Problem Set #7
ECE357 / ECE320
University of Toronto

1) Prove that electric field given as

$$\vec{E} = \hat{a}_x \, E_{10} \sin(\omega t - k \, z) + \hat{a}_y \, E_{20} \sin(\omega t - k \, z + \psi)$$

where E_{10}, E_{20} and ψ are arbitrary numbers is elliptically polarized.

2) Assume that the $z = 0$ plane separates two lossless dielectric regions (no free surface charges or currents) with $\varepsilon_{r1} = 2$ and $\varepsilon_{r2} = 3$. If we know that the $\vec{E}_1$ in region 1 in phasor form is given by

$$\vec{E}_1 = \left[\hat{a}_x \, 2y - \hat{a}_y \, 3x + \hat{a}_z \, (5 + z)\right] e^{j \omega t}$$

what are $\vec{E}_2$ and $\vec{D}_2$ at the interface.

3) The $\vec{E}$ field of a uniform plane wave propagating in a dielectric medium (assume perfect dielectric) is given by

$$\vec{E}(t,z) = \hat{a}_x \, 2\cos\left(10^8 \, t - \frac{z}{\sqrt{3}}\right) - \hat{a}_y \, \sin\left(10^8 \, t - \frac{z}{\sqrt{3}}\right) \quad [V/m]$$

a) Write the above expression for time harmonic fields.
b) Determine the frequency and the wavelength.
c) Describe the polarization of the wave.
d) Find the corresponding $\vec{H}$ field (express your results in both phasor and instantaneous form.)

4) Prove the following relations between group velocity (V_g) and phase velocity (V_p) in a dispersive medium:

a) $V_g = V_p + k \, dV_p / dk$

b) $V_g = V_p - \lambda \, dV_p / d\lambda$