Last Name:	First name:
Student number	Signature

Faculty of Applied Science and Engineering

ECE357 Electromagnetic Fields

Second Test, March 22, 2005

Examiners – M. Mojahedi

Only Calculators approved by Registrar allowed
Answer the questions in the spaces provided or on the facing page
A complete paper consists of answers to all questions
For numerical answers specify units

DO NOT REMOVE STAPLE

Do not write in these spaces

1	2	3	4	5	TOTAL

$\varepsilon_0 = 8.854\times10^{-12}\ [F/m]$, $\mu_0 = 4\pi\times10^{-7}\ [H/m]$, $c = 3\times10^{8}\ [m/s]$

Problem 1) Consider a lossy but simple medium (linear, isotropic, homogenous) which contains both electric and magnetic impressed (source) current densities ($\vec{J}_i$, and $\vec{M}_i$) and magnetic (fictitious) and electric charge density (ρ_{mv}, ρ_{ev}).

(a) For the above medium find the wave equation governing the dynamical behavior of the instantaneous magnetic field intensity, $\vec{H}(r,t)$. **(26 pts)**

(b) Give the time-harmonic form of the wave equation found in part (a). **(7 pts)**

Problem 1)

a) ① $\nabla \times \vec{E}(\vec{r},t) = -\frac{\partial \mu \vec{H}}{\partial t} - \vec{M_i}$

② $\nabla \times \vec{H}(\vec{r},t) = \sigma \vec{E} + \frac{\partial}{\partial t} \varepsilon \vec{E} + \vec{J_i}$ $\quad\quad \vec{J_c} = \sigma \vec{E}$

③ $\nabla \cdot \varepsilon \vec{E}(\vec{r},t) = \rho_{ev} \Rightarrow \nabla \cdot \vec{E} = \frac{\rho_{ev}}{\varepsilon}$

④ $\nabla \cdot \mu \vec{H}(\vec{r},t) = \rho_{mv} \Rightarrow \nabla \cdot \vec{H} = \frac{\rho_{mv}}{\mu}$

From (2) ⑤ $\nabla \times \nabla \times \vec{H} = \sigma \nabla \times \vec{E} + \varepsilon \frac{\partial}{\partial t} \nabla \times \vec{E} + \nabla \times \vec{J_i} \Rightarrow$

⑥ $\nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = \sigma\left[-\frac{\partial}{\partial t}\mu \vec{H} - M_i\right] + \varepsilon \frac{\partial}{\partial t}\left[-\frac{\partial}{\partial t}\mu\vec{H} - \vec{M_i}\right] + \nabla \times \vec{J_i}$

using (4) in (6)

$$\nabla\left(\frac{\rho_{mv}}{\mu}\right) - \nabla^2 \vec{H} = -\mu\sigma \frac{\partial}{\partial t}\vec{H} - \sigma \vec{M_i} - \varepsilon\mu \frac{\partial^2}{\partial t^2}\vec{H} - \varepsilon \frac{\partial}{\partial t}\vec{M_i} + \nabla \times \vec{J_i} \Rightarrow$$

$$\boxed{\frac{1}{\mu}\nabla(\rho_{mv}) + \sigma \vec{M_i} - \nabla \times \vec{J_i} + \varepsilon \frac{\partial}{\partial t}\vec{M_i} + \mu\sigma \frac{\partial}{\partial t}\vec{H} + \varepsilon\mu \frac{\partial^2}{\partial t^2}\vec{H} = \nabla^2 \vec{H}}$$

where $\vec{H} = \vec{H}(r,t)$, $\vec{M_i} = M_i(\vec{r},t)$, $\vec{J_i} = J_i(\vec{r},t)$ & $\rho_{mv} = \rho_{mv}(\vec{r},t)$

b) $$\boxed{\frac{1}{\mu}\nabla(\rho_{mv}) + \sigma \vec{M_i} - \nabla \times \vec{J_i} + j\omega\varepsilon \vec{M_i} + j\omega\mu\sigma \vec{H} - \omega^2 \varepsilon\mu \vec{H} = \nabla^2 \vec{H}}$$

where $\rho_{mv} = \rho_{mv}(\vec{r})$, $\vec{M_i} = M_i(\vec{r})$, $\vec{J_i} = \vec{J_i}(\vec{r})$, $\vec{H} = \vec{H}(\vec{r})$

Problem 2) A $75\,\Omega$ lossless transmission line of length 0.4λ is connected to a $Z_L = 100 + j150\,\Omega$ load. Using the Smith chart (attached) answer the following questions

a) Determine the reflection coefficient, Γ. **(6 Pts)**
b) Determine the standing wave ratio, S. **(6 Pts**)
c) Locate and give value of the load admittance, Y_L. **(6 Pts)**
d) Determine the line input impedance, Z_{in}. **(8 Pts)**
f) Determine the location of the first voltage maximum. **(7 Pts)**

Important Note: To receive full credit, you must mark the appropriate points on the chart and clearly describe the procedure used to obtain the values for the above parameters.

