Problem Set #8
ECE357 /ECE320
University of Toronto

Question 1) A uniform plane wave polarized along +*x*-direction is propagating in +*z*-direction in an unbounded lossy medium. The filed amplitude at *t=0* and *z=0* is E_0.

a) Give the appropriate expressions for the phasor and instantaneous electric field.
b) Give the appropriate expressions for the phasor and instantaneous magnetic field.
c) Give the appropriate expression for instantaneous and time average Poynting vector (power density).
d) Suppose that the frequency of operation is 3 [GHz], the field amplitude at *t=0* and *z=0* is 50 [V/m], the dielectric constant of the non-magnetic lossy medium is 2.5, and its loss tangent is 10^{-2}. Write the expression for the instantaneous magnetic field.

Question 2) Prove that
a) A circularly polarized plane wave can be obtained from a superposition of two oppositely directed elliptically polarized waves.
b) A linearly polarized wave can be obtained from the superposition of a left handed and right handed circularly polarized waves.

Question 3) The electrosatic energy associated with charge distribution ρ is given by $W_e = \frac{1}{2}\iiint_{v'} \rho V \, dv$ where V is the potential at the point where the volume charge density is ρ, and v' is the volume of the region where ρ exist. Show that this expression will simplify to $W_e = \frac{1}{2}\iiint_{v'} \varepsilon \left|\vec{E}\right|^2 dv$. Express the last expression in terms of the electric filed in intensity ($\vec{E}$) and electric filed flux density ($\vec{D}$).

Question 4) Make sure you understand Example 8-9, page 389 of the book by David Cheng

Question 5)
a) Show that for a dispersive medium the group velocity can be written as

$$V_g(\lambda_0) = \frac{c}{n(\lambda_0) - \lambda \, dn(\lambda_0)/d\lambda_0}$$

b) The dispersion in a certain material is described by its index of refraction as a function of frequency

$n(\omega) = n_0 - \frac{a(\omega - \omega_0)}{a^2 + (\omega - \omega_0)^2}$, where n_0, a, and ω_0 are constants.

What are the phase and group velocities in this medium?