Problem Set #6
ECE357 /ECE320
Spring 2005

1) In deriving the non-homogeneous wave equations for scalar potential (V) and vector potential ($\vec{A}$) we used the Lorentz-gauge given by $\nabla \cdot \vec{A} + \mu\varepsilon \frac{\partial V}{\partial t} = 0$. Obtain the differential equations relating V and $\vec{A}$ to the sources (ρ *and* $\vec{J}$) under different gauge for which $\nabla \cdot \vec{A} = 0$. This condition is called Coulomb gauge. Comment on the use of Coulomb gauge vs. Lorentz gauge. (Assume a simple medium)

2) In class the inhomogeneous wave equation for V and $\vec{A}$ were obtained for a simple medium (linear, isotropic, and homogeneous), subject to Lorentz gauge. Here, try to obtain the differential equations governing the dynamical behavior of V and $\vec{A}$, when ε and μ depend on position (as in the class assume Lorentz gauge). Comment on your results.

3) Derive the two divergence equations $\nabla \cdot \vec{B} = 0$ and $\nabla \cdot \vec{D} = \rho$ from the two curl equations (Faraday's and Ampere's law) and the continuity equation.

4) Obtain the time-dependent and time-harmonic wave equations for $\vec{E}$ and $\vec{H}$ in the case of linear, isotropic, homogeneous medium for which there are free charges and conduction current present.

5) Prove that $U(R,t) = f(t - R/c)$ is a solution of differential equation
$$\frac{\partial^2 U}{\partial R^2} - \frac{1}{c}\frac{\partial^2 U}{\partial t^2} = 0.$$

6) An electric field confined between two metallic plates (with air between the plates) is propagating along the z-direction with phase constant β. The instantaneous electric field is given by
$$\vec{E} = \hat{a}_y\ 0.1 \sin(10\,\pi\,x)\cos(6\pi\,10^9\,t - \beta\,z)\ \text{[v/m]}$$
Find the $\vec{H}$ between the two plates and the value of β (express $\vec{H}$ as an instantaneous field.)