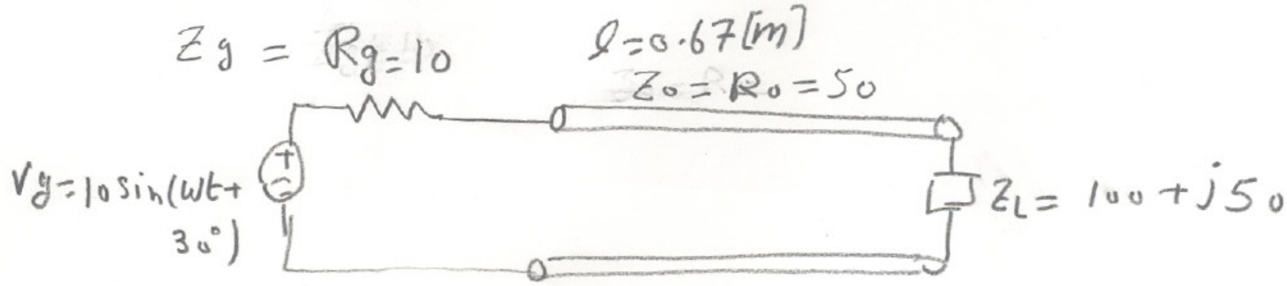
Do not write in these spaces

1	2	3	TOTAL

$\varepsilon_0 = 8.854 \times 10^{-12}$ $[F/m]$, $\mu_0 = 4\pi \times 10^{-7}$ $[H/m]$, $c = 3 \times 10^{8}$ $[m/s]$

Problem 1) A voltage source with series internal impedance of $R_g = 10\ \Omega$ generates a voltage wave form given by $v_g(t) = 10\ \sin(\omega t + 30^\circ)$. The source frequency is 1.05 GHz and it is connected to a load, $Z_L = (100 + j50)\ \ \Omega$ through a $50\ \Omega$ lossless line. The line is 67 cm long and the phase velocity on the line is $0.7\ c$, where c is the speed of light in vacuum. What is the instantaneous voltage, $v(z,t)$, along the line? **(Total points: 33)**

problem 1)



$\nu = 1.05\ GHz$

$V_p = 0.7C$

(1) $V_p = \dfrac{\omega}{\beta} \Rightarrow$ (2) $\beta = \dfrac{\omega}{V_p} = \dfrac{2\pi\nu}{0.7\,C} = \dfrac{2\pi \times 1.05 \times 10^9}{0.7 \times 3 \times 10^8} = 10\pi$

$V_g = 10\sin(\omega t + 30^\circ) = 10\cos(90^\circ - \omega t - 30^\circ) = 10\cos(60 - \omega t) =$

$10\cos(\omega t - 60) \Rightarrow$ (3) $\boxed{\tilde{V}_g = 10\angle -60^\circ = 10\angle -\pi/3}$

(4) $\Gamma_L = \dfrac{Z_L - Z_o}{Z_L + Z_o} = \dfrac{100 + j50 - 50}{100 + j50 + 50} = \boxed{0.4 + j0.2 = 0.447\, e^{j26.565^\circ} = 0.447\, e^{j0.464 (R)}}$

(5) $\Gamma_g = \dfrac{Z_g - Z_o}{Z_g + Z_o} = \dfrac{10 - 50}{10 + 50} = \dfrac{-40}{60} = \boxed{-2/3 = -0.667}$

(6) $V(z) = \dfrac{Z_o V_g}{Z_o + Z_g}\, e^{-\gamma z}\left(\dfrac{1 + \Gamma_L e^{-2\gamma z'}}{1 - \Gamma_g \Gamma_L e^{-2\gamma l}}\right) = \dfrac{R_o V_g}{R_o + R_g}\, e^{-j\beta z}\left(\dfrac{1 + \Gamma_L e^{-2j\beta z'}}{1 - \Gamma_L \Gamma_g e^{-2j\beta l}}\right)$

since (7) $z + z' = l \Rightarrow \boxed{z' = l - z}$ then

(8) $V(z) = \dfrac{R_o V_g}{R_o + R_g}\, e^{-j\beta z}\left(\dfrac{1 + \Gamma_L e^{-2j\beta l} e^{+2j\beta z}}{1 - \Gamma_g \Gamma_L e^{-2j\beta l}}\right)$ OR Finally