Problem 2) 1

a) $z_\ell = \dfrac{100+j150}{75} = 1.33+j2 = r+jx$ (normalized load)

This is point P on the smith chart. We draw a circle centered at O, with radius OP.

*Measuring the length $OP = 2.1''$ & $OQ = 3.2''$ we get

$\Gamma = \dfrac{\overline{OP}}{\overline{OQ}} = \dfrac{2.1''}{3.2''} = 0.66$

* The θ_Γ is found from $\theta_\Gamma = (0.25-0.195)\times 4\pi$

$= 0.69\ (\text{rad}) = 39.6^\circ \simeq 40^\circ$

check $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{100+j150-75}{100+j150+75} = 0.506+j0.424 = 0.66\angle 39.9^\circ$

b) The circle with radius OP cuts the $x=0$ line at A & B points. The r-circle going through A-point is the standing wave ratio. This is $r=5$ circle $\Rightarrow$

$S' = 5$

Check: $S = \dfrac{|\Gamma|+1}{1-|\Gamma|} = \dfrac{0.66+1}{1-0.66} = 4.88 \simeq 5$

c) The load addmitance is obtained by locating point P', diametrically across point P on the Γ circle (circle of radius OP centered at O). Reading this value on the chart we have

$y = g+jb = 0.22-j0.36 \Rightarrow Y = Y_0\, y = \frac{1}{75}(0.22-j0.36)$

$= 0.003 - j0.005\ [\frac{1}{\Omega}]$

Check: $Y_L = \frac{1}{Z_L} = \frac{1}{100+j150} = 0.003 - j0.005$

d) Input impedance at 0.4λ away from the load is found by moving away from the load (Point P at $\frac{\Delta z'}{\lambda} = 0.195$) a distance 0.4 toward generator. In other words, we move $0.4 + 0.195 = 0.595$ toward generator. This is equivalent to $0.595 - 0.5 = 0.095$. on the chart circumference and is marked by line O O'. Line o o' intersect the Γ-circle at R. We read the values for r & x at R ⇒

$$z_{in} = r_{in} + j x_{in} = 0.3 + j0.65 \Rightarrow$$

$$Z_{in} = 75(0.3 + j0.65) = 22.5 + j48.75 \ [\Omega]$$
$$= 53.7 \angle 65.2^\circ$$

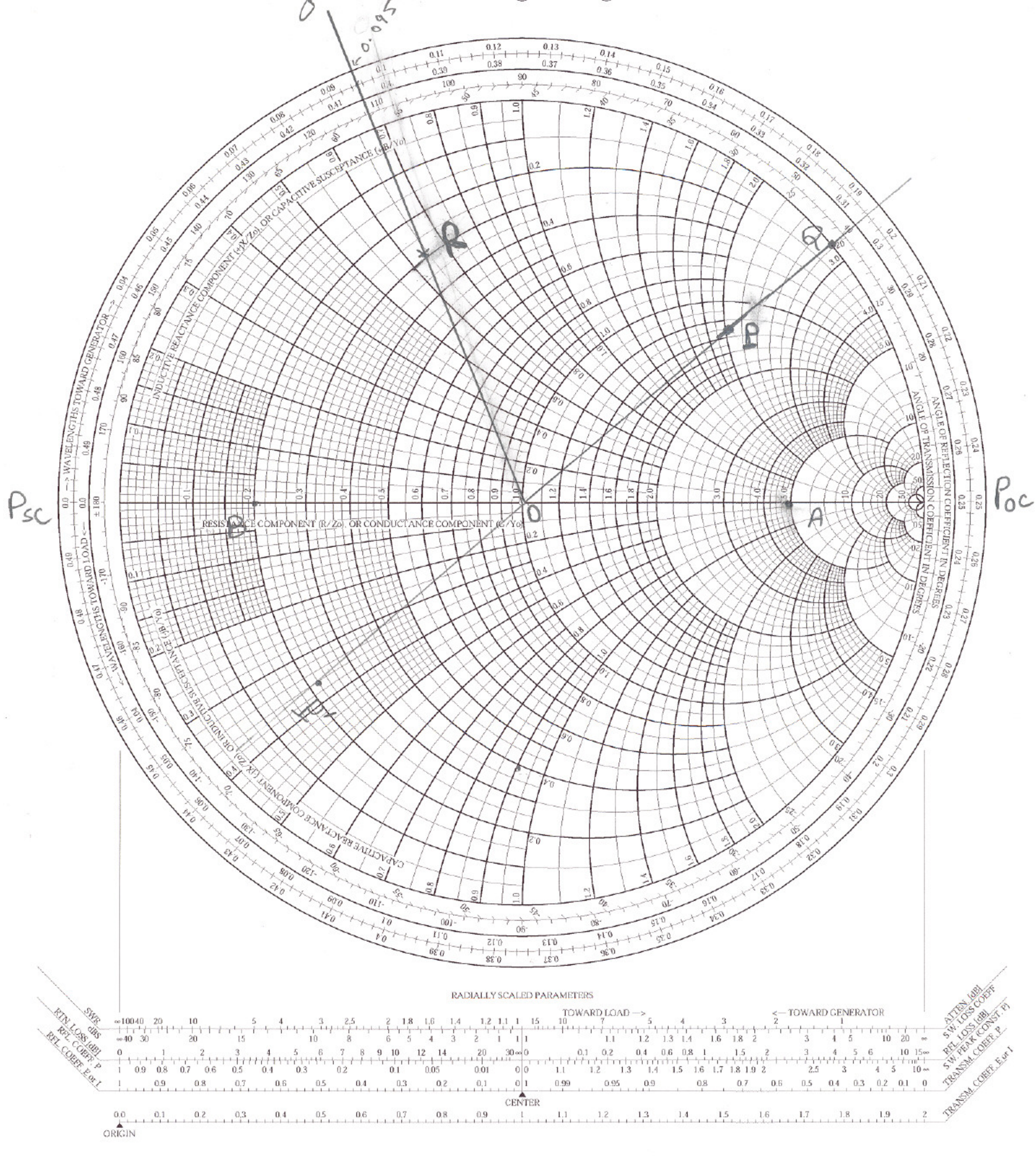
Test $$Z = R_0 \frac{Z_L + jR_0 \tan(\beta l)}{R_0 + jZ_L \tan(\beta l)} = 75 \frac{(100+j150) + j75 \tan(144^\circ)}{75 + j(100+j150)\tan(144^\circ)}$$

$$\beta l = \frac{2\pi}{\lambda} l = \frac{2\pi}{\lambda} 0.4\lambda = 0.8\pi = 144^\circ \qquad = 21.97 + j47.6$$
$$= 52.4 \angle 65.2$$

e) The first voltage maximum occurs at A (in going from P to R we cross the OP_{oc} at A where the voltage is maximum.) From the chart circumference this is at $(0.25 - 0.195)\lambda$
$= 0.055\lambda$ from the load

The Complete Smith Chart

Black Magic Design



Problem 3) In discussing the solution to the wave equation for scalar potential

$$\nabla^2 V(r,t) - \frac{1}{c^2}\frac{\partial^2 V(r,t)}{\partial t^2} = -\rho_{ev}(r,t)/\varepsilon_0\,, \qquad (1)$$

we began by considering the electrostatic case; stating that the solution to

$$-\nabla^2 V(r) = \rho_{ev}(r)/\varepsilon_0 = \phi(r) \qquad (2)$$

is given by

$$V(r) = \iiint \frac{\phi(r')}{4\pi\,|\vec{r}-\vec{r}'|}\,d^3r'\,. \qquad (3)$$

Here prove that indeed (3) is a solution of (2). **(34 Pts)** prob 3;

* We want to show that solution to ① $-\nabla^2\psi(r)=\phi(r)$ is

given by $\psi(r)=\frac{1}{4\pi}\iiint \frac{\phi(r')}{|\vec{r}-\vec{r}'|}d^3r'$ ②

③

In homework we have shown that $-\nabla^2\frac{1}{R}=4\pi\delta^3(\vec{R})$, ⑥

where ④ $R=|\vec{R}|$. Here, ⑤ $\vec{R}=\vec{r}-\vec{r}'$ & hence $|\vec{R}|=R=|\vec{r}-\vec{r}'|$

in otherwords (3) $\Rightarrow$ $\boxed{-\nabla^2\frac{1}{|\vec{r}-\vec{r}'|}=4\pi\delta^3(\vec{r}-\vec{r}')}$ ⑦

⑧

From (2) $\Rightarrow \nabla^2\psi(r)=\nabla^2\iiint\frac{\phi(r')}{4\pi|\vec{r}-\vec{r}'|}d^3r'=\iiint\nabla^2\left(\frac{1}{4\pi}\frac{\phi(r')}{|\vec{r}-\vec{r}'|}\right)d^3r'$

$=\iiint\phi(r')\nabla^2\left(\frac{1}{4\pi}\frac{1}{|\vec{r}-\vec{r}'|}\right)d^3r'$ From (7) $\Rightarrow$

⑨ $\nabla^2\psi(r)=\iiint\phi(r')(-1)\delta^3(\vec{r}-\vec{r}')\,d^3r'$

$=-\phi(r)$ $\Rightarrow$

⑩ $\nabla^2\psi(r)=-\phi(r)$, hence we see that if $\psi(r)$ is given by (2) it will satisfy (10) which is the Poisson equation