Problem 1)
(a)

$$V(z) = V_g \frac{R_o}{R_o + R_g} \frac{\left(e^{-j\beta z} + \Gamma_L e^{-2j\beta \ell} e^{+j\beta z}\right)}{\left(1 - \Gamma_g \Gamma_L e^{-2j\beta \ell}\right)}$$

substitute the values for V_g, R_o, R_g, Γ_L & Γ_g we here

$$V(z) = 10 e^{-j\frac{\pi}{3}} \frac{50}{50+10} \frac{\left(e^{-j\beta z} + 0.447 e^{j0.464} e^{-j2\times 10\pi \times 0.67} e^{+j\beta z}\right)}{\left[1 - \left(\frac{-2}{3}\right) 0.447 e^{j0.464} e^{-j2\times 10\pi\times 0.67}\right]}$$

$$= 8.333 e^{-j\frac{\pi}{3}} \frac{\left[e^{-j\beta z} + 0.447 e^{-j41.633} e^{j\beta z}\right]}{\left[1 + 0.298 e^{-j41.633}\right]} \Rightarrow$$

$$\boxed{V(z) = 10.178 e^{-j1.309} \left[e^{-j\beta z} + 0.447 e^{-j41.633} e^{j\beta z}\right]}$$

$$\boxed{V(z) = 10.178 e^{-j1.309} e^{-j\beta z} + 4.55 e^{-j42.942} e^{j\beta z}}$$ OR

$$\boxed{V(z) = 10.178 e^{-j75^\circ} e^{-j\beta z} + 4.55 e^{-j300.4^\circ} e^{+j\beta z}}$$

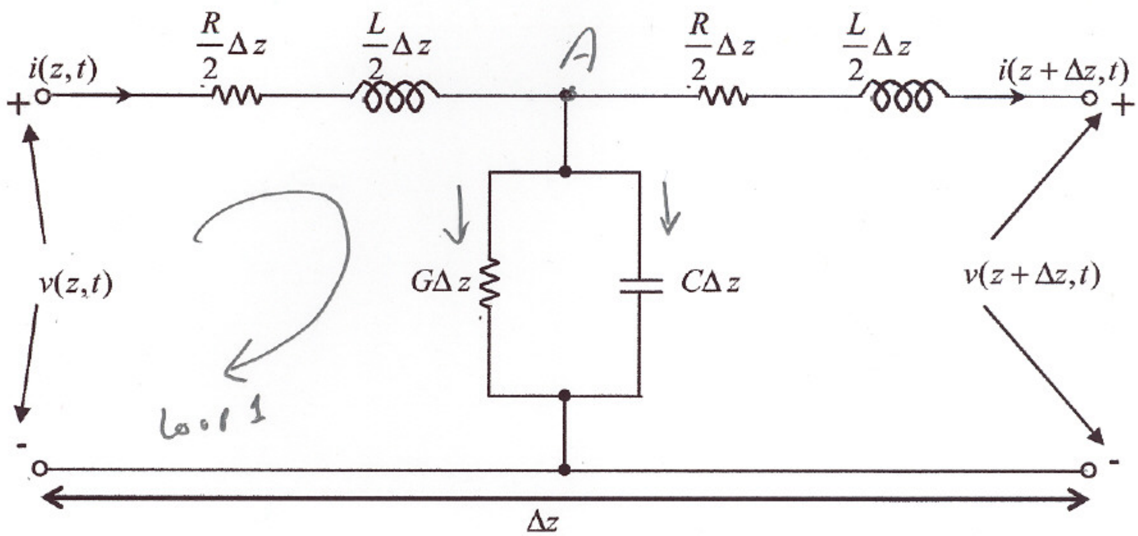
$$\boxed{V(z,t) = 10.178 \cos(\omega t - \beta z - 75^\circ) + 4.55 \cos(\omega t + \beta z - 300.4^\circ)}$$

where $\beta = 10\pi \ell$ $\quad \omega = 2\pi \times 1.05 \times 10^9 = 6.597 \times 10^9$ Hz

Problem 2)

1

Problem 2) A transmission line is modeled by an equivalent circuit shown below. For this transmission line find the general transmission line equations (the so called telegrapher's equation) in time (t) and space (z). Show all your work. **(Total point 33)**



KVL at loop 1

$$\Delta z \frac{R}{2} i(z,t) + \Delta z \frac{L}{2} \frac{\partial i(z,t)}{\partial t} + v(z+\tfrac{\Delta z}{2}, t) = v(z,t)$$

$$\frac{v(z+\frac{\Delta z}{2}, t) - v(z,t)}{\Delta z/2} = -R\, i(z,t) - L \frac{\partial i(z,t)}{\partial t}$$

$$\lim_{\frac{\Delta z}{2}\to 0} \left[\frac{v(z+\frac{\Delta z}{2}, t) - v(z,t)}{\Delta z/2} \right] = \lim_{\frac{\Delta z}{2}\to 0} \left\{ -R\, i(z,t) - L \frac{\partial}{\partial t} i(z,t) \right\}$$

$$\boxed{\frac{\partial v(z,t)}{\partial z} = -R\, i(z,t) - L \frac{\partial}{\partial t} i(z,t)}$$

KCL at node A

(5) $$i(z,t) = G\Delta z\, v(z+\tfrac{\Delta z}{2},t) + C\Delta z \frac{\partial}{\partial t} v(z+\tfrac{\Delta z}{2},t) + i(z+\Delta z,t) \Rightarrow$$

(6) $$\frac{i(z,t) - i(z+\Delta z,t)}{\Delta z} = G v(z+\tfrac{\Delta z}{2},t) + C\frac{\partial}{\partial t} v(z+\tfrac{\Delta z}{2},t)$$

(7) $$\lim_{\Delta z\to 0} \frac{i(z,t) - i(z+\Delta z,t)}{\Delta z} = \lim_{\Delta z\to 0} \left[G v(z+\tfrac{\Delta z}{2},t) + C\frac{\partial}{\partial t} v(z+\tfrac{\Delta z}{2},t)\right]$$

(8) $$-\frac{\partial}{\partial z} i(z,t) = \lim_{\Delta z\to 0} \left[G v(z+\tfrac{\Delta z}{2},t) + C\frac{\partial}{\partial t} v(z+\tfrac{\Delta z}{2},t)\right],$$

but physically, $\lim_{\Delta z\to 0} v(z+\frac{\Delta z}{2},t) \neq v(z,t)$ since otherwise there will not be a voltage drop from input to point A.

Use $V(z+\frac{\Delta z}{2},t)$ from (1) in (8) $\Rightarrow$

$$-\frac{\partial}{\partial z} i(z,t) = \lim_{\Delta z\to 0} \left\{G v(z,t) - G\Delta z \frac{R}{2} i(z,t) - G\Delta z \frac{L}{2}\frac{\partial}{\partial t} i(z,t) + C\frac{\partial}{\partial t}\left[v(z,t) - \Delta z\frac{R}{2} i(z,t) - \Delta z \frac{L}{2}\frac{\partial}{\partial t} i(z,t)\right]\right\}$$

$$\Rightarrow -\frac{\partial}{\partial z} i(z,t) = \lim_{\Delta z\to 0} \left\{G v(z,t) - \underbrace{G\Delta z\frac{R}{2} i(z,t)}_{\to 0} - \underbrace{G\Delta z\frac{L}{2}\frac{\partial}{\partial t} i(z,t)}_{\to 0} + C\frac{\partial}{\partial t} v(z,t) - \underbrace{C\frac{R}{2}\Delta z\frac{\partial}{\partial t} i(z,t)}_{\to 0} - \underbrace{C\frac{L}{2}\Delta z\frac{\partial^2}{\partial t^2} i(z,t)}_{\to 0}\right\} \Rightarrow$$

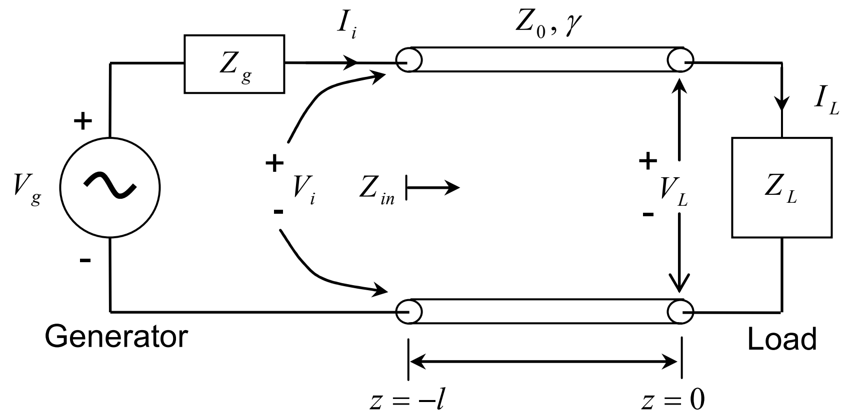
$$\boxed{\frac{\partial i(z,t)}{\partial z} = -G v(z,t) - C\frac{\partial}{\partial t} v(z,t)}$$

Problem 3) The diagram below shows a load connected to a generator by a transmission line. *In contrast to your notes and text, this coordinate system locates the load at z = 0 and the source at z = -l.*

a) For this configuration, define the reflection coefficient at the load (Γ_L) in terms of the positive and negative traveling voltages V_0^+, and V_0^-.

b) Show that for a properly defined reflection coefficient in part (a), the expression for Γ_L in terms of Z_L and Z_0 is the same that we found in class.

(Total Point 34)



problem 3)

a) For this configuration $\Gamma_L = \dfrac{V_0^-}{V_0^+}$ ①

b) we start with

$$V = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad ②$$

$$I = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z} \quad ③$$

At load $(z=0)$ & $V_L = I_L Z_L$ & ④

⑤ $V_L = V_0^+ + V_0^-$

⑥ $I_L = \dfrac{V_0^+}{Z_0} - \dfrac{V_0^-}{Z_0}$

From (4) $\Rightarrow$ ⑦ $Z_L = \dfrac{V_L}{I_L}$ use (5) & (6) in (7) $\Rightarrow$

⑧
$$Z_L = \frac{V_0^+ + V_0^-}{\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}} = \frac{Z_0 (V_0^+ + V_0^-)}{V_0^+ - V_0^-} = \frac{Z_0 \cancel{V_0^+} (1 + V_0^-/V_0^+)}{\cancel{V_0^+}(1 - V_0^-/V_0^+)} \Rightarrow$$

⑨ $Z_L = \dfrac{Z_0 (1 + \Gamma_L)}{(1 - \Gamma_L)} \Rightarrow$ ⑩ $Z_L - Z_L \Gamma_L = Z_0 + Z_0 \Gamma_L \Rightarrow$

$Z_L - Z_0 = \Gamma_L (Z_L + Z_0) \Rightarrow$ ⑪ $\boxed{\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}}$

Eq (11) is the same we found in class with $z=0$ at the source & $z=l$ at the load.